Errata for Book *Operations Research Models and Methods*
Paul A. Jensen and Jonathan F. Bard
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Corrections marked with a (1) superscript were added to the text after the first printing. Other corrections have not been made. Please send corrections or suggestions to one of the authors with addresses at www.ormm.net. Updated versions of the errata are maintained on the web site.

Chapter 2

Pages 42 – 43, car rental problem. The solution is not correct. The correct answer is: M = 1, W = 7, F = 4, MTuW = 2, Weekday = 2, Week = 5. The cost is $2,210.

Ex. 15. The solution to the LP is not integer. There is no guarantee of integrality when the series of 1's in each column of the LP is broken by 0's. An integer solution is obtained by requiring the Solver to give integer answers. This makes the model into an integer program.

On page 36 in 3rd paragraph change "hours" to "minutes".

Chapter 3

Page 61 in the last line should be "*m*-dimensional null vector" instead of "*k*-dimensional null vector"
Page 77 in the last of tables 3.6 (at the bottom of the page). Both instances of "–0.5" in the x_4 column should be positive.
Page 79, Table 3.7. The last basic variable should be x_5 not x_1.
Page 81, Table 3.11. The entry for x_3 in row 2 should be 0 rather than 1.
Page 82, In the middle of the page the reference to Definition 4 should be to Definition 5.
Page 83, Table 3.12. The row for x_5 (row 3) should be "0 1 0 0 0 1" rather than "1 0 0 0 1 1"
Page 83, Table 3.14. The circle should be around the 0.25 in the x_3 column and the arrow should be pointing at the x_3 column not the x_4 column.
Page 83, Table 3.15. x_I should be x_3.

Page 87. The model at the bottom should be

$$\text{Minimize } \hat{w} = \sum_{i \in F} \alpha_i$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ij} x_j + \alpha_i = b_i, \; i = 1,\ldots,m$$

$$x_j \geq 0, j = 1,\ldots,n; \; \alpha_i \geq 0, i = 1,\ldots,m$$

Page 88 in Phase 1 of the Example, write as

$$\begin{array}{llll} \textit{Phase 1:} & \text{Minimize} & w = & +\alpha_1 \quad +\alpha_2 \\ \text{(or} & \text{Maximize} & \hat{w} = & -\alpha_1 \quad -\alpha_2) \\ \textit{Phase 2:} & \text{Maximize} & z = & -7x_1 + 3x_2 \end{array}$$

Page 89, Table 3.22, the 0' row is not correct. The coefficients should be 7 and –3 rather than the other way.
Page 92, Table 3.25. In the Basic column, it should be x_4 and x_5, not x_1 and x_2.

On page 100 (last line) and 102, the table reference should be to Table 3.35 not Table 3.24.
Page 125, Table 4.6. Bases 2, 3, 4, and 5 have typos:
#2 should be x_1, x_{s2}, x_{s3} not x_{s1}, x_{s2}, x_{s3};
#3 should be x_2, x_{s2}, x_{s3} not x_{s2}, x_{s2}, x_{s3};
#4 should be x_1, x_{s1}, x_{s3} not x_{s1}, x_{s1}, x_{s3};
#5 should be x_{s1}, x_2, x_{s3} not x_{s1}, x_{s2}, x_{s3}.

Exercise 20, page 106, should be $x_j \geq 0$, not $0 \leq x_j \leq 1$. [1]

Chapter 4
Exercise 11. The column in the tableau for x_4 should be 0,0,1, rather than 0,1,0.[1]

Exercise 13(g). Change 4th to 3rd

Exercise 13(k). Say that x_3 has an upper bound of 1.5 and x_3= 1.5. The problem is …[1]

Chapter 6
Page 210, top of the page π_4 should equal 28 not 27.

Chapter 7
Page 247. In demand constraint for Days-Off Scheduling problem, the index range should be i = 1,…,7

Page 258, Exercise 11(a). In definition of x_{ij}, change to "… covers the demand for the next j months"

Page 258, Exercise 11(b). Change the inventory variable from z_j to I_j. Also, change the instructions to "Give an MILP formulation for the problem using the new notation."

