Optimal Scheduling of Trains on a Single Line Track

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This paper describes the development and use of a model designed to optimize train schedule on a single line rail corridor. The model has been developed for two major applications, namely: a decision support tool for trains dispatchers to schedule trains in real time, and a planning tool to evaluate the impact of timetable changes, as well as railroad infrastructure changes. The mathematic programming model is presented to describe the scheduling process over a single line track.

Past Work

Work on an optimum solution to the train scheduling problem origins to the early seventies by Szpigel (1973), who developed a linear programming model to determine the best overtaking and crossing positions given the departure times and upper velocities of the trains. A branch and bound method is used to resolve the conflicts using a lower bound, which relaxes the remaining conflicts. Along this line, Kraft (1987) designed an efficient heuristic algorithm based on the given trains priority and the delay penalties of each train. Mees (1991) and Mills et al (1991) formulated the train scheduling problem as a discrete project scheduling problem with network structure, where the traveling of trains on each section can be viewed as activities. By doing this way, network flow programming algorithm can be used to get a feasible solution.

Model Formulation

This paper models the train scheduling problem as a non-linear mixed integer program, in particular the integer part is solved by an intelligent branch and bound procedure. The problem can be defined as follows. Given Q railroad stations and N trains, and half trains go inbound and half go outbound. Suppose the departure times for each train on the first station are provided, the goal is to find a minimal delay and train operation costs such that each section only can be occupied by (1) one train at each time and (2) the departure safety headway and arrival headway should be satisfied. In this model, only train operation cost is modeled as a non-linear function, the other constrains and delay cost employ linear function form. Moreover, the decision variables are the arrival time and departure time of the study trains at intermediate stations and the final station (terminal). The train delay can be expressed as a function of the arrival time at the terminal.

The model is subject to various constrains in order to ensure safe operation, speed restriction and permit stops. From an integer programming model standpoint, the following and overtaking requirement are represented as logical constraints by introducing binary variables to model two interrelated decisions. Specifically, the
binary variable indicates which train goes first for two conflicting trains. The remaining constraints are straightforward expression for the speed limit and required stop time.

In the bound and bound algorithm, a simple lower bound relaxing the remaining conflict constraints are first tested. Even for 20 trains on 8 stations, which correspond to about 20 conflicts, this lower bound method cannot find the optimum solution in a minimal possible time. Extending from fixed priority method used in Kraft, a new lower bound is presented in this paper. Basically, this bound is intended to assign the priority of trains intelligently based on an estimate of the remaining delay, that is, the estimation of the least cost path for each train is calculated, assuming that the path is independent of accumulation of earlier delay. Due to this independency, the sum of these least cost paths will provide a lower bound to the cost of the actual remaining delay. Even the calculation of this lower bound is a little time-consuming compared to the simple lower bound, the resulting saving in the branching procedure is considerable significant. For example, a problem of 30 trains and 12 stations with 50 conflicts can be solved in real-time. Based on the new lower bound, a number of applications are illustrated, such as evaluating the impact due to changing of station position and introducing new fast freight trains.

**Conclusions**

A new lower bound using the estimate of the remaining overtake and crossing delay is presented in order to reduce the search space in the branch and bound tree. The calculation of the lower bound is shown to be of low order polynomial time. This has allowed the optimal solution to realistic size problems to be found in a reasonable time.