Measurement of fluid contents and wetting front profiles by real-time neutron radiography

M.R. Deinert\textsuperscript{a}, J.-Y. Par lange\textsuperscript{b}, T. Steenhuis\textsuperscript{b}, J. Throop\textsuperscript{b}, K. Ünlü\textsuperscript{c}, K.B. Cady\textsuperscript{d,*}

\textsuperscript{a}Nuclear Science and Engineering, Cornell University, Ithaca, NY 14853, USA
\textsuperscript{b}Biological and Environmental Engineering, Cornell University, Ithaca, NY 14853, USA
\textsuperscript{c}Mechanical and Nuclear Engineering, Pennsylvania State University, College Station, PA, 16802-1412, USA
\textsuperscript{d}Theoretical and Applied Mechanics, Cornell University, 219 Kimball Hall, Ithaca, NY 14853, USA

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Abstract

Few methods exist for measuring rapidly changing fluid contents at the pore scale that simultaneously allow whole flow field visualization. We present a method for using real-time neutron radiography to measure rapidly changing moisture profiles in porous media. The imaging technique monitors the attenuation of a thermal neutron beam as it traverses a flow field and provides measurements every 30 ms with an image area $410 \text{ cm}^2$ and a spatial resolution $0.05 \text{ cm}$. The technique is illustrated by measuring the variation in moisture content across a wetting front moving at constant velocity through SiO\textsubscript{2} sand. The relative contributions of the hydraulic conductivity and diffusivity terms in Richards' equation to the total fluid flux within the wetting front region were also measured. The diffusivity was found to rise from zero to a peak value within the wetting front region before falling off while the conductivity was found to rise monotonically. The reliability of the technique was checked via mass balance.

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1. Introduction

The use of non-invasive imaging to study multi-phase flows has gained considerable popularity in recent years (Chaouki et al., 1997). The first techniques for doing so, however, date to the advent of X-ray radiographs in the late 19th century. While non-invasive imaging has advanced considerably since that time, many of the currently available methods are unable to image transient flow phenomena in porous media (Darnault et al., 2001).

Synchrotron X-rays have been used to probe such flows but are limited to measurements of a few square millimeters (e.g. Lui et al., 1992; DiCarlo et al., 1997). Light transmission has been used for quantitative imaging of transient flows but requires the use of translucent media and thin experiment systems, usually 3/4 1 cm (e.g. Darnault et al., 2001; Mortensen et al., 2001). Magnetic resonance imaging has also found application in studying fluids in porous media but quantitative measurements are not always possible, especially at low saturation levels (Votrubová et al., 2000).
Neutron radiography is a real-time, non-invasive imaging method that overcomes the above limitations for many laboratory applications. The process involves the attenuation of a thermal neutron beam by a substance, and the recording of the beam’s attenuation pattern as an image by a ccd or tube camera. The technique works especially well when the substance under study contains hydrogen or another element with good neutron attenuating properties (Jasti and Folger, 1992; Tullis et al., 1990). Neutron radiography has also been used to measure water diffusion coefficients in materials as well as variations in matric potential in unconsolidated sand (Nemeca et al., 1999; Prazak et al., 1990; Pel et al., 1993; Deinert et al., 2002). Neutron radiography has been used to image water transport in building materials on time scales $\geq 1.0$ s. It has also been used to measure water diffusion coefficients in materials as well as variations in matric potential in unconsolidated sand (Nemeca et al., 1999; Prazak et al., 1990; Pel et al., 1993; Deinert et al., 2002). Neutron radiography has also been used to image reactive dissolution, immiscible dispersion and fingering in porous media (Jasti and Folger, 1992, 1987; Tullis et al., 1992, Fredd et al., 1996). While these later works illustrate applications, they do not measure fundamental soil properties or analyze rapidly evolving fluid phenomena.

This work extends the application of neutron radiography to quantitative imaging of transient flows in porous media on a sub-second time scale. We illustrate the technique by measuring the hydraulic conductivity and diffusivity in an unconsolidated porous medium using a 'multistep' experiment in which the flow rate was stepped up from 2.5 to 20.0 ml/min. The experiment was performed on air-dried 20–30 sieve sand. At each flow rate the system was allowed to come to steady state after which measurements of $\frac{d\theta}{d\xi}$ were made. Eq. (2) can be rearranged to give an explicit expression for $D(\theta)$:

$$
\frac{v\theta - k(\theta)}{d\theta/d\xi} = D(\theta)
$$

3. Methods

3.1. Experiments: overview

We measured the hydraulic conductivity of a porous medium using a ‘multistep’ experiment in which the flow rate was stepped up from 2.5 to 20.0 ml/min. The experiment was performed on air-dried 20–30 sieve sand. At each flow rate the system was allowed to come to steady state after which the average moisture content in the flow field (the region actively conducting fluid), as well as the area through which the flow moved, were measured using neutron radiography (see Figs. 1a and 2a). Water was withdrawn from the bottom of the experiment chamber through underflow drains attached to a vacuum pump (see Fig. 1b). The underflow drains were covered with a mesh screen to prevent media from leaving the chamber and the vacuum was adjusted so that water would pass through them. The slopes $d\theta/d\xi$ were also measured and found to be effectively zero in all cases. Eq. (2a) was used to measure $k(\theta)$. 

