Cavity dynamics in high-speed water entry

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A method is presented for modeling the cavity formation and collapse induced by high-speed impact and penetration of a rigid projectile into water. The approach proposes that high-speed water-entry is characterized by a cavity that experiences a deep closure prior to closure at the surface. This sequence in the physical events of the induced cavity dynamics is suggested by the most recent high-speed water-entry experimental data, by results from numerical experiments using a hydrocode, and by an understanding of the fundamental physics of the processes that govern surface closure. The analytical model, which specifies the energy transfer for cavity production as equivalent to the energy dissipated by velocity-dependent drag on the projectile, provides accurate estimates for variables that are important in characterizing the cavity dynamics, and reveals useful knowledge regarding magnitudes and trends. In particular, it is found that the time of deep closure is essentially constant and independent of the impact velocity for a given projectile size, while the location of deep closure has a weak dependence on impact velocity. Comparison of these analytical results with experimental results from the literature and with results from numerical simulations verifies the analytical solutions. © 1997 American Institute of Physics. [S1070-6631(97)02903-6]

I. INTRODUCTION

The cavity formed during high-speed water entry by a projectile can exhibit a wide range of dynamical response characteristics, which depend primarily on the type of projectile and its impact velocity. Of particular interest are physical events that occur at various stages of the entry that not only influence the projectile motion but also the nature of induced pressure waves in the late stages of a cavity collapse. By focusing attention on these characteristic measures, namely the closure events, it is possible to evaluate how well analytical models capture the relevant physics of these complex processes.

Cavity formation has been a topic of interest for some time, especially in the area of hydroballistics, and related water entry work can be found dating back to World War II. Most of the work conducted before this period is for very low speeds, whereas some hydroballistics studies, including some conducted recently that were concerned with obtaining design information for ordnance applications, examined impact velocities in the range of hundreds of meters per second. Information on water entry beyond these subsonic speeds is not available in the open literature, even though recent experiments have determined that the ambient conditions at the free surface, and the order in which these events occurs influences the subsequent physical problem, and experimental studies have determined that the ambient conditions at the free surface, the impact velocity, and shape of the projectile are critical parameters. Deep closure, or seal, occurs in very low-speed water entry problems, and was observed as early as 1918 by Mallock.1 This velocity regime, which leads to a deep closure roughly midway between the location of the projectile and the free surface, is classified by Birkhoff and Zaratunelloa as very low-speed. They suggested that this velocity regime be classified by a range of Froude number, 20<Fr<70, where Fr=V^2/D, V is the characteristic length (e.g., the diameter of a sphere).

These regimes are not well-defined, but for roughly Fr>150, classified as low-speed,2 deep closure is preceded by a surface closure. Under these conditions, the cavity formed by the projectile continues to grow after a surface...
closure has occurred. This type of water entry cavity has been the subject of many experimental investigations. The closure mechanisms were studied for low-speed impact by Gilbarg and Anderson. They investigated the dependence of air–water entry cavities on the atmospheric pressure at the free surface for impact velocities between 15 ft/s and 100 ft/s, and for air pressures between 1/60 and 3 atmospheres. They found that a main factor in controlling cavity formation at low-speed entry was the nature of surface closure and that Froude scaling was not useful in parameterizing the cavity phenomena. Gilbarg and Anderson deduced from their experimental results that the product $V_i \cdot T_s$ was constant, where $T_s$ is the time of surface closure following impact of the surface. The value of $T_s$ depends on the parameters of the problem and serves to signal the end and beginning of certain critical events in the life of this type of cavity. A general trend indicated by existing experimental data in the literature, for example, is that increasing impact velocity lowers the time of surface closure.

Modeling the closure mechanisms requires verification of some basic assumptions about the cavity motion. In support of such endeavors, low-speed experiments were conducted by Birkhoff and Caywood, who used a photographic technique (novel at the time) to evaluate velocity fields in water. They studied water entry by missiles and other cavity motion problems and deduced that the motion of the cavity wall is primarily radial or perpendicular to the cavity axis. May investigated the vertical entry of steel spheres, examining the effect of atmospheric density and pressure (above the free surface), the impact velocity, and the size and shape of the missile (projectile). He found that at increasing depths more energy appears to go into cavity production than is lost by the missile. May’s observation was based on a model using a constant drag coefficient, which might be valid for low speeds, and he suggested the difference might be due to the radial motion of the fluid from the cavity wall, as observed by Mallock and by Birkhoff and Caywood in their photographic studies.