Exercise 21(a). Should be "… shortest path tree problem …"

Chapter 8
Page 299. In Step 3 of the algorithm, Equation 14 should be Equation 13.

Chapter 9
Page 321. Change Definition 3 to:

Definition 3: A function $f(\mathbf{x})$ is convex on the convex set $S \subseteq \Re^n$ if and only if

$$f(\lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) \le \lambda f(\mathbf{x}_1) + (1-\lambda) f(\mathbf{x}_2)$$

for all $\mathbf{x}_1, \mathbf{x}_2 \in S$ and for all $0 < \lambda < 1$. It is *strictly convex* if the inequality sign $\le$ is replaced with the sign $<$.

Page322. Change *any* to *every* in Definition 5

Page 327. Change solution: If the equality constraint is removed the solution beocmes $\mathbf{x}^*$= (2, sqrt(2)) with $f(\mathbf{x}^*) = 0.586$.

Page 335, Figure 9.13. The coordinates of the center point should be (3, 2.115).

Page 354, Exercise 16. In the equation for $C_i(x_i)$, it should be "for $i = 1,\ldots,6$"

Definition 5: A set $S \subseteq \Re^n$ is convex if <u>every</u> point…

Page 340, middle of page. Replace the sentences starting with the one that begins with "The arc from …" with:
"The arc from node 2 to node 3 allows flow from station B to station E. A nonlinear cost function of the type given in Equation (6) is associated with each station arc. For example, the flow through the arc from node 1 to node 2 is the flow rate entering station B, f_B, where $f_B \equiv \lambda_B$. The cost measure is the average number of units at that station, which can be written as $L_B = f_B/(3 - f_B)$.

Page 348: In the table in the array describing geometric programming, the word polynomial should posynomial both with respect to $f(\mathbf{x})$ and $g_i(\mathbf{x})$

Page 349: posinomial should be spelled posynomial

Exercise 3(a), page 351: last term should be x_2^2

Exercise 9(f): In second summation, change $c(x_j)$ to $c_j(x_j)$

Exercise 11, page 353; rephrase: "Use the definition of convexity and induction to prove Lemma 1."[1]

Exercise 14, page 353; should read

"Following the suggestions in Section 9.3, prove that $f(\mathbf{x})$ is convex if and only if

$$f(\mathbf{x}_1) \geq f(\mathbf{x}_2) + \nabla^T f(\mathbf{x}_2)(\mathbf{x}_1 - \mathbf{x}_2) \text{ for all } \mathbf{x}_1, \mathbf{x}_2 \in S$$

where S is a convex set." [1]

Exercise 16. Replace with the following.

You are in charge of providing labor for a manufacturing shop for the next 6 months. Currently, there are 20 employees in the shop. Each worker costs \$1000 per month and can manufacture five units of product during the month. The cost of hiring and training a new worker is \$500. The cost of laying off a worker is \$1000. At the end of the 6-month period you want 20 persons working in the shop. All products must be sold in the month they are produced.

Demand in each month is a random variable uniform distributed over the ranges specified in the table below. If demand is lower than production capacity, some workers will be idle. If demand is higher than capacity, sales are lost with a penalty of \$200 per item not sold. To compute the expected cost of lost sales, consider month *i* with minimum demand a_t and maximum demand b_t. Say x_t is the capacity for production in month *t*. Then the expected cost of lost sales is

$$C_t(x_t) = \begin{cases} 200(b_t - a_t)/2 + 200(a_t - x_t), \ \text{for } 0 \leq x_t \leq a_t \\ \dfrac{200}{2(b_t - a_t)}(b_t - x_t)^2, \ \text{for } a_t \leq x_t \leq b_t \\ 0, \ \text{for } b_t \leq x_t \end{cases} \qquad t = 1,\ldots,6$$

Set up and solve the NLP to determine how many workers to hire or lay off at the beginning of each month so that the expected cost of lost sales plus the cost of providing the workforce is minimized. Workers may also be laid off at the end of month 6. Assume that sufficient capacity is provided to meet the minimum demand.