Following the method of Selker et al. (1992) the one-dimensional (1D) flux within a constant velocity flow field is given by:

$$
q = \frac{d\theta}{d\xi} = k(\theta(\xi)) \frac{\partial h(\theta(\xi))}{\partial \xi} + k(\theta(\xi))
$$

where $q$ is the flux {cm/s}, $v$ is velocity {cm/s}, $\partial h(\theta(\xi))/\partial \xi$ is the soil moisture content {cm$^3$/cm$^2$}, $k(\theta(\xi))$ is the hydraulic conductivity {cm/s}, $h(\theta(\xi))$ is the matric pressure {cm}, and $\xi = vt - z$, where $t$ is time [s], $z$ is the distance [cm] from the point of infiltration (positive down). For a wetting front moving at constant velocity Selker et al. (1992) derived an expression relating the 1D flux in a flow field to the hydraulic conductivity and diffusivity of the media:

$$
v\theta = k(\theta) + D(\theta) \frac{d\theta}{d\xi}
$$

Here $D(\theta)$ is the diffusivity {cm$^2$/s}, $\theta$ is understood to be a function of $\xi$ and all other terms are as previously defined. When $d\theta/d\xi = 0$ Eq. (1) reduces to:

$$
q = k(\theta) = Q/A = v\theta
$$

where $Q$ is the total fluid flux {cm$^3$/s}, and $A$ is the area through which the flow moves {cm$^2$}. Once $k(\theta)$ is known, $D(\theta)$ can be obtained if measurements of $d\theta/d\xi$ are made. Eq. (2) can be rearranged to give an explicit expression for $D(\theta)$: 

$$
\frac{v\theta - k(\theta)}{d\theta/d\xi} = D(\theta)
$$
Once \( k(\theta) \) was known, the chamber was dried with air pulled through the underflow drains by the vacuum pump. The dryness of the chamber was checked by comparing its original dry weight to its weight after drying. A series of infiltration experiments were next conducted (see Fig. 2b). Neutron radiography was used to measure the velocity of the wetting fronts, \( v \), the average moisture content in the flow field as a function of location relative to the wetting front, \( u(\xi) \), and the area through which the flow moved. Equation (4) was used along with data on \( v \), \( \frac{d\theta}{d\xi} \), and \( k(\theta) \) to obtain \( D(\theta) \). In between each experiment the chamber was dried as above. The details of how the above measurements were performed follows below.

3.2. Experiments: details

3.2.1. Imaging system

A Precise Optics neutron image intensifier connected to a Vidicon camera was used for this work. The real-time images were recorded using NIH Image 1.61 running on a G3 Macintosh with a Scion LG-3 interface. The system was shown to register variations in water density within the experiment chamber of less than 0.007 cm\(^3\)/cm\(^3\). This is greater than the maximum density resolution of 0.004 cm\(^3\)/cm\(^3\) available with the 8 bit images used in this work. The maximum spatial resolution was 0.051 mm\(^2\) for a field of view of 80 cm\(^2\) and 0.20 mm\(^2\) for a 410 cm\(^2\) field of view. The temporal resolution of the system was 33 ms (Deinert et al., 2002b). The chamber to image plan separation used for this work limited the spatial resolution < 1.0 mm\(^2\) (defined as the ability to completely distinguish adjacent points) with a field of view of 410 cm\(^2\).

The neutron source for this work was a TRIGA Mark II research reactor running at a steady state power of 300 kW. The beam L/D was \(~110\) and its intensity at 300 kW was \(~5 \times 10^6\) neutrons/cm\(^2\)s at the imaging plane. The gamma contamination of
the beam was negligible and the cadmium ratio was \( \sim 21 \) (Deinert et al., 2002b).