Birkhoff and Isaacs obtained a rough theoretical estimation of the time of surface closure. They found that $\rho s V_i T_s / D$ should be approximately constant, where $\rho s$ is the density of air at the free surface. The two major forces leading to the surface closure are an under-pressure caused by the flow of air into the cavity behind the projectile, referred to as the Bernoulli effect, and surface tension. For low-speed water entry, the local under-pressure at the splash neck is commonly estimated by the dynamic pressure term $\frac{1}{2} \rho s V_i^2$. This under-pressure is a dominant factor in early surface closure. After a surface closure, the cavity below the free surface continues to expand due to inertial effects and from energy transfer by the projectile at its instantaneous location. The expansion leads to a drop in the cavity pressure (usually estimated by a isotropic process). Eventually, deep closure occurs as the cavity walls at a certain depth are driven to a closed state by the pressure difference between the surrounding fluid and the cavity. Therefore, the time for deep closure depends on the time at which the surface closure occurred.

In related studies, Abelson focused on measurement of the pressure in the cavity for projectiles entering vertically and obliquely into water. He measured actual cavity underpressures (at two different heights along the cavity) and found that the estimation by $\frac{1}{2} \rho s V_i^2$ was an order of magnitude too low. Abelson assumed that the cavity pressure would drop by 1 atm when the impact velocity of the projectile was greater than 165 m/s based on extrapolation of experimental data he obtained for impact velocities up to 76 m/s. More recent work by Wolfe and Gutierrez has also examined cavity pressures for water entry with velocities in the range of 150 to 300 m/s. The focus of these studies, classified as high-speed, has been to provide information for predicting the cavity dynamics for those cases leading to an early surface closure.

There does not appear to have been any experimental or theoretical studies on the cavity dynamics for very high-speed water entry; that is, beyond 400 m/s and above 1 km/s. A study by Metzger includes analysis of one particular experiment for water entry with $V_i = 356$ m/s, which is classified here as high-speed. This test involved an oblique water entry at an angle of 70° from the surface, and is the highest impact velocity reported for tests conducted by Albert May in the late 1960s. Interestingly, this water entry led to a deep closure rather than a surface closure. While the occurrence of deep closure prior to a surface closure is typically associated with very low-speed water entry, cavities for which deep closure precedes surface closure at higher speeds were observed by Gilbarg and Anderson for cases in which the air density was reduced. In these experiments, the suppression of a surface closure was attributed to an effective reduction in the Bernoulli effect. For high-speed water entry, the pressure drop can not be estimated using the incompressible Bernoulli equation. Those cases where impact velocity is greater than $V_i = 345$ m/s, and classified as high-speed, correspond to a transition regime, and conditions may not induce an accelerated surface closure. Indeed, this velocity corresponds closely to Mach 1 for air flowing into the cavity from the free surface, and the prevailing conditions would induce a choked flow. Consequently, for very high-speed water entries, it will be assumed that this combination of effects will result in the generation of a deep closure cavity before surface closure. First, there will be a relatively high-energy deposition into the fluid that induces large velocities in the cavity wall motion. Second, there will be a choked flow condition that will inhibit early surface closure, and instead promote early deep closure.