Month, t	Minimum demand, a_t	Maximum demand, b_t
1	50	150
2	100	200
3	75	175
4	50	150
5	200	250
6	100	200

Note that the above expression for $C_t(x_t)$ is a piecewise nonlinear function and does not lend itself to an NLP model in continuous variables. Also, it would be better to define capacity in terms of the number of employees. This can be done by letting $5w_t = x_t$, where w_t is the size of the workforce in month t. Next you should derive a continuous expression for the expected cost of lost sales in terms of w_t for the case where $5w_t \le b_t$, $t = 1,\ldots,6$. Because the expression that you derive will not be valid when $5w_t > b_t$, it will be necessary to break w_t into two variables, u_t and v_t, where u_t is the number in the workforce who are idle (i.e., not producing because there is no demand) and v_t is the number in the workforce who are contributing to production.

Exercise 20, page 356; Add the following at the after the values of μ.

Note that from Little's law discussed in Chapter 16, the average time in the system is equal to the average number in system / entering flow rate, where the entering flow rate here is equal to 0.5/min. Therefore minimizing the average time in the system when the entering flow rate is constant is equivalent to minimizing the sum of the average number at each station.

Chapter 10

Page 403; missing parenthesis in equation. Should be

$$f(\mathbf{x}) \cong q(\mathbf{x}) = \ldots$$

Chapter 12

Page 429, Computer Repair example, paragraph 2, sentence 2; change to:
"Computers that fail during the day are picked up the next morning, repaired, and then returned to the office the following morning."

Page 432, definition of s_2 at bottom should be:
s_2 = status of the second machine (working or failed)

Page 433, line 2. Add the following sentence after the first sentence:
"For s_2, we assign a value of 0 if the second machine has not failed and a value of 1 if it has."

Page 433, line 8, change from

"One failure leads to (1,0) and two failures lead to (2,0) ..."
to
"One failure leads to (1,0) and two failures lead to (1,1) ..."

Page 433. In Table 12.3, change the following state definitions:

$\mathbf{s}^1$ = (1,0): One machine has failed and will be in the first day of repair today.

$\mathbf{s}^2$ = (2, 0): One machine has failed and will be the second day of repair today.

$\mathbf{s}^3$ = (1,1): Both machines have failed, one will be in the first day of repair today and the other is waiting.

$\mathbf{s}^4$ = (2,1): Both machines have failed, one will be in the second day of repair today and the other is waiting.

Page 439. In matrix equation for **q** after 2 transitions, should be: **q**(2) = …

Exercise 1. Update the text as follows:
"… Assume that the order is placed just after taking inventory at the end of the week. Cars on order arrive just before inventory is taken so there is always at least one car on the lot …"

Exercise 12. Change problem statement as follows.

"Consider a process in which a single worker must perform five stages of a manufacturing process, as indicated in the figure below. For purposes of analysis, divide time into one-hour segments. When the worker is idle raw material enters the system during the next hour with probability p_A. At stage *i*, the probability of completing the corresponding tasks and moving on to stage *i* + 1 during the current hour is $p_{C(i)}$, *i* = 1,...,5. Assume that no more than one stage can be completed in an hour."

Exercise 16. Hint: For part (c), you must consider two states when there are *k* printers is in the shop, *k* = 1, 2, 3. The first is associated with the repair work that is just starting; call it *repair k_start*. The second is associated with the repair work about to finish; call it *repair k_finish*.

Exercise 26 is meaningless as written. It should be replaced with the following. [1]

26. Heart patients at a local hospital can be found in one of two places: the coronary care unit or in a regular room.

   a. If we assume that the number of heart patients remains constant and that the 1-day transition probabilities are as shown, what are the steady-state probabilities for an individual patient?

One-day transition probabilities — heart patients

	CCU	Hospital rehabilitation	Not hospitalize d
Coronary care unit (CCU)	0.700	0.200	0.100
Hospital rehabilitation	0.050	0.800	0.150
Not hospitalized	0.015	0.005	0.980

   b. Assume persons leaving the hospital from the CCU actually die. For each fatality, a new heart patient enters a competing hospital. There is a 1-day probability of 0.05 that a patient leaves the competing hospital and enters the CCU. How would you change the 1-day transition matrix? Compute steady-state probabilities.