3.2.2. Setup

A 16-gauge aluminum chamber with interior dimensions of 35 cm by 20 cm by 1.27 cm was loaded with 20–30 industrial quartz (Unimin) with a mean grain size of 0.6 mm and packed by vibration (see Fig. 1a). The only preparation of the media was a thorough rinse with distilled water followed by drying. The total porosity was 0.33, determined by weighing the chamber and using a bulk density of 2.65 gm/cm\(^3\) for SiO\(_2\). The interior surfaces of the chamber were sprayed with Dow-Corning Silicone-20 release compound to render them hydrophobic. Degassed, demineralized water was used for the experiments, and the pump was calibrated by plotting its flow rate vs. time. The chamber to image plane separation was 5 cm and the chamber was leveled so as to stand dead vertical. The saturated moisture content \( u_{\text{sat}} \) \( \text{cm}^3/\text{cm}^3 \) was determined by measuring the attenuation of a neutron beam on a flooded portion of the chamber and found to be equal to 0.31.

3.2.3. Measuring moisture content

Using the general analysis of image brightness given by Habibi and Mishima (1996), Deinert et al. (2002b) showed that the response of the real-time imaging system could be expressed as:

\[
\frac{I(x, z)}{I_o(x, z)} = a e^{-\int d(x, z)} + (1 - a)
\]  

(4)

Here \( a \) is a system response coefficient, see also Fig. 3, \( \Sigma_i \) is the total neutron linear attenuation coefficient of the material \( 1/\text{cm} \) and \( d(x, z) \) \( \text{cm} \) is the total distance that neutrons had to travel through that material at location \((x', z')\), see Fig. 1b. The resulting intensity change is registered at location \((x, z)\). Eq. (4) can be rearranged to give:

\[
d(x, z) = -\ln\left(\frac{I(x, z)I_o(x, z)}{a} - 1\right) / \sum_i
\]  

(5)

which provides a way to measure the amount of material attenuating the neutron beam. Because the images used in this study were digital they can be viewed as 2D data arrays, where a location \((x, z)\) corresponds to the point \((i, j)\) in the array and the image brightness corresponds to the pixel value at that location (integer values ranging from 1 to 256).

The average moisture content was measured by summing Eq. (5) over \( i, j \) in the flow field, and dividing by the cross sectional area through which the flow moves:

\[
\theta(j) = \frac{1}{A} \sum_{i=m}^{i=n} d(i, j) w
\]  

(6)
Here \( n \) and \( m \) are the values of \( i \) for which \( I(i,j)/I_o(i,j) \leq 0.99 \), \( w \) is the width of a pixel \((0.045 \text{ cm})\) and \( A = \pi r^2 \) where \( r \) is the radius of the flow field at location \( j \).

### 3.2.4. Images used for measurements

Before each experiment a 500 frame sequence of the dry chamber was taken, averaged and used as the \( I_o(x,z) \) image for that experiment. A similar sequence of the calibration standard was taken and averaged before each experiment. The radiographs from the multistep experiments, used for measuring \( k(\theta) \), were also 500 frame averages. In all cases the frame averaging was done to reduce noise and error in the data.

The images taken of the infiltration experiments were captured at 10 frames per second for 50 s each and none were averaged. A 100 frame sequence was chosen from these 500 and the average moisture content in the flow field, \( \theta(j) \), was measured at one pixel intervals from 0.25 cm in front of the wetting front to 4.5 cm back from it and this was done for all 100 images. These data were then averaged to obtain a 100 frame average of \( \theta(j) \) for the flow as it infiltrated the chamber.

### 3.2.5. Measuring wetting front velocity

For the infiltration experiments the progression of the wetting front was measured by recording the location \((i,j)\) on each image for which \( I(i,j)/I_o(i,j) \leq 0.98 \). The value of \( i \) varied little during the experiments and \( j \) vs. time was plotted to obtain the wetting front velocity.

### 3.2.6. Measuring flow area

The width of the flow field was measured in all experiments and in all cases found to be less than the chamber thickness. For the multistep experiments the data array \( I(i,j)/I_o(i,j) \) was read from left to right and the indices \((i,j)\) were recorded for which \( I(i,j)/I_o(i,j) \) went below 0.99 and then above it again giving the values \( i = m \) and \( i = n \) in Eq. (6). The interval was then multiplied both by the pixel width \((0.045 \text{ cm})\), and the flow area was taken to be \( \pi r^2 \) where \( r \) was one half of the flow field width. This was done for each \( j \) in the data array.

The flow area in the infiltration experiments was obtained using the same algorithm but measurements were made from the tip of the wetting front to 4.5 cm back from it.
4. Results and discussion

The sand filled chamber remained virtually unchanged over the course of the experiments and always dried to within 0.1 gm of its original weight. All experiments formed nearly identical flow paths and measurements from neutron images were able to account for the water entering the chamber to within better than 1% (Deinert et al., 2002b). A single, 20 ml/min infiltration experiment was conducted where the flow field was allowed to come to steady state and evolve for 2 h. The results showed no measurable change in the width of the flow field, a strong indication that fluid only moved vertically and that the 1D flow assumption in Eqs. (1)–(3) was reasonable.

