Stability and Performance Limits of Latency-Prone Distributed Feedback Controllers

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Abstract—Robotic systems are increasingly relying on distributed feedback controllers to tackle complex sensing and decision problems such as those found in highly articulated human-centered robots. These demands come at the cost of a growing computational burden and, as a result, larger controller latencies. To maximize robustness to mechanical disturbances by maximizing control feedback gains, this paper emphasizes the necessity for compromise between high- and low-level feedback control effort in distributed controllers. Specifically, the effect of distributed impedance controllers is studied where damping feedback effort is executed in close proximity to the control plant and stiffness feedback effort is executed in a latency-prone centralized control process. A central observation is that the stability of high impedance distributed controllers is very sensitive to damping feedback delay but much less to stiffness feedback delay. This study pursues a detailed analysis of this observation that leads to a physical understanding of the disparity. Then a practical controller breakdown gain rule is derived to aim at enabling control designers to consider the benefits of implementing their control applications in a distributed fashion. These considerations are further validated through the analysis, simulation and experimental testing on high performance actuators and on an omnidirectional mobile base.

Index Terms—Distributed Feedback Control, High Impedance Control, Feedback Delays, Mobile Robotics.

I. INTRODUCTION

As a result of the increasing complexity of robotic control systems, such as human-centered robots [1–3] and industrial surgical machines [4], new system architectures, especially distributed control architectures [5, 6], are often being sought for communicating with and controlling the numerous device subsystems. Often, these distributed control architectures manifest themselves in a hierarchical control fashion where a centralized controller can delegate tasks to subordinate local controllers (Figure 1). As it is known, communication between actuators and their low-level controllers can occur at high rates while communication between low- and high-level controllers occurs more slowly. The latter is further slowed down by the fact that centralized controllers tend to implement larger computational operations, for instance to compute system models or coordinate transformations online.

A. Control architectures with feedback delays

One concern is that feedback controllers with large delays, such as the centralized controllers mentioned above, are less stable than those with small delays, such as locally embedded controllers. Without the fast servo rates of embedded controllers, the gains in centralized controllers can only be raised to limited values, decreasing their robustness to external disturbances [7] and unmodelled dynamics [8].

As such, why not remove centralized controllers altogether and implement all feedback processes at the low-level? Such operation might not always be possible. For instance, consider controlling the behavior of human-centered robots (i.e. highly articulated robots that interact with humans). Normally this operation is achieved by specifying the goals of some task frames such as the end effector coordinates. One established option is to create impedance controllers on those frames and transform the resulting control references to actuator commands via operational space transformations [9]. Such a strategy requires the implementation of a centralized feedback controller which can utilize global sensing data, access the state of the entire system model, and compute the necessary

Fig. 1. Depiction of various control architectures. Many control systems today employ one of the control architectures above: a) Centralized control with only high-level feedback controllers (HLCs); b) Decentralized control with only low-level feedback controllers (LLCs); c) Distributed control with both HLCs and LLCs, which is the focus of this paper.
models and transformations for control. Because of the aforementioned larger delays on high-level controllers, does this imply that high gain control cannot be achieved in human-centered robot controllers due to stability problems? It will be shown that this may not need to be the case. But for now, this delay issue is one of the reasons why various currently existing human-centered robots cannot achieve the same level of control accuracy that it is found in high performance industrial manipulators. More concretely, this study proposes a distributed impedance controller where only proportional (i.e., stiffness) position feedback is implemented in the high-level control process with slow servo updates. This process will experience the long latencies found in many modern centralized controllers of complex human-centered robots. At the same time, it contains global information of the model and the external sensors that can be used for operational space control. For stability reasons, our study proposes to implement the derivative (i.e., damping) position feedback part of the controller in low-level embedded actuator processes which can therefore achieve the desired high update rates.

B. Analysis of sensitivity to delay

To focus the study on the physical performance of the proposed distributed control approach, our study first focuses on a single actuator system with separate stiffness and damping servos and under multiple controller delays. Then the physical insights gained are used as a basis for achieving high impedance behaviors in single actuator systems and in an omnidirectional mobile base. Let us pose some key questions regarding distributed stiffness-damping feedback controllers considered in this study: (A) Does controller stability have different sensitivity to stiffness and damping feedback delays? (B) If that is the case, what are the physical reasons for such a difference?

