

Network Models S.1

Determining Economic Life

Consider an investment in a machine with an initial purchase price of \$1000. The yearly operating costs and salvage value of the machine depend on its age as shown in the table below. We anticipate requiring the use of the machine far into the future. Given that the salvage value is decreasing and operating costs are increasing, there must be some optimal time to replace it. The optimal replacement time is called the economic life of the machine.

Year of ownership	Operating cost during year	Salvage value at year end
1	\$1000	\$500
2	1200	300
3	1500	100
4	1900	0
5	2500	0

Investment analysis recognizes that money spent or earned in the future has less value when viewed from the present. This is called the time value of money principle. We compute the present value of an amount c_n received n years from now as

$$P = \frac{c_n}{(1 + i)^n}$$

The quantity i is a percentage expressed as a decimal, and is variously called the interest rate, discount rate, or minimum acceptable rate of return. The term $1/(1 + i)^n$, is the discount factor. When i is a positive quantity the discount factor is less than 1.

The network representation of decision problems involving time value of money is not obvious. In this section, we describe the problem of determining the *economic life* of an investment. The approach is illustrative of a wide variety of problems that can be handled using network flow programming.

The investment can be viewed as a series of cash flows in time, as illustrated in Fig. 25. Assume we keep the machine for three years. We identify the present as time 0. The operating costs are expended at the end of each year and the salvage value is a revenue received at the end of the life of the machine, three years in this case. Costs are shown as positive in the figure and revenues as negative.

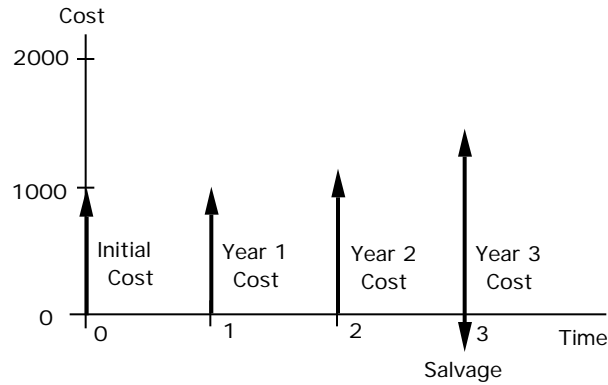


Figure 25. Cash flows for a three year life

To find a general expression for the present value of the cash flow as a function of the time of disposal, let c_n be the cost expended at time n , let N be the life of the machine, and let s_N be the salvage revenue of the machine when we dispose of it at time N . The net present value for one lifetime, P_1 , is then

$$P_1 = \sum_{n=0}^N \frac{c_n}{(1+i)^n} - \frac{s_N}{(1+i)^N}$$

For the specific case of the example and assuming i is 20%, we have

$$P_1 = 1000 + 833.33 + 833.33 + 810.19 = \$3476.85$$

The network representation of this calculation uses gains to model the discount factor, as shown in Fig. 26 for the general case with $N = 3$. The net cash flow in year 3 considering both operating cost and salvage value is denoted by c'_3 . The gain factor, g , is equal to $1/(1+i) = 0.8333$.

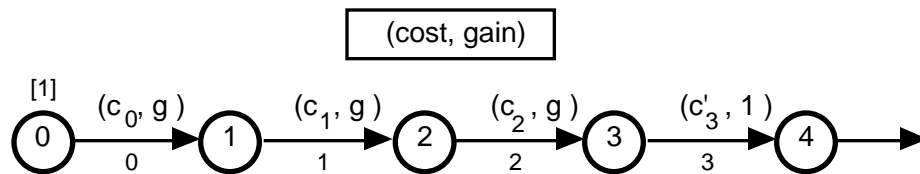


Figure 26. Network model for present value calculation

There is no optimization in the network of Fig. 26. The fixed flow at node 0 is forced on the network resulting in the arc flows:

$$x_0 = 1, x_1 = 0.833, x_2 = 0.833^2, x_3 = 0.833^3, P_1 = 3476.85.$$

The cost associated with the network is the present value of the cash flows for the machine.

The fact that the present value can be computed as the cost of a generalized network allows us to address various decision issues associated with investments. We might ask for the present value of a series of replacements over an infinite period for the cash flows depicted in Fig 27.

