Economic Life and Replacement Problems key

1a. The table below shows the operating cost and salvage value for a machine that was purchased for $50,000 and has a useful live of 3 years. Find its economic life using an MARR of 10%.

<table>
<thead>
<tr>
<th>Year</th>
<th>Operating cost</th>
<th>Salvage value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10,000</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$40,000</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>$70,000</td>
<td>0</td>
</tr>
</tbody>
</table>

NAC 1 = 50,000(A/P, 0.1, 1) + 10,000 = 50,000 × 1.1 + 10000 = 65,000
NAC 2 = 50,000(A/P, 0.1, 2) + 10,000 + 30,000(A/G, 0.1, 2)
      = 50,000 × 0.5762 + 10,000 + 30,000 × 0.476 = 28810 + 10,000 + 14,280
      = 53,090
NAC 3 = 50,000(A/P, 0.1, 3) + 10,000 + 30,000(A/G, 0.1, 3)
      = 50,000 × 0.4021 + 10,000 + 30,000 × 0.937 = 20,105 + 10,000 + 28,110
      = 58,215

The economic life is 2 years.

b. The table below shows the operating cost and salvage value for a machine for a three-year period. Find the economic life of the machine. The machine has an initial cost of $50,000. The MARR for this analysis is 10%.

<table>
<thead>
<tr>
<th>Year</th>
<th>Operating cost</th>
<th>Salvage value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10,000</td>
<td>$30,000</td>
</tr>
<tr>
<td>2</td>
<td>$10,000</td>
<td>$20,000</td>
</tr>
<tr>
<td>3</td>
<td>$10,000</td>
<td>0</td>
</tr>
</tbody>
</table>

NAC 1 = 50,000(A/P, 0.1, 1) + 10,000 − 30,000(A/F, 0.1, 1) = 50,000 × 1.1 − 20,000
     = 35,000
NAC 2 = 50,000(A/P, 0.1, 2) + 10,000 − 20000(A/F, 0.1, 2)
      = 50,000 × 0.5762 + 10000 − 20,000 × 0.4762 = 28,810 + 10,000 − 9,524
      = 29,286
NAC 3 = 50,000(A/P, 0.1, 3) + 10,000 = 50,000 × 0.4021 + 10,000
      = 20,105 + 10,000 = 30,105

The economic life is 2 years.
2. A piece of equipment was purchased one year ago for $100,000. The annual cost of operating the equipment is $20,000. It is expected to last another 5 years during which the operating cost will remain constant.

Several months after the purchase, a remarkable new process was discovered that uses waste material to run the equipment. The result is that the same output can be obtained with $0 operating cost. The cost of purchasing and installing the new equipment is $200,000, and is expected to last 10 years. At that time, it will have $0 salvage value. Note that under these conditions the economic life of the new equipment is 10 years.

You must have either the new or old equipment. Both options perform the function with equal quality. You are now considering selling the old equipment and replacing it with the new one. There is a buyer who is willing to pay you $30,000 for the old equipment. If you don't make the deal now, you doubt that you will be able to find a buyer at any price in the future. Therefore, after this year, you will have to pay $10,000 to get rid of the old piece of equipment. The minimum acceptable rate of return is 10%.

a. What is the Net Annual Cost of the challenger?

The challenger has an NAC = $200,000(A/P, 0.1, 10) = $200,000 × 0.1627 = $32,540

b. What is the economic life of the defender?

For the defender:

\[
\begin{align*}
\text{NAC}(1) &= 30,000(A/P, 10\%, 1) + 20,000 + 10,000(A/F, 10\%, 1) \\
&= 30,000(1.1) + 20,000 + 10,000(1.0) \\
&= $63,000 \\
\text{NAC}(2) &= 30,000(A/P, 10\%, 2) + 20,000 + 10,000(A/F, 10\%, 2) \\
&= 30,000(0.5762) + 20,000 + 10,000(0.4762) \\
&= $42,048 \\
\text{NAC}(3) &= 30,000(A/P, 10\%, 3) + 20,000 + 10,000(A/F, 10\%, 3) \\
&= 30,000(0.4021) + 20,000 + 10,000(0.3021) \\
&= $35,084 \\
\text{NAC}(4) &= 30,000(A/P, 10\%, 4) + 20,000 + 10,000(A/F, 10\%, 4) \\
&= 30,000(0.3155) + 20,000 + 10,000(0.2155) \\
&= $31,620 \\
\text{NAC}(5) &= 30,000(A/P, 10\%, 5) + 20,000 + 10,000(A/F, 10\%, 5) \\
&= 30,000(0.2638) + 20,000 + 10,000(0.1638) \\
&= $29,552
\end{align*}
\]
The economic life is 5 years and the associated annual cost is $29,552.

c. What is the best decision?