Figure 1 contains results that indicate how the time of surface closure depends on impact velocity. The data points in the low-speed range represent experimental measurements made by Gilbarg and Anderson for water entry by a 1-in. sphere. The experimental results are compared with predictions made using a model developed in this paper. For impact velocities less than 20 m/s, the cavity pressure is determined by the pressure drop due to the Bernoulli effect, which was assumed to be $0.5 \rho s V_i^2 \cdot n$, where $n$ is 50. The value of $n$ was chosen based upon the results of Abelson who found that the under-pressure $0.5 \rho s V_i^2$ was an poor approximation and an order of magnitude smaller than measured values. The use of $n = 50$ at low-speeds is obtained from Abelson’s experimental results. For $V_i \geq 165$ m/s (high-speed), the
pressure drop increases to 1 atm, and it is then held constant at 1 atm for all cases based on the assumed choked condition. Results from experiments for high-speed water entry by projectiles indicate the cavity pressure is close to the value for water vapor pressure.$^9$ Consequently, the results in Fig. 1 incorporate the assumption that the cavity pressure is zero for high-speed cases. The trend shown in the transition region from the low-speed region up to 165 m/s and to the high-speed region requires attention. The curve is drawn through this region, however experimental results should be conducted to confirm this predicted trend.

III. THEORY

A. Projectile dynamics

Consider a projectile with an initial velocity $V_i$ penetrating into a fluid along a straight trajectory in the $+z$ direction. For normal penetration of the free-surface of a half-plane, the deceleration from the impact velocity can be described by Newton’s second law,

$$m \frac{d^2 z}{dt^2} = m \frac{dV_p}{dt} = \sum F_i = mg - \frac{1}{2} \rho_w A_0 C_d V_p^2,$$  \hspace{1cm} (1)

where $m$ is the projectile mass, $z$ is the penetration axis, $t$ is time, $V_p$ is the penetration velocity of the projectile, $\rho_w$ is the fluid density, $A_0$ is the projected area of the projectile, $C_d = C_d(V_p)$ is a velocity-dependent drag coefficient, and $g$ is gravitational acceleration. Except for very low-speed water entry, the effect of $g$ will be negligible, and so it will be ignored in the remaining analysis. The rate of change of kinetic energy with respect to depth, $dE_p/dz_b$, can then be expressed as

$$\frac{dE_p}{dz_b} = -m V_p \frac{dV_p}{dz_b} = m \beta V_p^2,$$ \hspace{1cm} (2)

where $E_p = 0.5 m V_p^2$, and $z_b$ is the projectile location. The velocity decay coefficient, $\beta(V_p)$, is defined as

$$\beta = \beta(V_p) = \frac{\rho_w A_0 C_d(V_p)}{2 m}.$$ \hspace{1cm} (3)

Low velocity impact studies typically use a constant drag coefficient. At higher velocities, the drag coefficient is highly dependent on Mach number, $M$, especially in the transonic regime. For spheres, the correlation of the drag coefficient$^{11}$ shows that there are three ranges classified by velocity, roughly “subsonic” up to $M = 0.5$, “transonic” from 0.5 to 1.4, where $C_d = 0.6396 + 0.5974(M - 1)^2 - 0.1618(M - 1)^2 - 0.7212(M - 1)^3$, and “supersonic” beyond $M = 1.4$, where $C_d = 0.7624 + 0.2398(M^{-1} - 1.275)^2 - 0.475(M^{-1} - 1/2.75)^2$.

By numerically integrating Eq. (1), the velocity-dependence of the drag forces can be considered, and the velocity decay and kinetic energy loss of the projectile can be evaluated as functions of the penetration depth, $z$. Using these results, an equation for the cavity dynamics will be determined in the next section.

B. Equation of cavity dynamics

The combined effect of the projectile and the cavity on the fluid motion is approximated by using distributed point sources along the axis of penetration. These sources include a time delay that accounts for the motion of the projectile along its computed trajectory [via integration of Eq. (1)]. The source strength at any point along this trajectory is determined by employing energy conservation, where it is assumed that the kinetic energy loss of the projectile equals the total energy (kinetic plus potential) in a fluid section,$^2$ as illustrated in Fig. 2. Internal energy changes due to thermal effects in the fluid are assumed negligible in this analysis. This model is based on the work of Lundstrom,$^{12}$ who drew upon the basic cavity model of Birkhoff and Zarantonello,$^2$ to model the pressure field generated by a subsonic tumbling projectile in fuel tanks. The analysis presented here and in a subsequent section extends this approach to describe the dynamics of the water entry cavity, with special attention given to water entry at very high-speeds. Consequently, the methodology is useful, in principle, for estimating the cavity dynamics for any impact velocity and for any shape of the projectile (with suitable accounting for the drag coefficients).