Exercise 29. In table, letter "*n*" should be italic only, not bold. [1]

Chapter 13

Ex. 10(e). The expected cost vector should be: $\mathbf{C} = (1250, 1400, 900, 0)^T$. [1]

Ex. 12(a). Add to the problem statement of part a:

"The stock price is currently $39." [1]

Ex. 13. Labels on parts d and c are switched. [1]

Chapter 14

Page 501. Second **P** matrix is better written as:

$$= \begin{bmatrix} 1-\Delta 2 & \Delta 2 & 0 & 0 & 0 & 0 \\ \Delta 2.5 & 1-\Delta 4.5 & \Delta 2 & 0 & 0 & 0 \\ 0 & \Delta 2.5 & 1-\Delta 4.5 & 2\Delta & 0 & 0 \\ 0 & 0 & \Delta 2.5 & 1-\Delta 4.5 & \Delta 2 & 0 \\ 0 & 0 & 0 & \Delta 2.5 & 1-\Delta 4.5 & \Delta 2 \\ 0 & 0 & 0 & 0 & \Delta 2.5 & 1-\Delta 2.5 \end{bmatrix}$$

Page 503. The proportion of customers who wait; should be: $1 - \pi_0^{\mathrm{P}} - \pi_5^{\mathrm{P}}$ (note error is subscript "5").

Page 504: First heading should be: **Adding ATMs**

Page 506. Paragraph before rate matrix. Should be: $\mu_1 = 1$ and $\mu_2 = 2.5$.
Should be: matrix element $r_{43} = \mu_1 + \mu_2 = 1 + 2.5 = 3.5$.

Page 516, Table 14.9. In summation term, index "*K*" on π should be lower case. [1]

Page 518. Replace last sentence in last paragraph of Section **Transition Probabilities** with:

If event x occurs with rate γ_x, then the rate assigned to the transition would be

$$r_{ij} = \gamma_x p(i, j \mid x)$$

Exercise 14, parts c, d, e: The data in the tables for "Mean Time" should be shifted to the left to line up with the column headers. [1]

Exercise 15, parts a and b: The data in the tables for "Mean Time" should be shifted to the left to line up with the column headers. [1]

Chapter 16

Page 563, in table at bottom for $M/M/2$, change $\rho = \lambda/s\mu$ to $\rho = \lambda/2\mu$. Also, give second formula for L:

$$L = L_q + L_s = L_q + \lambda/\mu = 0.8727^1$$

The corrections for pages 563, 572 and 573 were suggested by R.G. Vickson, University of Waterloo.

Page 563. In the expression for $\Pr\{T_{sys} > t\}$ the term in the inner parentheses becomes indeterminate when $s-1-s\rho=0$. To address this situation, change the sentence before the expression for $\Pr\{T_{sys} > t\}$ to:

Without going through the details, the result for $s-1-s\rho \neq 0$ is

$$\Pr\{T_{sys} > t\} = e^{-\mu t}\left[1 + \frac{(s\rho)^s\pi_0}{s!(1-\rho)}\left(\frac{1-e^{-\mu t(s-1-s\rho)}}{s-1-s\rho}\right)\right],\ t \geq 0 \text{ and } 0 < \rho < 1$$

(add the following)

When $s-1-s\rho=0$, a singularity occurs in the denominator of the inner term in parentheses. To analysis this case, it is more convenient to write the above expression in terms of λ and μ by substituting $\rho = \lambda/s\mu$ everywhere. Now, taking the limit of the fraction in the inner parentheses as $\lambda \rightarrow \mu(s-1)$ gives

$$\Pr\{T_{sys} > t\} = e^{-\mu t}\left[1+\frac{(\lambda/\mu)^s\pi_0}{s!(1-\lambda/s\mu)}(\mu t)\right],\ t \geq 0 \text{ for } \frac{\lambda}{\mu} = s-1$$

where

$$\pi_0 = \left[\sum_{n=0}^{s-1}\frac{(\lambda/\mu)^n}{n!}+\frac{(\lambda/\mu)^s}{s!(1-\lambda/s\mu)}\right]^{-1}$$

in both equations for $\Pr\{T_{sys} > t\}$. Note that when taking the limit, it is necessary to use L'Hospital's rule because the fraction is 0/0 at $s-1-s\rho=0$.