To answer these questions, this paper studies the physical behavior of the proposed real-time distributed system using control analysis tools, such as the phase margin stability criterion, applied to the system’s plant. Using these tools our study reveals that system closed-loop stability and performance are much more sensitive to damping feedback delays than to stiffness feedback delays.

C. Benefits of the proposed distributed control architecture

As it will be empirically demonstrated, the benefit of the proposed split control approach over a monolithic controller implemented at the high-level is to increase control stability due to the reduced damping feedback delay. As a direct result, closed-loop actuator impedance may be increased beyond the levels possible with a monolithic high-level impedance controller. This conclusion may be leveraged on many practical systems to improve disturbance rejection by increasing gains without compromising overall controller stability. As such, these findings are expected to be immediately useful on many complex human-centered robotic systems.

To demonstrate the effectiveness of the proposed methods, this study implements tests on a high performance actuator followed by experiments on a mobile base. First, a position step response is tested on an actuator under various combinations of stiffness and damping feedback delays. The experimental results show high correlation to their corresponding simulation results. Second, the proposed distributed controller is applied to an implementation into an omnidirectional base. The results show a substantial increase in closed-loop impedance capabilities, which results in higher tracking accuracy with respect to the monolithic centralized controller counterpart approach.

Consequently, our main contribution is to analyze, provide control system solutions, implement and evaluate actuators and mobile robotic systems with latency-prone distributed architectures to significantly enhance their stability and trajectory tracking capabilities. In particular, a new study is performed to reveal that system stability and performance is more sensitive to damping servo latencies than stiffness servo latencies. Then a novel servo breakdown rule is proposed to evaluate the benefits of using a distributed control architecture. As a conclusion, this paper proposes to use stiffness servos for centralized operational space control while realizing embedded-level damping servos as joint space damping processes for stability and tracking accuracy.

The organization of this paper is as follows: related work of our study is presented in Section II; Then Section III proposes a distributed control architecture, which is simulated with observations of phase margin sensitivity to feedback delays; Then the fundamental reasons of this sensitivity discrepancy are analyzed in detail and a servo breakdown gain rule is proposed correspondingly in Section IV; To validate this discrepancy, experimental evaluations are shown in Section V. Finally, Section VI concludes this study and discusses future directions.

II. RELATED WORK

Advances in distributed control technologies [6, 10, 11], have enabled the development of decentralized multiple input and multiple output systems such as humanoid systems and highly articulated robots [2, 3, 12]. Distributed control architectures combine centralized processes with self-contained control units that are in close proximity to actuators and sensors. In [5], servo motor controllers are used as sub-controllers coordinated via CAN communications by a central controller. As a result, computation burden on the central controller is reduced based on this distributed architecture. Recently, a distributed motion control system [13] has been developed to experimentally demonstrate the capabilities of real-time communications and synchronous tracking control. Analogous to human muscle actuation and neural systems, a bio-inspired distributed control infrastructure [14] reduces controller task complexity by off-loading parts of the controller into the robot’s limb processors. Another typical distributed control architecture, related to neuroscience and robotics, is locomotor central pattern generators (CPGs) [15, 16]. CPGs are modeled as distributed neural networks of multiple coupled oscillators to control articulated robot locomotion. This distributed architecture has the advantage of reducing time delays in the motor control loop (i.e., fast feedback loops through the spinal cord), in order to efficiently coordinate mechanical movements with rhythms.
However, the effect on controller performance due to the ever-growing computational demand on feedback servos and latency-prone serial communications in human-centered robots has been largely overlooked on distributed controller studies. A detailed analysis, exploration, and implementation of the high impedance capabilities of distributed controllers with latency-prone centralized processors has not been previously performed. To this end, our study focuses on how a distributed controller improves control system stability and performance over monolithic centralized control approaches.