The radial and axial velocity components are found by solving the linearized potential flow equation for distributed point sources. The solutions obtained by Lee$^{13}$ involve complex integral equations that must be solved numerically. However, on the wall of the cavity the radial velocity has a very simple form which is proportional to the source strength, $\xi$, and is given by

$$u = \frac{2}{w} \cdot \xi(\xi, t),$$ \hspace{1cm} (4)

where $w$ is the radial distance, $\xi$ is the source strength, $t$ is time, and $\xi$ is the penetration index, a variable that measures the distance along the trajectory. The kinetic energy, $E_w$, of the fluid in the section within a finite radius, $\Omega$, is then
If the cavity wall velocity is denoted as $\dot{a}$, then the kinematic boundary condition at the cavity wall is $u_{wn}=\dot{a}$. Therefore, the approximate velocity from potential theory [Eq. (4)] and the kinematic boundary condition together imply

$$\zeta = \frac{1}{2} a(z) \dot{a}(z).$$

Combining Eqs. (9) and (10), an equation for the cavity dynamics is determined as

$$a(z) \dot{a}(z) = \pm B(z) \sqrt{[A(z)]^2 - [a(z)]^2}.$$  \hspace{1cm} (11)

Integration of Eq. (11) with boundary condition $a=0$ at the time of projectile arrival, $t_b$, yields

$$\pm \sqrt{[A(z)]^2 - [a(z)]^2} - [A(z)] - [B(z)](t-t_b).$$

To account for the finite diameter of a projectile, $a=D/2$ instead of $a=0$, can be used for the integration of Eq. (11), if necessary. Equation (12) specifies the radial motion of the cavity as a function of time at any depth, $z$. In the next section, this method is extended to study the cavity dynamics in detail.

IV. EXTENSION OF THEORY AND RESULTS

A. Cavity radius

In this section, the previous analysis is used to describe the cavity dynamics. From Eq. (12), an analytical expression for the cavity radius can be derived as

$$a(z) = \sqrt{[A(z)]^2 - [A(z) - B(z)(t-t_b)]^2}, \hspace{1cm} t > t_b,$$

which is valid at any depth, $z$. This equation describes the cavity formation as a projectile reaches the depth, $z$. The collapse of the cavity wall at any depth begins after it reaches the maximum allowed radius. Note that a modified form of Eq. (13) can also be derived by changing the boundary condition from $a=0$ to $a=D/2$ for the integration of Eq. (11).

Figure 3 shows the evolution of the cavity formed in water at different times after an impact at the surface by a projectile. The results reported in this paper with an impact velocity of $V_i=0.5$ km/s. The analytical model estimates that deep closure will occur at a depth of 58 cm and between 38 ms and 40 ms following the surface impact. After this initial closure, the cavity continues to close in the immediate vicinity both above and below the location of deep closure. However, at
z = 0, as shown in the figure, the cavity continues to increase in size due to the large amount of energy initially transferred during impact.

The cavity reaches a maximum radius when the velocity of the cavity wall becomes zero. From Eq. (11), we find the maximum radius, $w_m$, along the depth by setting $a$ to zero to yield

$$w_m = A(z) = \sqrt{\frac{1}{\pi P_g} \frac{dE_p}{dz}} = \sqrt{\frac{\rho_s C_d A_0 V_p^2}{2 \pi P_g}} \approx V_p. \quad (14)$$

Note that the maximum cavity radius is proportional to the local penetration velocity, assuming $C_d = \text{const}$ and ignoring the variation at $P_g$ with depth. Equation (14) considers that the total energy contained in the fluid when the maximum cavity radius is reached is potential energy alone; this energy subsequently acts to initiate the cavity collapse. Such a scenario represents a dynamical energy conversion process between the kinetic energy imparted by the projectile and potential energy stored in cavity formation.

