Reviewed on
“Cutting Plane Algorithms For Integer Programming” Paper
by John E. Mitchell

Integer programming is a way to solve an optimization problem with integer variable solutions. One of the integer programming applications is to solve decision-making problems with 1/0 variables to represent on/off decision. Others applications of integer programming are for solving optimization problems where all or some of the variables are integers. Integer programming problems are relatively harder to solve than linear programming problems since it has a tighter constraint (the variable has to be integer). This paper mainly talks about “cutting Plane” algorithm to solve integer programming problem.

Motivation
According to the author, John Mitchell, cutting plane methods were exact algorithms for integer programming problems. The cutting plane methods worked by solving a sequence linear programming relaxations of the original integer programming model, and the relaxations would gradually improve the approximations to the integer programming problem. For hard integer programming problem that could not be solved to optimality, the cutting plane method provided approximations to the optimal solution that guarantees on the distance to optimality in moderate time.

Paper Summary
In this paper Mitchell used a simple problem for example. Mitchell then used this example to explain the cutting plane method proposed by R.E. Gomory. Mitchell explained that even though at first experts felt reluctant towards Gomory’s cutting plane because it was slow to converge to an optimal solution, after the development of polyhedral theory in the eighties, the Gomory’ cutting plane could be useful in solving integer programming problems. Mitchell also explained that Gomory originally derived constraints using the linear programming optimal simplex tableau. The rows of the tableau are combination of the original objective functions and constraints. Using these rows one can generate the cutting planes. The sequences of Gomory’s cutting plane algorithm is solving the linear programming relaxation to optimality, generating a cutting plane from a row of the tableau if necessary, adding the cutting plane as a new constraint to the linear programming relaxation, solving the new linear programming relaxation, and repeating this process until all integral variables were achieved. It can also be noted that Gomory’s cutting plane method could also be used when an interior point method is used to solve the linear programming relaxation since most of the information in simplex tableau could still be obtained easily. Even though the optimal solution appears to
converge slowly, Gomory’s cutting plane algorithm will solve an integer program in a finite number of iterations.

Mitchell also explained Chvatal-Gomory cutting planes. This method weighted combination of the inequalities from linear programming relaxation to generate new valid inequalities. The right hand side of these new inequalities then rounded down (assuming all the inequalities are less than or equal to inequalities) to give valid inequalities for integer programming problem. Mitchell suggested that adding as many Chavatal-Gomory cuts at once would speed up the rate of convergence.

Mitchell then moved to explain strong cutting planes method from polyhedral theory. This method uses integral feasible points as cuts. By definition, a facet is a face of a polytope that has a dimension one less than a dimension of the polytope. Therefore, to have all linear inequality description of the polytope, it needed to have an equality that represented each facet. The idea of this method was creating facets to make a convex hull (polytope) based on linear programming relaxation polyhedral. If complete description of the convex hull for the integer problem was known, then the problem could be solved as linear programming problem by optimizing the linear objective over this convex hull.

**Conclusion**

Mitchell’s paper explains mostly Gomory, Chvatal-Gomory cutting plane method, and strong cutting planes method from polyhedral theory to solve integer programming problem. This paper is suitable only for a reader who has a fair knowledge about cutting plane methods, since Mitchell does not provide thorough explanation in his paper that sometimes can lose the readers in reading his paper. Mitchell in my opinion uses too many reference, and expects the reader to find the information to understand better about what he said in the reference. It helps that Mitchell uses example to visualize the problems. Unfortunately, Mitchell also likes to jump from one subject to another, and then jump back to the first subject (his explanation for Gomory’s cutting plane method for example. Mitchell first starts talking about Gomory cut, then he start talking about Chvatal- Gomory, and then go back to Gomory cut). I found this very uncomfortable and confusing at first.