Robustness and effects of delay have often been studied in work regarding Proportional-Integral-Derivative (PID) controller tuning. A survey of PID controllers including system plants using phase margin techniques with linear approximations is conducted in [17]. The works [18, 19] study autotuning and adaption of PID controllers while the work [20] furthers these techniques by developing optimal design tools applied to various types of plants which include delays. The study in [21] proposed an optimal gain scheduling method for DC motor speed control with a PI controller. In [22], a backstepping controller with time-delay estimation and non-linear damping is considered for variable PID gain tuning under disturbances. The high volume of studies on PID tuning methods highlight the importance of this topic for robust control under disturbances. However, none of those studies considers the sensitivity discrepancy between stiffness and damping feedback loops nor propose solutions to increase performance based on this discrepancy.

III. Basic Distributed Control Structure

This section describes the actuator model used to analyze closed-loop system stability, propose a basic distributed control architecture that delocalizes stiffness and damping servo loops, and analyzes the sensitivity of these control processes to loop delays.

A. Actuator plant model

Many rigid electrical actuators like the ones used in modern robots can be approximately modeled as a second-order plant with a force acting on an inertia-damper pair (as shown in Figure 2).

Considering a current-controlled motor, the control plant from current, $i_M$, to position, $x$, is

$$P(s) = \frac{x(s)}{i_M(s)} = \frac{F_M(s)}{i_M(s)} = \frac{\nu}{ms^2 + bs},$$

where $F_M$ is the applied motor force, $\nu \equiv F_M/i_M = \eta N k_\tau$, $\eta$ is the drivetrain efficiency, $N$ is the gear speed reduction and $k_\tau$ is the motor torque constant.

B. Closed-loop distributed controller model

Figure 2 shows our proposed distributed controller built using a proportional-derivative feedback mechanism. It includes velocity feedback filtering ($Q_v s$), stiffness feedback delay ($T_s$), damping feedback delay ($T_d$), with $T_s \neq T_d$, stiffness feedback gain ($K$) and damping feedback gain ($B$). Excluding the unknown load ($F_d$), the desired motor force
where \( s \) in the Laplace domain are the desired output position and velocity, as closed-loop transfer function from desired to output positions respectively. Using Equations (1) and (2), one can derive the cutoff frequency \( f_n \) where \( f_n \triangleq \omega_n/2\pi \) is the so-called natural frequency defined in (10) and the servo delays shown in Figure 2. A simulations of the open loop transfer function shown in (6) as a function of the natural frequency defined in (10) and the servo delays shown in Figure 2. A simulations indicate that phase margins less than 50 degrees exhibit oscillatory behavior [31]. The phase margin of 0 degrees is considered marginally stable. Simulations indicate that the stiffness and damping gains using an idealized second order characteristic polynomial [32]

\[
\begin{align*}
    & s^2 + 2\zeta\omega_n s + \omega_n^2, \\
    & s = \omega_n(1 + \zeta^2),
\end{align*}
\]

where \( \omega_n \) is the so-called natural frequency and \( \zeta \) is the so-called damping factor. In such case, the idealized characteristic polynomial (i.e. ignoring delays, \( T_d = T_s = 0 \), and filtering, \( Q_v = 1 \)) associated with our closed loop plant of Equation (3) would be

\[
\begin{align*}
    & s^2 + (B + b)/m \cdot s + K/m. \tag{8}
\end{align*}
\]

Choosing the second order critically damped rule, \( \zeta = 1 \) and comparing Equations (7) and (8) one can get the gain dependency

\[
\begin{align*}
    & B = 2\sqrt{mK} - b, \tag{9}
\end{align*}
\]

and the natural frequency,

\[
\begin{align*}
    & f_n \triangleq \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{K}{m}}. \tag{10}
\end{align*}
\]

The second order dependency of Equation (9) will be used for the rest of this paper for deriving new gain selection methods through the thorough analysis of the oscillatory behavior of the closed loop plant of Equation (3). In particular our study will use the phase margin criterion and other visualizations tools to study how the complete system reacts to feedback delays and signal filtering. Phase margin is the additional phase value above \(-180^\circ\) when the magnitude plot crosses the 0 dB line (i.e., the gain crossover frequency). It is common to quantify system stability by its phase margin.

