Problem 1.

A pure aluminum highway sign is 1 m x 1 m x 0.005 m and has thermal properties of thermal conductivity, $k=237 \text{ W/mK}$, density, $\rho=2702 \text{ kg/m}^3$, and specific heat, $c_p=903 \text{ J/kgK}$. Overnight, the sign reaches equilibrium with the surrounding air ($T_\infty=0^\circ\text{C}$ and $h=100 \text{ W/m}^2\text{K}$). When the sun rises in the morning, the sign receives a uniform net radiation heat flux of $q_{\text{s}}''=1,000 \text{ W/m}^2$ on one of its sides. Assume no convection or radiation on the thin edges of the sign. A thin layer of paint covering the sign provides a uniform conduction resistance of $R''=0.001 \text{ m}^2\text{K/W}$, but does not affect thermal capacitance.

(a) What type of analysis is appropriate for determining the transient temperature? Justify your approach numerically.

(b) Draw a diagram showing the related heat flows and derive the transient energy balance that includes all heat transfer mechanisms involved in the problem. Write the energy balance symbolically in terms of known parameters.

(c) Determine the final sign temperature at steady state, $T_f =$

(d) Determine the sign temperature equation in symbolic form, $T(t) =$

(e) Plot $T(t)$ labelling $T_i$, $T_\infty$, $T_f$, $\tau$, and the temperature at one time constant, $T(t=\tau)$. 
Problem 2.

Some infrared (IR) cameras use an array of microbolometer detectors to measure the IR emission from the object. One of the detectors in the array is shown in the figure below. It consists of a flat plate made of a thin silicon nitride layer covered by a vanadium oxide layer, which is used to absorb the IR radiation and to detect the plate temperature based on the measured electrical resistance of the vanadium oxide layer. Two long and narrow beams are used to suspend the flat plate above a semiconductor substrate, which consists of transistor devices for electronic signal conditioning.

(a) If a single detector is placed in close proximity to the 38°C forehead of a person without any optical lens or vacuum packing materials in between the flat plate and the person, discuss the heat transfer mechanisms that can influence the temperature of the vanadium oxide resistance thermometer layer. Write an energy balance equation for the flat plate. It can be assumed that the large semiconductor substrate remains at room temperature, which is 25°C.

(b) For given plate and substrate temperatures, length, cross section, and thermal properties of each of the two long and narrow beams, calculate the heat conduction from the flat plate to the two beams.

(c) If all surfaces are diffuse, gray, and opaque with known emissivities, calculate the net radiation heat transfer between the flat plate and the person, and the net radiation heat transfer between the flat plate and the large semiconductor substrate.