Because the NAC is greater than for the defender, do not make the replacement and keep the defender for 5 years.

3. Your company purchased a machine for $14,000 with a 6-year tax life. The sum-of-the-years digits method is used for depreciation and the tax salvage value is zero.

a. After the third year of use, the machine is sold for $10,000. How much does the company get from the sale after taxes assuming the tax rate on capital gains is 40%.

For a 6-year tax life, the SOYD is $6 \times 7/2 = 21$. The depreciation for the first 3 years is

1: $14,000 \times 6 / 21 = 4,000$
2: $14,000 \times 5 / 21 = 3,333$
3: $14,000 \times 4 / 21 = 2,667$.

The book value after 3 years is $14,000 – 4,000 – 3,333 – 2,667 = 4,000$.

Selling the asset for $10,000, results in a taxable gain of $6,000. The tax is $2,400, so the net receipts are $7,600.

b. Neglect taxes in this part. After the third year of life, the company is thinking about replacing the machine with a new one. It can be sold now for $10,000. Next year it will only be worth $6,000 and in two years, only $4,000. Three years from now the machine will have no resale value. The operating cost of the machine is expected to be constant for the next three years at $1,000 per annum. The new machine has a life of 10 years with a NAC of $5,000. Should the old machine be replaced with the new one if the company’s MARR is 10%?

The NAC for 1 year is $(10,000 – 6,000)(A/P, 0.1, 1) + 6,000 \times 0.1 + 1,000$

$= 4,000 \times 1.1 + 600 + 1,000 = 6,000$

The NAC for 2 years is $(10,000 – 4,000)(A/P, 0.1, 2) + 4,000 \times 0.1 + 1,000$

$= 6,000 \times 0.5762 + 400 + 1,000 = 3,457.20 + 400 + 1,000 = 4,857.20$

The NAC for 3 years is $(10,000)(A/P, 0.1, 3) = 10,000 \times 0.4021 + 1,000$

$= 4,021 + 1,000 = 5,021$

The economic life of the old machine is 2 years. It's NAC at this life is less than that of the challenger. Keep the old machine.
4. You are considering replacing your car with a new one. After much bickering, the dealer offers you the new car for $10,000 with your car as a trade-in or $12,000 without your car. An acquaintance offers you $3,500 for your car if you fix the air conditioner. That will cost you $1,000. The expected maintenance cost of your car for the next year is $1200 not including the air conditioner. You think you can sell it for $1,000 at this time next year without fixing the air conditioner.

Assume that the life of the new car is 10 years with annual maintenance cost of $800 per year. The new car will have $2,000 salvage value at the end of the 10 years. Show the cash flows you would use in an analysis. Your MARR is 10%. What is your most economic course of action?

We will use the following capital recovery formula to compute the equivalent annual cost of the cars.

\[
CR(i) = (I - S)(A/P, i, N) + iS
\]

(See page 38 of the workbook.)

For the old car:  
\[ P = I - S = $3,500 - $1,000 = $2,500, \text{ life } = 1 \text{ year}, S = $1,000, \]

Maintenance = $1,200

\[
NAC_{\text{OLD}} = 1,500(A/P, 0.1, 1) + 1,000 \times 0.1 + 1,200 = 1,500 \times 1.1 + 100 + 1,200
\]
\[ = 1,650.0 + 100 + 1,200 = $2,950 \]

\[
NAC_{\text{NEW}} = (12,000 - 2,000)(A/P, 0.1, 10) + 2,000 \times 0.1 + 800
\]
\[ = 10,000 \times 0.1627 + 200 + 800
\]
\[ = 1,627 + 200 + 800 = $2,627 \]

Buy the new car and sell the old one.

5. A company faces the following equipment replacement problem. An existing piece of equipment is 4 years old and was originally bought for $8,000. The economic life of this equipment is 8 years and it has no salvage at the end of its useful life. The machine has been depreciated by the straight-line method and its current market value is equal to its book value. If the machine is replaced, $500 must be spent to remove it and ready it for sale. Neglect any tax effects associated with the sale or removal costs. The net annual before tax benefit of $2,000 from the existing equipment is expected to continue for another four years. Use four years as the economic life, and assume the salvage value is zero at the end of four years.
The new machine being considered as a replacement costs $10,000. It has an economic life of 10 years at the end of which it can be sold for $2,000. The net annual before tax benefit expected from this machine is $3,000. Again the straight-line method is used for depreciation using a tax salvage of $2,000.

The tax rate is given to be 50%. The after tax MARR is 10%. Should you replace the existing machine with a new one? Show all calculations.