For the proposed distributed controller in Figure 2, the damping feedback loop is labeled as low level (e.g. embedded) to emphasize that it is meant to be locally implemented to take advantage of high servo rates. On the other hand the stiffness loop is implemented in a high-level computational process close to external sensors and centralized models, for operational space control purposes. Operational space control is normally used in human-centered robotic applications where controllers use task coordinates and global models for their operation. The simplified controller in Figure 2 is used to illustrate the discrepancies in sensitivity to latencies between the servo loops. It does not correspond to a practical robot controller as it contains only a single degree of freedom. After analyzing this structure, we implement a similar distributed controller into a multi-axis robotic base shown in Figure 8, which results in the simultaneous improvement of system stability while achieving operational space control.
C. Phase margin sensitivity comparison

This subsection focuses on utilizing frequency domain control methods to analyze the phase margin sensitivity to time delays on the distributed control architecture shown on Figure 2. Different delay range scales are considered: (1) a small range scale (1 – 5 ms) to show detailed variations, and (2) a larger range scale (5 – 25 ms) to cover practical delay ranges. These scales roughly correspond to embedded and centralized computational and communication processes found in highly articulated robots such as [1]. Phase margin plots, are subsequently obtained for the controller of Equation (3) and shown on Figures 3 and 4 as a function of the natural frequency given in Equation (10) and using the gain relationship of Equation (9). All the simulations are carried out using Matlab® software. Feedback delays are represented by the exponential term $e^{-Ts}$ in frequency domain. Phase margin is computed by using Matlab’s margin() command based on the open loop transfer function.

In Figure 3, delays ranging between 1 ms and 5 ms are simulated for both the stiffness and damping servos. The simulations are performed based on identical actuator parameters than those used in the experimental section, Section V, i.e. passive output inertia $m = 256$ kg and passive damping $b = 1250$ Ns/m. Equations (9) and (10) can subsequently be used to derived the stiffness and damping feedback gains. It is noticeable that reducing either stiffness or damping feedback delays will increase the stability of the controller. But more importantly, it is clearly visible that phase margin behavior is much more sensitive to damping servo delays ($T_d$) than to stiffness servo delays ($T_s$). Not only there is a disparity on the behavior with respect to the delays, but phase margin is fairly insensitive to stiffness servo delays in the observed time scales. Such disparity and behavior is the central observation that motivates this paper and the proposed distributed control architecture. Figure 4 simulates step position response of the controller for a range of relatively large stiffness delays and for two choices of damping delays, a short and a long one. The first point to notice here is that the phase margin values for subfigure (a) are significantly lower than for (d) due to the larger damping delay. Secondly, both (a) and (d) show small variations between the curves, corroborating the small sensitivity to stiffness delays that will be studied in Section IV. Corresponding step responses are shown along for various natural frequencies. It becomes clear that reducing damping delay significantly boosts stability even in the presence of fairly large stiffness delays. These results emphasize the significance of implementing damping terms at the fastest possible level (e.g. at the embedded level) while proportional (i.e. stiffness) servos can run in latency prone centralized processes. This conclusion is also validated by Nichols diagram [33].

IV. BASIS FOR SENSITIVITY DISCREPANCY

In the previous section it was observed different behavior of the controller’s phase margin depending on the nature of
delay. Damping delay seems to affect much more the system’s phase margin than stiffness delay. This section will analyze this physical phenomenon in much more detail and reveal the conditions under which this disparity occurs.

A. Equations expressing phase margin sensitivity to delays

Detailed mathematical analysis is developed to find further physical structure for the causes of stability discrepancies between damping and stiffness delays. Let us re-visit the open loop transfer function of Equation (6). The resulting open loop transfer function, including the low pass velocity filter of Equation (4), in the frequency domain \( s = j\omega \) is

\[
P_{OL}(\omega) = \frac{jA_1(\omega) + A_2(\omega)}{j\omega(jm\omega + b)(j\tau_v\omega + 1)},
\]

with

\[
A_1(\omega) \triangleq B\omega \cos(T_d\omega) - K\sin(T_s\omega) + K\tau_v\omega \cos(T_s\omega),
\]

\[
A_2(\omega) \triangleq B\omega \sin(T_d\omega) + K\cos(T_s\omega) + K\tau_v\omega \sin(T_s\omega).
\]

Note that Euler’s Formula \((e^{-j\omega} = \cos\omega - j\sin\omega)\) has been used to obtain the above results.

