Homework 1
Chapter 2 in the textbook: Problems 9, 11, 19 and 37

1. In each case there is a cash flow shown on the left. Write the formula for the quantity requested on the right. Use as few time-value-of-money factors in the formula as possible.

Find P

\[ P = [-50(F/P, i, 1) + 50](A/F, i, 2)(P/A, i, 10) \]

Find A. Series of five payments.

\[ A = [300(F/A, i, 3) + 100(P/G, i, 3)(F/P, i, 6)](A/P, i, 5) \]

Find A

\[ A = 100 + 50(A/G, i, 3) \]

Find A. Series Continues forever.

\[ A = [300(F/A, i, 6) - 300(F/P, i, 3)](A/P, i, \text{inf}) \]
2. To purchase your first house you must borrow $100,000 from the bank. The term of the loan is 30 years, and the interest on the loan has a nominal rate of 12% per year. The payments are monthly with the first payment occurring one month after the loan is taken out. Based on these facts, the monthly payment of principal and interest is $1,028.61.

a. How much of the second payment is interest?
   a. For the first payment the interest is 100000*.01 = $1000
   The principal payment is 28.16
   The interest on the second payment is (100000 – 28.16)*0.01 =
   99971.39*0.01 = 999.7139

b. You live in the house for 10 years and make regular monthly payments of $1,028.61. After 120 payments how much do you still owe?
   b. After 120 payments the principal will be:
   $1028.61(P/A, .01, 240) = 93418

   c. What is the effective annual interest rate for this loan?
   c. effective rate = (1 + .01)12 - 1 = 0.126825 or 12.68%

3. A local symphony club offers membership as follows:

   Continuing membership, cost per year $ 60
   Patron lifetime membership $ 400

   In either case you must join and make the first payment today. No additional payments are necessary for the lifetime membership. If the minimum acceptable rate of return is believed to be 16%, how long would it be necessary to live to make the lifetime membership worthwhile?

   The capitalized cost here is not $400 but only $340 now for n equal payments of $60 and we can compute that n no. of years as follows:
   $340 = 60(P/A, 16%, n)
   or (P/A, 4%, n) = 5.67

   From the 16% tables we get n = 16 years. Thus one would go for the lifetime membership if one expects to be active for another 16 years.

4. John wishes to set aside some amount every month to meet the annual maintenance expense of his home. He estimates the maintenance costs for his home to be $1000
for first year and increase by $100 each year for the subsequent years. These costs are assumed to occur at the end of each year.

What is the uniform monthly amount he is to deposit into his savings account if he is to meet annual maintenance costs for the next 5 years? The bank pays a nominal annual interest rate of 12% compounded monthly.

\[ i = 1\% \text{ per month.} \]

Effective annual interest \( i_{\text{eff}} = (1 + .01)^{12} - 1 = 12.68\% \)

Annualizing all costs
\[ A = 1000 + 100 \times (A/G, .1268, 5) \]
\[ = 1176.27 \]

Monthly deposit \[ = 1176.27 \times (A/F, 1\%, 12) = 1176.27 \times (.0788) \]
\[ = \$92.69 \]

Alternatively
\[ P = 1000(P/F, 1\%, 12) + 1100(P/F,1\%,24) + 1200(P/F,1\%,36) \]
\[ + 1300(P/F,1\%,48) + 1400(P/F,1\%,60) \]
\[ A = P(A/P, 1\%, 60) = 4169.39 \times (0.0222) = \$92.56 \]

5. A Christmas savings fund plan is offered by the credit union. The plan is as follows. You will deposit $20 at the beginning of January, $30 at the beginning of February, and $40 at the beginning of March. The payments continue in this fashion growing by $10 in each month. The last payment is at the beginning of November. The credit union pays the nominal interest rate of 6% a year for money deposited in the plan; however, the interest is computed and added to the account each month (the compounding period is one month). Write a formula that can be used to determine the amount that can be withdrawn from the account at the beginning of December. Use as time-value-of-money factors as possible. Be sure to indicate the interest rate used in your formula. Compute the final amount that can be withdrawn.

The monthly equivalent of the plan for beginning of the month payments is:
\[ A = 20 + 10(A/G, i, 11) = 69.50 \]

The future worth at the beginning of November is: \( F_N = A(F/A, i, 11) = 783.94 \)
At the beginning of December the worth is: \( F_D = F_N(F/P, i, 1) = 784.94 \).

The complete formula is then: \( F_D = [20 + 10(A/G, i, 11)](F/A, i, 11)(F/P, i, 1) \)

In all these formulas: \( i = 0.5\% \text{ per month.} \)

6. Capital Metro is considering a light rail system. The system will require an initial investment of $1 billion ($500 million). The annual operating cost of the system will be $50 million, and this cost not depend on the number of riders using the system. The annual revenue depends on the number of riders.

An analysis suggests that the number of riders depends on the ticket price. With a price of $2 per ticket, the daily number of riders will be 80,000. If Capital Metro
offers free fares, the number of riders will rise to 200,000 a day. Assume a linear relationship between the number of riders and price in the price range of 0 to $2.

For this problem, assume there 365 days per year, that the light rail system has a 30-year life, and that the salvage value is zero after 30 years. Use an 12% interest rate for time value of money calculations.

a. What price per ride will maximize the annual revenue for Capital Metro?

\[ R = 200,000 - 120,000\left(\frac{P}{2}\right). \]

Revenue = R*P = (200 – 60P)P = 200P – 60P²

Choose P to maximize the revenue: First derivative: 200 – 120P = 0 or \( P = \frac{100}{60} = \$1.67 \)
Number of riders at this price = 200,000 – 60,000(1.67) = 100,000 per day or 36,500,000 per year

b. What annual profit does the city make with this price?

Daily Revenue = 100,000*1.67 = $167,000
Annual Revenue from fares: 83,000*365 = 60,833,333
Annual Costs = 500(A/P, .12, 30) + 50 = 112,071.
Net profit = -51,237,666.7