B. Cavity wall velocity

The cavity wall velocity as a function of depth and time is given by

$$\frac{da}{dt} = \frac{B(z)[A(z) - B(z)(t-t_b)]}{\sqrt[A(z)]{A(z)^2 - [A(z) - B(z)(t-t_b)]^2}}, \quad \text{for } t > t_b,$$

which results from differentiating Eq. (13) with respect to time. From this formulation, the predicted cavity wall velocity can be compared to measurements from an experimental study as shown in Fig. 4(a). This plot displays cavity velocity at a depth of 22 in. for vertical water entry by a 1-in. sphere that has impacted the free surface at $V_i = 73$ ft/s. Although this is not a high-speed impact, it represents the only data set found in the literature permitting this comparison. The analytical prediction is contrasted with the experimental results of May6 reproduced in Fig. 4(b). In May’s experiments, the air pressure was reduced to 1/27 of an atmosphere in order to prevent surface closure. Note that a negative cavity wall velocity corresponds to a collapse stage for the cavity. As a consequence of neglecting dissipative effects in the analytical cavity formation model, Eq. (15) has an inherent symmetry in periods for cavity formation and collapse at any depth. Note, however, that the experimental results show that at a depth near deep closure the collapsing period is larger than the formation period by about 25%. This asymmetry is attributed to losses during the cavity formation where some energy is converted into both pressure waves and dissipated in the form of heat.

Figure 5 contains plots of the velocity of the cavity wall for various depths as predicted by Eq. (15). There is a decrease in the period for cavity formation and collapse as depth increases because cavities formed at relatively larger depths receive less energy from the projectile than those formed at shallower locations. In other words, the kinetic energy transfer at deeper locations is significantly reduced.

It is appropriate, at this point, to recall the assumption made in the analytical model that water is to be treated as an incompressible fluid. This assumption is valid insofar as the radial velocity of the cavity wall is small compared to the speed of sound in water. Note that even though a large cavity wall velocity may occur near the wall surface, the assumption of incompressibility is quite reasonable since this level
is only maintained for a relatively short period of the cavity formation process. Note: Actually, the incompressible assumption is valid up to $V_i = 700$ m/s.

V. NUMERICAL SIMULATION

A. Description of hydrocode

Before presenting analytical results for deep closure, the results presented thus far will be compared to those from numerical simulations using an impact and penetration hydrocode. Hydrocodes are computer programs that numerically simulate nonlinear fluid–structure interaction, particularly physical problems that include shock generation and propagation. Commercially available hydrocodes have become quite efficient, and in this study we used AUTODYN-2D to study the complex physical phenomena that arises in high-speed water entry.

The conservation equations of mass, momentum, and energy, are coupled with specified constitutive laws and equations of state to arrive at a direct numerical solution of the (fully nonlinear) problem. Commonly an Eulerian mesh is generated to model the target, which is water in this case, and a Lagrangian mesh is used for the projectile. This mesh generation technique is useful for solid and fluid interaction problems. The numerical mesh moves and distorts in the Lagrangian scheme as the material flows, and so it has the advantage of easily tracking the material interface. A disadvantage is that large deformation of the mesh requires rezoning and erosion algorithms to continue the calculation. With an Euler grid, the material flows through a fixed mesh, so there will be no numerical mesh deformation. The disadvantage of this scheme is the advection error associated with multiple materials in a cell.

In this study a two-dimensional axisymmetric calculation was performed for normal water entry. The projectiles were modeled by specifying a relatively high yield strength so deformation would not occur. The water was modeled as a fluid with no strength, with the equation of state (EOS) following a Mie–Gruneisen approximation. The parameters of the EOS for water used in the numerical simulation are summarized in Table I.

B. Verification of hydrocode

A difficulty faced in the study of very high-speed water entry problems is that experimental results are scarce or not available in the open literature. Indeed, a motivation for the present study was a need to develop numerical and analytical tools that would guide modern, large-scale experiments. It is necessary to refer to studies conducted during the 1940s, 1950s, and 1960s, which reveal some critical characteristics about the water entry problem. Specifically, useful information about the cavity formation and the shock stand-off distance can be inferred from the work of McMillen et al. Conduction experiments in which the pressure fields generated by high-speed projectiles fired into water were measured. A shadowgraph technique was used to...
measure the pressure field surrounding and propagating from a sphere as well as the instantaneous velocity of a sphere as it penetrated (normally) into a water target.