The phase margin, \( PM = 180^\circ + \angle P_{OL}(\omega_g) \), of the plant (11), where \( \angle \) is the angle of the argument, is

\[
PM = \tan\left[ \frac{A_{1g}}{A_{2g}} \right] + 90^\circ - \tan\left[ \frac{m\omega_g}{b} \right] - \tan\left[ \tau_v\omega_g \right],
\]

with \( \omega_g \) being the gain crossover frequency [32] and \( A_{ig} \triangleq A_i(\omega_g), \ i = \{1,2\} \). After performing several manipulations – details are ignored due to space constraints – we obtain the sensitivity equations below expressing variations of the phase margin with respect to stiffness and damping delays,

\[
\frac{\partial PM}{\partial T_s} = \frac{-K^2(\tau_v\omega_g^2 + 1) + KB\omega_g M}{A_{1g} + A_{2g}} \omega_g,
\]

\[
\frac{\partial PM}{\partial T_d} = \frac{-B^2\omega_g^2 + KB\omega_g M}{A_{1g} + A_{2g}} \omega_g,
\]

where

\[
M = \sqrt{(\tau_v\omega_g)^2 + 1} \cdot \sin((T_s - T_d)\omega_g + \phi)
\]

where the phase shift \( \phi \triangleq \tan(-\tau_v\omega_g) \).

B. Gain crossover sensitivity condition

From the control analysis of the distributed plant performed in previous sections, increasing damping delays decreases the phase margin. This observation means that the sensitivity of the phase margin to damping delays must be negative, i.e.

\[
\frac{\partial PM}{\partial T_d} < 0.
\]

Also from those analysis, it is observed that the phase margin is more sensitive to damping than to stiffness delays. This observation can be formulated as

\[
\frac{\partial PM}{\partial T_d} < \frac{\partial PM}{\partial T_s}.
\]

Let us re-organize the numerator of Equation (15) to be written in the alternate form

\[
\frac{\partial PM}{\partial T_d} = \frac{[B\omega_g + KM]B\omega_g^2}{A_{1g} + A_{2g}}.
\]

An upper bound of the above equation occurs when the maximal condition \( \sin((T_s - T_d)\omega_g + \phi) = 1 \) is met, i.e.

\[
\frac{\partial PM}{\partial T_d} \leq \frac{[B\omega_g + K\sqrt{(\tau_v\omega_g)^2 + 1}]B\omega_g^2}{A_{1g} + A_{2g}}.
\]

Based on the above inequality, (17) is met if the following gain crossover sensitivity condition is met,

\[
\omega_g > \frac{K}{\sqrt{B^2 - K^2\tau_v^2}}.
\]

The above equation is only a sufficient condition for fulfilling Condition (17). Obtaining a closed form solution for that condition would be very complex due to the presence of trigonometric terms. Therefore, the remainder of this section is to study under what circumstances Condition (21) holds.

At the same time, Inequality (18) can be re-written in the form

\[
\frac{\partial PM}{\partial T_d} - \frac{\partial PM}{\partial T_s} = \frac{[B^2\omega_g^2 + K^2(\tau_v\omega_g^2 + 1)]\omega_g}{A_{1g}^2 + A_{2g}^2} < 0,
\]

where it has been subtracted the right hand sides of Equations (14) and (15) for the derivation. Notice that in that subtraction the sine functions cancel out. Coincidentally, the above inequality is also met if the gain crossover sensitivity condition (21) is fulfilled. In other words, that condition is sufficient to meet both Inequalities (17) and (18).
variable that allows to write (21) as an equality, which Condition (21) holds. Let us start by creating a new the parameters and numerically derives parameter ranges for

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Using the plant (11), it can be shown – once more we omit

Using Equation (25) it can be further demonstrated that

C. Servo breakdown gain rule

To validate the gain crossover condition (21), this study solves for the gain crossover frequency, which consists of the frequency at which the magnitude of the open loop transfer function is equal to unity, i.e.