(It may be a good idea to derive the above expression for the case where $s-1-s\rho=0$.)

Page 572-573. The section of Finite Input Source Systems has several errors. The last sentence of the first paragraph of the section should read.
We assume arrivals balk when $n = K$ and $K \leq N$. The results of the section also hold when the maximum number in the system is equal to the population.

The expression for q_n should be:

$$q_n = \frac{(N-n)\pi_n}{N-L-(N-K)\pi_K} \quad \text{for } n = 0, \ldots, K-1,$$

Page 573. In Figure 16.10, the expression for P_B should be:

$$P_B = \sum_{n=s}^{K}\pi_n$$

The expression for the average arrival rate should be

$$\bar{\lambda} = \lambda\left[N - L - \pi_K(N - K)\right] \text{ for } K \le N$$

Page 578. In Table 16.9 for Case 3, should be: $L_q = 5.657$

Exercise 7. Change last sentence in first paragraph to: "Assume that the system is in steady state, and in parts $a - e$,…"

Exercise 7. Change part (b) to:

b. When a motorist is filling up, all other customers must wait on the street. How many spaces for cars should be made available on the station property to assure that there is sufficient room to wait there 85% of the time?

Exercise 15. Remove second occurrence of sentence "The company has two technicians who can … to effect a repair." Also, change "affect" to "effect" in first occurrence. [1]

Exercise 17. Change the service rate to 8 customers per hour for a better problem. [1]

Chapter 18

Exercise 4(d). Use $t = 1$. [1]

Exercise 10(b) and (c). Should ask for 12 replications rather than 10. [1]

Exercise 15. Add the sentence: Simulate the process of passing from system 1 to system 2 with a Bernoulli random variable. [1]

Exercise 21. Should refer to Table 18.19 rather than Table 18.20. Note that Table 18.19 in Chapter 18 is in error as well as Appendix A1 of the simulation chapter. [1]

Table 18.19 is in error. The correct table should be: [1]

Table 18.19 Error as Function of Sample Size for Inventory Simulation

Measures	Demand (D)	Lead time (T_L)
Estimated mean	11.65	2.8
Estimated standard deviation	2.594	0.980
Estimated size (n) for 1% error	3290	8127
Sample size (n) for 5% error	77	188
Sample size (n) for 10% error	14	33

Supplement

The discussion concerning confidence limits in the Simulation supplement distributed on the original edition student disk is in error. It is corrected on the Teach ORMM CD and on the web. The correct confidence limit discussion is below.

Confidence Intervals

Once $\overline{x}$ and the *standard error of the mean*, $\sigma_{\overline{X}}$, are determined, the confidence interval and maximum error for μ_X are given by

$$\mu_x = \overline{x} + z_{\alpha/2}\sigma_{\overline{x}} \qquad \text{(A.6)}$$

or

$$\mu_x = \overline{x} + t_{\alpha/2}\hat{\sigma}_{\overline{x}} \qquad \text{(A.7)}$$

as the case may be, where $\hat{\sigma}_{\overline{X}}$ is the estimated standard error when (A.5) is used in place of σ_X^2 in (A.3) or (A.4). For A.7, the value of $t_{\alpha/2}$ depends on the number of degrees of freedom, *df*, where $df = n - 1$. If σ_X is known, then the maximum error ε for a given level of confidence can be found from

$$\varepsilon = z_{\alpha/2}\sigma_{\overline{x}} \text{ or } z_{\alpha/2}\sigma_x/\sqrt{n}$$

when (A.3) applies. It follows that

$$n = \left(\frac{z_{\alpha/2}\sigma_X}{\varepsilon}\right)^2 \qquad \text{(A.8)}$$

provides the required sample size which satisfies a given maximum error and confidence level.

Error in Probability Supplement

Page 24

The c.d.f. of the Triangular Distribution

$$F(x) = \begin{cases} 0, & \text{for } x \le 0 \\ \dfrac{x^2}{c}, & \text{for } 0 < x \le c \\ \dfrac{x(2-x)-c}{(1-c)}, & \text{for } c < x < 1 \\ 1, & \text{for } x \ge 1 \end{cases}.$$