Numerical experiments were conducted to simulate the physical experiments reported by McMillen et al. Figure 6 summarizes the results from these numerical simulations along with the shadowgraphs measured by McMillen et al. Two useful bases for comparison of these results are the shock stand-off (SO) distance at a point in the time history of the water entry and the instantaneous velocity (V). These measures are contrasted in Fig. 6, and the numerical results are found to compare very well with those measured and reported by McMillen et al.

A study was also conducted to evaluate the applicability of the drag coefficients from Charters and Thomas, which were determined for spheres moving in air, to the water entry problem. This evaluation compared the projectile motion computed using Eq. (1) (with a velocity dependent drag coefficient from Charters’ and Thomas’ correlation) to the motion predicted by the hydrocode, where there decay is governed by a simulation of the governing equations taking into account the material properties of the water and the impact parameters. That is, the projectile dynamics predicted by the hydrocode are dependent on fluid forces that are the integrated effect of the pressure field computed in a direct simulation of the Navier–Stokes equations. The results shown in Fig. 7 show that, for high-speed water entry, the drag from a simulation is comparable to that predicted using the Charters and Thomas correlation.

C. Contrasting analytical and numerical results

Figure 8 summarizes results from a numerical simulation of the same high-speed entry problem studied using the analytical model. There is an overall agreement with the analytical predictions shown in Fig. 3. In particular, the numerical simulation predicts a deep closure, thus supporting the initial assumption in the analytical model that a surface closure would not precede the deep closure of the cavity. The results in Fig. 8 do indicate, however, that the location of deep closure is about 48 cm below the undisturbed free surface—a value less than that predicted by the analytical model. This difference in the two results can be explained by identifying that a portion of the total kinetic energy lost from the projectile produces a splash on the free surface in the hydrocode.

<table>
<thead>
<tr>
<th>TABLE I. Water EOS parameters.</th>
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<tbody>
<tr>
<td>$\rho_o = 999$ kg/m$^3$</td>
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<tr>
<td>$c = 1480$ m/s</td>
</tr>
<tr>
<td>$s = 1.75$</td>
</tr>
<tr>
<td>$G = 0.28$</td>
</tr>
<tr>
<td>$C_v = 4.5$ kJ/kg·K</td>
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<tr>
<td>$U_s = c + s \cdot u_p$, where</td>
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<td>$u_p$ = particle velocity</td>
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<table>
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<tr>
<th>TABLE II. Results of deep closure produced by a 2 cm sphere.</th>
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<tr>
<td>$V_i$ (km/s)</td>
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<tr>
<td>$T_d$ (ms)</td>
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<tr>
<td>$z_d$ (cm)</td>
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<tr>
<td>$V_d$ (m/s)</td>
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<td>$t_d / T_d$</td>
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FIG. 6. Comparison of pressure fields generated during very high-speed water entry by a sphere. Data shown gauge the validity of results from numerical simulation using AUTODYN in (a) and (c) against experimental results in (b) and (d) as reported by McMillen et al. Cases (a) and (b) are for subsonic impact by a sphere at $V_i = 1.3$ km/s, and cases (c) and (d) are for supersonic impact by a sphere at $V_i = 1.8$ km/s.

FIG. 7. A comparison of the velocity decay as found from a numerical simulation using the hydrocode versus Eq. (1) with a drag coefficient dependent on Mach number (values from Charters and Thomas).

FIG. 8. Results from a numerical simulation of the same high-speed entry problem studied using the analytical model. There is an overall agreement with the analytical predictions shown in Fig. 3. In particular, the numerical simulation predicts a deep closure, thus supporting the initial assumption in the analytical model that a surface closure would not precede the deep closure of the cavity. The results in Fig. 8 do indicate, however, that the location of deep closure is about 48 cm below the undisturbed free surface—a value less than that predicted by the analytical model. This difference in the two results can be explained by identifying that a portion of the total kinetic energy lost from the projectile produces a splash on the free surface in the hydrocode.
A. Time and location of deep closure

The closures produce closed cavities whose subsequent collapse can be a significant source of pressure waves. In the class of high-speed water entry problems currently under discussion, the deep closure signals the end of the cavity formation, and a description of this process where many bubbles are generated. The splash effect was not included in the analytical model, so the effect of increasing the surrounding pressure will not be evident in the predictions.