Using the plant (11), it can be shown – once more we omit the manipulations due to space constraints – that the above equation results in the equality

The above equation is intractable in terms of deriving a closed loop expression of the gain crossover frequency. To tackle a solution this study introduces transformations of the parameters and numerically derives parameter ranges for which Condition (21) holds. Let us start by creating a new variable that allows to write (21) as an equality,

Thus, demonstrating the gain crossover sensitivity condition (21) is equivalent to demonstrating that \( \delta > 0 \). Rewriting Equation (9) as \( K = (B + b)^2/4m \) and substituting \( K \) in the above equation, (25) can be further expressed as

\[
\omega_g = (1 + \delta) \frac{(B + b)^2}{16B^2m^2 - (B + b)^4\tau_v^2}. \tag{26}
\]

Dividing Equation (24) by a new term \( K^2U\cdot V \), with \( U \triangleq \tau_v^2\omega_g^2 + 1 \), and \( V \triangleq B^2\omega_g^2/K^2 \), while substituting \( \omega_g \) on the right hand side of Equation (24) by Equation (26), and using \( M \) as shown in Equation (16), Equation (24) becomes

\[
\frac{1}{U} + \frac{1}{V} = \frac{2\sin((T_s - T_d)\omega_g + \phi)}{\sqrt{U\cdot V}} = \frac{(1 + \delta)^2(B + b)^4}{16B^4 - B^2(3B + b^3)\tau_v^2/m^2} + \left(\frac{b}{B}\right)^2. \tag{27}
\]

Using Equation (25) it can be further demonstrated that \( V = (\tau_v\omega_g)^2 + (1 + \delta)^2 \). Thus \( U \) and \( V \) are only expressed in terms of \( (\tau_v\omega_g)^2 \). To further facilitate the analysis, let us introduce three more variables

\[
\alpha \triangleq \sin((T_s - T_d)\omega_g + \phi) \in [-1, 1], \tag{28}
\]

\[
\beta \in (0, \infty) \quad \text{s.t.} \quad B = \beta m, \tag{29}
\]

\[
\gamma \in (0, \infty) \quad \text{s.t.} \quad B = \gamma b. \tag{30}
\]

Notice that \( \alpha \) can be interpreted as an uncertainty, \( \beta \) is the ratio between damping gain and motor drive inertia and \( \gamma \) is the ratio between damping gain and motor drive friction. Using these variables, (27) simplifies to

\[
\frac{U + V - 2\alpha\sqrt{U\cdot V}}{U\cdot V} = \frac{(1 + \delta)^2(1 + \gamma)^4}{16\gamma^4 - (1 + \gamma)^4\beta^2\tau_v^2} + \frac{1}{\gamma^2}. \tag{31}
\]

Using Equations (26), (29) and (30), the term \( (\tau_v\omega_g)^2 \) appearing in the variables \( U \) and \( V \) on Equation (31) can be expressed as

\[
(\tau_v\omega_g)^2 = \beta^2\tau_v^2 \frac{(1 + \delta)^2(1 + \gamma)^4}{16\gamma^4 - (1 + \gamma)^4\beta^2\tau_v^2}. \tag{32}
\]

Thus, Equation (31) does not contain direct dependencies with \( \omega_g \) and therefore can be represented as the nonlinear function

\[
f(\alpha, \beta, \gamma, \delta, \tau_v) = 0 \tag{33}
\]

Let us demonstrate under which conditions \( \delta > 0 \), which will imply that Equation (21) holds. In this study, velocity filters with \( \tau_v = 0.0032\, s \) are commonly used for achieving high performance control [31], and therefore Equation (31) will be solved for only that filter. Notice that it is not difficult to try new values of \( \tau_v \) when needed. Additionally, when sampling Equation (31) for the values of \( \alpha \) shown in Equation (28), it is observed that not only \( \delta \) is fairly invariant to \( \alpha \) but the lowest value of \( \delta \) occurs for \( \alpha = 1 \). These behaviors are omitted here for space purposes. Therefore, as a particular solution, Equation (31) is solved for the values

\[
f(\alpha = 1, \beta, \gamma, \delta, \tau_v = 0.0032) = 0. \tag{34}
\]

The above function is solved numerically and the solution surface is plotted in Figure 5. As it can be seen, \( \delta > 0 \) for \( \gamma > 2 \), allowing us to state that using a distributed PD feedback control law like the one in Figure 2 with the particular choice of the filter \( \tau_v = 0.0032\, s \) and with damping gains greater than

\[
B > 2b, \tag{35}
\]

causes the phase margin to be more sensitive to damping delays than to stiffness delays. The threshold above can therefore be interpreted as a breakdown gain rule which is sufficient to meet the gain crossover sensitivity condition (21), and from which the aforementioned phase margin sensitivity discrepancy follows.