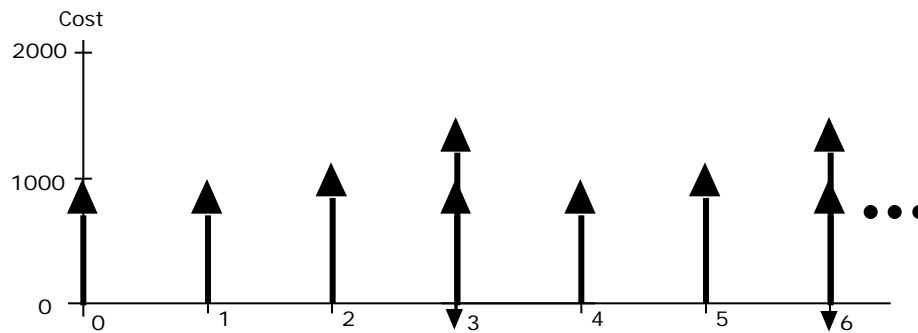


Figure 27. Cash flows for an infinite series of replacements

Without proving this result, we observe that the network model of a continuous series of replacements can be represented by the cycle shown in Fig. 28.

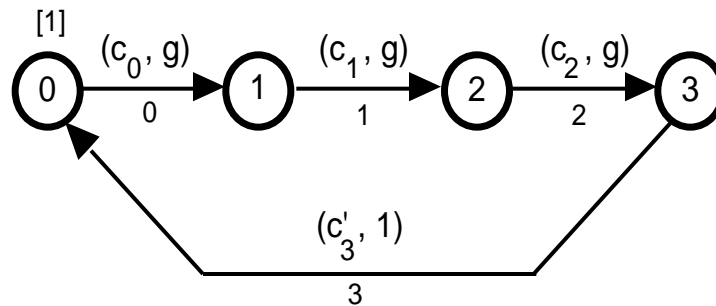


Figure 28. Network model for an infinite series of replacements

The economic life of a machine is that life which minimizes the present value of ownership over an infinite time horizon. To find this value an optimization problem must be solved. The network model for our example is depicted in Fig. 29. There are five cycles or alternative paths representing replacement after years 1 through 5, respectively.

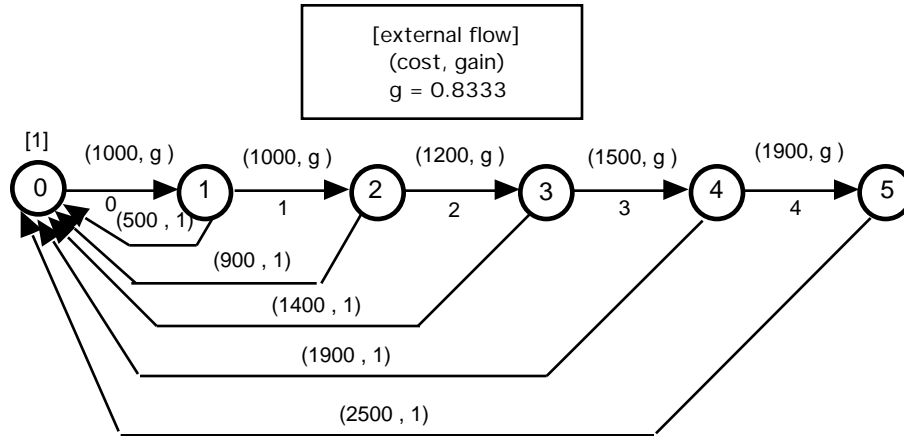


Figure 29. Network model of the economic life problem

The solution to this problem, shown in Fig. 30, has positive flows on only one cycle. The cycle indicates that the machine should be replaced every two years. The cost of the optimum flow solution, \$8045.46, is the present value of the costs incurred by following the optimum policy forever.

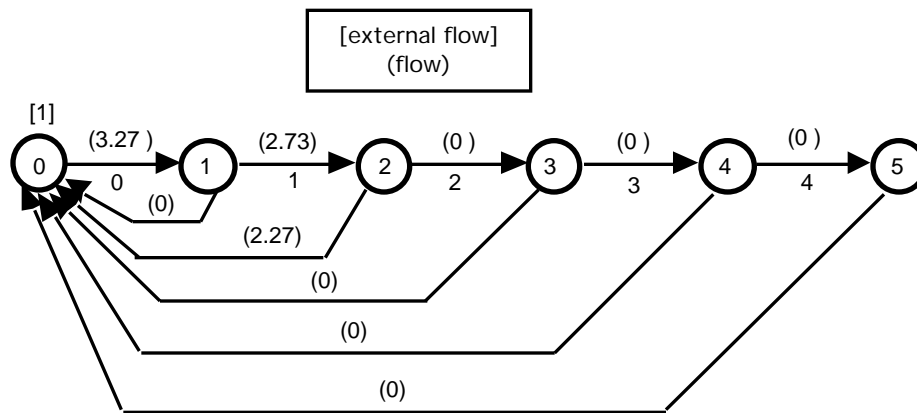


Figure 30. Optimal solution to the machine replacement problem