Assembly System Design: - A Branch and Cut Approach
Authors: Anulark Pinnoi and Wilbert E Wilhelm, Management Science Vol. 44, No. 1,

This paper addresses the single product assembly design problem (ASDP). ASDP seeks to minimize total cost by optimally integrating design and operating issues. The authors propose an effective branch and cut approach for solving single product ASDPs, adapting inequalities known to be valid for embedded line balancing structures to form inequalities that are valid for the ASDPs. The implementation also involves a specialized preprocessor, a heuristic, separation procedures, and an enumeration scheme.

Global competition, shorter product life cycles, product complexity, and technological innovation—all are driving significant changes in assembly methods. In particular, computer-aided systems provide the flexibility to operate profitably in today's environment of continuous change. The purpose of this paper is to describe an implementation of a cutting plane approach to the assembly system design problem (ASDP). Their formulation of the ASDP prescribes the minimum cost design, integrating design decisions, the number of stations and type of machine located at each station with operating issues such as assigning tasks to each station while observing precedence relationships among tasks and cycle time restrictions at all stations. Cycle time (in seconds/product) is determined from the reciprocal of the production rate (in products/planning period), which is specified to satisfy customer demand, so that 
\[ c = \frac{1}{\text{Production rate}} \times \text{planning period} \]

The ASDP is related to the classical assembly line balancing problem (ALBP), which prescribes the minimum number of (identical) stations, assigning tasks to stations while observing precedence relationships and cycle time restrictions. The ASDP contains the ALBP as an embedded component but also requires that appropriate hardware be specified and deals with a cost-based objective instead of a technological one (i.e., the number of stations). Recently, cutting plane methods have performed successfully in a variety of applications motivating the authors to study underlying polyhedral structures, seeking an effective solution approach for the complex ASDP.

The following assumptions are made in the ASDP problem considered in this paper (A1) Cycle time, processing times, and precedence relationships are known deterministically. (A2) No task processing time is larger than the cycle time. (A3) Task processing times are additive and independent of the task sequence. (A4) A system consists of a series of stations. (A5) Setup time for a task is negligible (or included in task time). (A6) There is no zoning restrictions or other special-case constraints. (A7) Each task must be assigned to one of a specified set of alternative machines. (A8) One machine must be assigned to each activated station. Assumptions (A1)-(A6) are fundamental ones that are typically invoked to structure even the ALBP. Assumption (A7) reflects the fact that machine types must be selected in the ASDP, and assumption (A8) provides a convenient viewpoint for interpreting the model in this paper.

The authors establish that ASDP is NP-hard and that NPP (Node Packing Problem) is a relaxation of the ALB-prototype and use that relationship to identify valid inequalities for the ALBP. They adapt inequalities valid for the ALBP to yield valid inequalities for the ASDP. Their preprocessor attempts to simplify the problem, for example, by determining
if a task can be pre assigned to a particular station and computes values for parameters. Parameter values determine problem size and influence the effectiveness of branch and cut. Since the ASDP is \textit{NP-hard}, a heuristic is used to provide a starting solution. The number of stations in this solution, \( SU \), gives an upper bound on the optimum number of stations and, consequently, the size of the problem, since the number of variables is determined by \( E_t \) the earliest station at which task \( t \) may be processed and \( L_t \) the latest station, and \( L_t \) is a function of \( SU \). A separation problem is used to identify inequalities that are violated by a fractional solution to the current LP relaxation, \( x \). The algorithms for these problems are all polynomial-time procedures: some are optimizing and others (i.e., heuristics) are not guaranteed of identifying an optimal solution to the separation problem. The separation problem associated with clique and clique related inequalities is a maximum weight clique problem in which \( xu \) represents the weight of node \( u \) and \( u = (s, t) \). The maximum weight clique corresponds to a most violated clique inequality. Even though this problem is \textit{NP-hard}, some special cases of clique and clique-related inequalities can be identified efficiently by complete enumeration. By evaluating both sides of the inequalities they search for the most violated inequalities associated with each pair of tasks \( s \) and \( t \). Sequential Lifting is used to strengthen the violated clique inequality. Packing cover inequalities require separation problems that involve knapsack, GUB, and node packing constraints. The authors have devised polynomial separation 20 heuristics patterned after the algorithm which Johnson and Padberg (1981) devised for the knapsack problem with disjoint, special-ordered sets. The branch and cut algorithm relies on the framework provided by the IBM Optimization Subroutine Library (OSL), including preprocessing, branching, and super node processing. The cuts are identified by separation algorithms included in a user exit routine and added to the matrix by the supernode processing routine at nodes throughout the tree. The user may also apply knowledge about the problem to control branching by fixing variables expected to speed up fathoming.

This paper reports the first implementation of cutting planes for solving ASDPs. Computational results demonstrate that branch and cut is a powerful approach for solving ASDPs. The authors solved three sets of test problems with task processing times and precedence relationships taken from line balancing problems that appear in the literature, actual industrial cases, and randomly generated problems. Test problems systematically evaluated the approach relative to four design factors: alternative machines, precedence graph structure, distribution of processing times, and cycle time. Computational results establish benchmarks for the application of cutting plane methods to ASDPs, showing that their approach achieved an order of magnitude improvement in the runtime required to resolve each of the more challenging problems. Dealing with the ASDP through a family of hierarchical models starting with the ALBP allows other, more complex, design problems to be addressed using the solution algorithms presented in this paper. For example, one important case relaxes assumption (A8), allowing identical machines to operate in parallel at a station to process a "long" task (i.e., with processing time greater than cycle time). Several types of improvements could be pursued by future research. For example, the heuristic could be improved. Most importantly, the polyhedral structure of the ASDP may be studied to identify additional families of valid inequalities.