This threshold hints towards a general rule for breaking controllers down into distributed servos, as was illustrated in Figure 2. Namely, if the maximum allowable feedback damping gain for a given servo rate is significantly larger than twice the passive actuator damping, then the controller’s stiffness servo can be decoupled from the damping servo to a
Fig. 7. Step response experiment with distributed controller. Subfigures (a) through (d) show various implementations on our linear rigid actuator corresponding to the simulations depicted on Figure 4. Overlapped with the data plots, simulated replicas of the experiments are also shown to validate the proposed models. The experiments not only confirm the higher sensitivity of the actuator to damping than to stiffness delays but also indicate a good correlation between the real actuator and the simulations.

slower computational process without hurting the controller’s stability.

In the next section we study in detail the implementation of the proposed distributed control strategy in a new high performance linear rigid actuator and an omnidirectional mobile base.

V. EXPERIMENTAL EVALUATION

The proposed controller of Figure 2 is implemented in our linear rigid actuator shown in Figure 6. This actuator is equipped with a PC-104 form factor computer running Ubuntu Linux with an RTAI patched kernel [34]. The PC communicates with the actuator using analog and quadrature signals through a custom signal conditioning board. Continuous signal time derivatives are converted to discrete form using a bilinear Tustin transform written in C. A load arm is connected to the output of the ball screw pushrod. Small displacements enable the actuator to operate in an approximately linear region of its load inertia. At the same time, the controller is simulated by using the closed loop plant given in Equation (5). Identical parameters to the real actuator are used for the simulation, thus allowing us to compare both side by side.

A. Step response implementation

First, a test is performed on the actuator evaluating the response to a step input on its position. The results are shown in the bottom part of Figure 7 which shows and compares the performance of the real actuator versus the simulated closed loop controller. All the experimental tests are performed with a $1 \text{ kHz}$ servo rate. Additional feedback delays are manually added by using a data buffer. A step input comprising desired displacements between $0.131 \text{ m}$ and $0.135 \text{ m}$ of physical pushrod length is sent to the actuator. The main reason for constraining the experiment to a small displacement is to prevent current saturation of the motor driver. With very high stiffness, it is easy to reach the $30 \text{ A}$ limit for step responses.
If current is saturated, then the experiment will deviate from the simulation. The step response is normalized between 0 and 1 for simplicity. Various tests are performed for the same reference input with varying time delays. In particular large and small delays are used for either or both the stiffness and damping loops. The four combinations of results are shown in the figure with delay values of 1 ms or 15 ms.

The first thing to notice is that there is a good correlation between the real and the simulated results both for smooth and oscillatory behaviors. Small discrepancies are attributed to unmodelled static friction and the effect of unmodelled dynamics. More importantly, the experiment confirms the anticipated discrepancy in delay sensitivity between the stiffness and damping loops. Large servo delays on the stiffness servo, corresponding to subfigures (a) and (b) have small effects on the step response. On the other hand, large servo delays on the damping servo, corresponding to subfigures (c) and (d), strongly affect the stability of the controller. In fact, for (c) and (d) the results corresponding to $f_n = 12$ Hz are omitted due to the actuator quickly becoming out of control. In contrast, the experiment in (b) can tolerate such high gains despite the large stiffness delay.

B. Distributed operational space control of a mobile base

As a concept proof of the proposed distributed architecture on a multi-axis mobile platform, a Cartesian space feedback Operational Space Controller [9] is implemented on an omnidirectional mobile base. The original feedback controller was implemented as a centralized process [35] with no distributed topology at that time. The mobile base is equipped with a centralized PC computer running Linux with the RTAI real-time kernel. The PC connects with three actuator processors embedded next to the wheel drivetrains via EtherCat serial communications. The embedded processors do not talk to each other. The high level centralized PC on our robot, has a roundtrip latency to the actuators of 7 ms due to process and bus communications, while the low level embedded processors have a servo rate of 0.5 ms. Notice that 7 ms is considered too slow for stiff feedback control. To accentuate even further the effect of feedback delay on the centralized PC, an additional 15 ms delay is artificially introduced by using a data buffer. Thus, the high level controller has a total of 22 ms feedback delay.