A plot of \( k \) vs. \( \theta/\theta_{sat} \) is shown in Fig. 4. In total, \( k \) was measured in three different multistep experiments all giving the same results to within \( \sim 5\% \). The error bars on the graph are the standard deviations calculated using the error propagation formula (Knoll, 1989) applied to Eq. (2a). A curve fit to the data shows a power law relation like that of Brooks and Corey (1964).

A series of single step experiments were conducted to assess the relative contributions of \( k(\theta) \) and \( D(\theta) \frac{d\theta}{d\xi} \) to the total fluid flux across the wetting front. Eq. (3) was used along with \( k(\theta) \) as given by the equation for the curve fit shown in Fig. 4 and measurements of \( v \) and \( \theta \). The results are given in Fig. 5 and show that \( D(\theta) \frac{d\theta}{d\xi} \) rises from 0 at the wetting front to a maximum value before decreasing again. In total, three 10.0 ml/min infiltration experiments were conducted and all yielded the same results for \( v \) and \( \theta \) to within \( \sim 5\% \). The spatial resolution of the data is sufficient to provide a high degree of detail in the wetting front region. In particular, the above measurements discern the relative contributions of the conductivity and diffusivity to the total fluid flux within that region. Fig. 6 plots wetting front position vs. time for one of these experiments and shows a nearly constant velocity.

The diffusivity was found using Eq. (3) and the results are shown in Fig. 7a. While the average moisture contents shown in Fig. 7a are well below saturation, the peak values in the flow field are likely...
near to it. While moisture content varies radially within the flow field, its vertical gradient is relatively insensitive to radial position near the wetting front, see Fig. 7b. As a result, measurements of $D$ based on Eq. (3), and field averaged moisture contents, are confined to the boxed region shown in Fig. 7b. The error bars on the data in Fig. 7a were calculated using the error propagation formula (Knoll, 1998) applied to

![Fig. 5](image1)

Fig. 5. The hydraulic conductivity, $k(\theta/\theta_{sat})$, comes from the curve fit shown in Fig. 4. Data on $(\theta/\theta_{sat})$, and $q$ are from a 10.0 ml/min infiltration experiment and both are flow field averages over $x$.

![Fig. 6](image2)

Fig. 6. The wetting front velocity. The graph plots wetting front position vs. time for a 10.0 ml/min infiltration experiment. The slope of the line is the velocity. The horizontal position of the wetting front ($\xi$) was ~ constant.
Eq. (3) with standard deviations for velocity and moisture content. The spatial resolution of the data are sufficient to show how $D$ varies in the region near the leading edge of the wetting front.

Fig. 8 shows the variation in $u$ relative to the flow field centerline, for selected flow rates, from one of the multistep experiments used to determine $k(t)$. The moisture content rises from 0.0 to a peak value before decreasing again. Fig. 8 supports the assumption made in Section 3.2.6 that fluid moved through cylinder like flow paths. Had the flow fields spanned the distance between the walls of the experiment chamber the profiles shown in Fig. 8 would instead rise from 0.0 to a plateau before decreasing again.

Fig. 9 shows a 100 image frame average of the wetting front profile and the profile as it appears on an individual image frame. The scatter in the data from the single image is substantially due to variations in $\theta(j)$. By contrast, if we consider a region where fluid is not present, then the frame to frame variation $I(x,z)/I_0(x,z) \sim 0.009$, (Deinert et al., 2002b). For a 1 cm diameter flow this corresponds to standard deviation in $\theta \sim 0.003$, far less then the scatter seen in Fig. 9.

While the work presented here used a thin experiment chamber, the flow fields appear to have developed fully 3D structure. Neutrons have the ability to penetrate larger volumes of dry media. While it would not be easy to use neutron radiography to study fluid movement, such as oils, in water saturated media, it could be used to study fluid movement in D$_2$O (heavy water) saturated...
media. Heavy water has a neutron attenuation coefficient that is approximately seven times smaller than that of normal water but has largely the same physical properties otherwise (Kirschenbaum, 1951).

5. Conclusions

Real-time neutron radiography was used for quantitative imaging of transient flows in porous media. The spatial and temporal resolution of
the system, coupled with its sensitivity to variations in moisture content and its ability to image non-translucent media, make it an ideal tool for detailed laboratory studies when the flow varies rapidly in space and time.

We considered the specific case where the flow field can be well approximated by a traveling wave solution to Richards’ Equation. Measurements of the variation in moisture content across the wetting front were made and the data used to assess the relative contribution of \( k \) and \( D(\theta) \frac{d\theta}{d\xi} \) to the total flux within the wetting front region. Rapid local variations in \( \theta \) were measurable on individual image frames, especially apparent when compared to time averaged views of the wetting front profile as seen in Fig. 9.

References