Existing Machine (Defender)

We use the NAW analysis here because the economic lives of the two options are different. PW and ROR analysis would not be valid for this problem due to the fact that identical replacement for the existing machine is not feasible.

<table>
<thead>
<tr>
<th>Year</th>
<th>BTCF</th>
<th>Depr.</th>
<th>TI</th>
<th>Tax</th>
<th>ATCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>−$3,500</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>−$3,500</td>
</tr>
<tr>
<td>1 - 4</td>
<td>$2,000</td>
<td>$1,000</td>
<td>$1,000</td>
<td>$500</td>
<td>$1,500</td>
</tr>
</tbody>
</table>

NAW = −$3,500(A/P, 10%, 4) + $1,500 = $395.75

Challenger

<table>
<thead>
<tr>
<th>Year</th>
<th>BTCF</th>
<th>Depr.</th>
<th>TI</th>
<th>Tax</th>
<th>ATCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>−$10,000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>−$10,000</td>
</tr>
<tr>
<td>1 - 9</td>
<td>$3,000</td>
<td>$800</td>
<td>$2,200</td>
<td>$1,100</td>
<td>$1,900</td>
</tr>
<tr>
<td>10</td>
<td>$3,000</td>
<td>$800</td>
<td>$2,200</td>
<td>$1,100</td>
<td>$1,900</td>
</tr>
</tbody>
</table>

NAW = −$10,000(A/P, 10%, 10) + $1,900 + $2,000(A/F, 10%, 10)

= −$1,627 + $1,900 + $125.4

= $398.4

Because the NAW of new machine is higher, we replace the existing machine with a new one.

6. A milling machine (machine A) in your company’s shop has a current market value of $30,000. It was bought nine years ago for $54,000 and has since been depreciated by the straight-line method assuming a 12-year tax life. If the decision is made to keep the machine at this point it time it can be expected to last another 12 years (measured from today). At the end of the 12 years it will be worthless. The operating costs of this machine are $7,500 per year and are not expected to change for its remaining life.
Alternatively, machine A can be replaced by a smaller machine B, which costs $42,000 and is expected to last 12 years. Its operating costs are $5,000 per year and would be depreciated by the straight-line method over the 12-year period with no salvage value expected.

Both income and capital gains are taxed at 40%. Compare the after-tax equivalent uniform annual costs of the two machines and decide whether machine A should be retained or replaced by machine B. Use a 10% after-tax MARR in your calculations.

Current book value of machine A = Cost – Depreciation to date
= $54,000 – (9/12)($54,000 – 0) = $13,500.

Long term capital gain if sold now = $30,000 – $13,500 = $16,500
Net return if machine A is replaced = $30,000 – $16,500 × 0.4 = $23,400

Machine A annual depreciation = (P – S)/N = ($54,000 – 0)/12 = $4,500 for 3 more years.

Machine A:

<table>
<thead>
<tr>
<th>Year</th>
<th>BTCF</th>
<th>Depr.</th>
<th>Taxable income</th>
<th>Income tax</th>
<th>ATCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>–30,000</td>
<td></td>
<td></td>
<td>–23,400</td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>–7,500</td>
<td>4,500</td>
<td>–12,000</td>
<td>–4,800</td>
<td>–2,700</td>
</tr>
<tr>
<td>4-12</td>
<td>–7,500</td>
<td>0</td>
<td>–7,500</td>
<td>–3,000</td>
<td>–4,500</td>
</tr>
</tbody>
</table>

After tax annual cost:

\[
\text{NAC} = \left[\$23,400 – 1,800(P/A, 10\%, 3)\right]\left[(A/P, 10\%, 12)\right] + 4,500
\]
\[
\text{NAC} = \left[\$23,400 – 1,800(2.487)\right][0.1468] + 4,500 = 7,278
\]

The after tax cash flow in year 0 reflects the loss of income after capital gains tax from not selling machine A.

Machine B:

Annual depreciation = (P – S)/N = ($42,000 – 0)/12 = $3,500

<table>
<thead>
<tr>
<th>Year</th>
<th>BTCF</th>
<th>Depr.</th>
<th>Taxable income</th>
<th>Income tax</th>
<th>ATCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>–42,000</td>
<td></td>
<td></td>
<td>–42,000</td>
<td></td>
</tr>
<tr>
<td>1-12</td>
<td>–5,000</td>
<td>3,500</td>
<td>–8,500</td>
<td>–3,400</td>
<td>–1,600</td>
</tr>
</tbody>
</table>

After tax annual cost = $42,000(A/P, 10\%, 12) + 1,600 = 7766

Choose the alternative with smaller annual cost \(\rightarrow\) **Keep Machine A**