An operational space controller (OSC) is implemented in the mobile base using two different architectures. First, the controller is implemented as a centralized process, which will be called COSC, with all feedback processes taking place in the slow centralized processor and none in the embedded processors. In this case, the maximum stiffness gains should be severely limited due to the effect of the large latencies. Second a distributed controller architecture is implemented inspired by the one proposed in Figure 2 but adapted to
a desired operational space controller, which will be called DOSC. In this version, the Cartesian stiffness feedback servo is implemented in the centralized PC in the same way than in COSC, but the Cartesian damping feedback servo is removed from the centralized process. Instead, our study implements damping feedback in joint space (i.e. proportional to the wheel velocities) on the embedded processors. A conceptual drawing of these architectures is shown in Figure 8. The metric used for performance comparison is based on the maximum achievable Cartesian stiffness feedback gains, and the Cartesian position and velocity tracking errors.

To implement the Cartesian stiffness feedback processes in both architectures, the Cartesian positions and orientations of the mobile base on the ground are computed using wheel odometry (more details are discussed in [35]). To achieve the highest stable stiffness gains, the following procedure is followed: (1) first, Cartesian stiffness gains are adjusted to zero while the damping gains in either Cartesian space (COSC) or joint space (DOSC) – depending on the controller architecture – are increased until the base starts vibrating; (2) the Cartesian stiffness gains, on either architecture, are increased until the base starts vibrating or oscillating; (3) a desired Cartesian circular trajectory is commanded to the base and the position and velocity tracking performance are recorded.

Based on these experiments, DOSC was able to attain a maximum Cartesian stiffness gain of 140 N/(m kg) compared to 30 N/(m kg) for COSC. This result means that the proposed distributed control architecture allowed the Cartesian feedback process to increase the Cartesian stiffness gain ($K_x$ in Figure 9) by 4.7 times with respect to the centralized controller implementation. In terms of tracking performance, the results are shown in Figure 8. Both Cartesian position and velocity tracking in DOSC are significantly more accurate. The proposed distributed architecture reduces Cartesian position root mean error between 62% and 65% while the Cartesian velocity root mean error decreases between 45% and 67%.

VI. CONCLUSIONS AND DISCUSSIONS

The motivation for this paper has been to study the stability and performance of distributed controllers where stiffness and damping servos are implemented in distinct processors. These types of controllers will become important as computation and communications become increasingly more complex in human-centered robotic systems. The focus has been first on studying the physical performance of a simple distributed controller. Simplifying the controller allows us to explore the physical effects of time delays in greater detail. Then the proposed architecture has been leveraged to a mobile base system as a proof of concept. Our focus on this paper has been on high impedance behaviors. This focus contrasts with our previous work on low impedance control [34]. However, both high and low impedance behaviors are important in human-centered robotics. For instance, high impedance behaviors are important to attain good position tracking in the presence of unmodelled actuator dynamics or external disturbances.

Using the phase margin frequency technique allowed us to reveal the severe effects of delays on the damping loop and appreciate the discrepancy with respect to the stiffness servo behavior. However, to reveal the physical reasons for this discrepancy, in-depth mathematical analysis is performed based on phase margin sensitivity to time delays. This analysis allowed us to derive the physical condition for the discrepancy between delays. Further analysis revealed that the previous condition is met for high impedance controllers, in which the
damping feedback gain is significantly larger than the passive damping actuator value. To confirm the observations and analytical derivations, two experiments are performed by using an actuator and a mobile base. In particular, the results have shown that decoupling stiffness servos to slower centralized processes does not significantly decrease system stability. As such, stiffness servo can be used to implement operational space controllers which require centralized information such as robot models and external sensors.

The next step is to develop a similar study for controllers with an inner torque loop, such as those used for series elastic actuators [34]. For this type of actuators our interest lies in exploring both high and low impedance capabilities (i.e., impedance range) under latencies and using distributed control concepts similar to those explored in this paper. The challenge is that system dynamics become high order instead of second order and more advanced gain selection rules need to be designed for both impedance and torque gains. Some preliminary results are shown in [37].

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