VI. DEEP CLOSURE ANALYSIS

The late stage of the cavity dynamics can involve a succession of closures along the cavity length, leading to a "pinching" process where many bubbles are generated. The closures produce closed cavities (or bubbles) whose subsequent collapse can be a significant source of pressure waves. In the class of high-speed water entry problems currently under discussion, the deep closure signals the end of the cavity formation, and a description of this first deep closure is examined in this section. The time and location of deep closure are determined by a direct numerical solution of Eq. (1). In addition, scaling laws for these parameters are developed using an approximation based on a constant $\beta$ value.

A. Time and location of deep closure

If it is assumed that the cavity begins to collapse at a particular depth when the opening process of the cavity is complete, the condition $a=A$ in Eq. (12) will yield an estimation for the cavity collapse initiation time at that depth, $T_c(z)$, given by

$$T_c(z) = t_b(z) + \frac{A(z)}{B(z)},$$

where $t_b(z)$ is the time it takes for the projectile to travel to $z$, which is determined by integrating Eq. (1). The ratio $A(z)/B(z)$ is the cavity opening time (the time the cavity to reach the maximum radius from zero), where the values of $A(z)$ and $B(z)$ are found from the numerical solution of Eq. (1) along with Eq. (2).

One difficulty in determining and confirming an estimate for the cavity collapse completion time is that there is not much information available on the cavity collapse process, especially for very high-speed water entry. As a first approximation, it is assumed that the cavity collapse period is equal to the cavity formation period, even though this is not entirely realistic. Based on this assumption, however, an estimate for the cavity collapse completion time, $T_c$, along the depth of penetration is obtained from Eq. (16),

$$T_c = t_b(z) + 2\cdot \frac{A}{B}.$$  \hfill (17)

Now, the time of deep closure, $T_{d}$, is defined as the minimum value for the cavity collapse completion time along the entire depth of penetration. The solution of the projectile dynamics using Eq. (1), along with Eq. (17), leads to a numerical assessment of the time of deep closure. After the first deep closure, the cavity is divided into two parts: one is an open cavity from the location of deep closure to the free surface, the other is a closed cavity that surrounds the sphere, as shown in Fig. 3 or Fig. 8.

Figure 9 shows the time of deep closure for spheres impacting water at high speed as predicted by the method described. It is of interest to note that the time of deep closure as estimated by Eq. (17) is essentially independent of the impact velocity. This invariance in the time of deep closure was observed in the low-speed experiments of Gilbarg and Anderson and May, under conditions where the pressure in the cavity is about the same as that at the free surface ($P_c = P_{\text{atm}}$). By reducing the pressure above the free surface, the deep closure event can be made to occur for low-speed water entry before an early surface closure.

The numerical simulations predict that the time of deep closure will slightly decrease with impact velocity. This effect can be attributed to the increase in hydrostatic pressure from splash at the free surface, which would increase the pressure in the fluid surrounding the cavity and induce an early deep closure. It should be pointed out that 5 days of computational time were typically required for each water entry simulation, while the analytical predictions were computed in 30 s on an IBM RS/6000.

Plots of the velocity decay of a 2.0 cm sphere after water entry at different impact velocities are given in Fig. 10, with the results obtained from direct integration of Eq. (1) and taking into account the velocity-dependence in the drag coefficient. After about 5 ms following impact, each case shows that the projectile velocity, $V_p$, approaches the same value regardless of the impact velocity. Therefore, the cavity formation and collapse periods show a weak dependence on the initial impact velocity.
The ratio of the traveling time to the location of deep closure, \( t_d \), to the time of deep closure, \( T_d \), was computed for spheres of different diameter using the analytical model. Figure 11 indicates the invariance of this ratio with respect to sphere diameter and impact velocity. For different impact velocities, the predicted values for \( z_d \) and \( V_d \) (the velocity at deep closure) are summarized along with \( t_d \) and \( T_d \) in Fig. 6. The time of deep closure, \( T_d \), is essentially independent of the impact velocity, and the location of deep closure, \( z_d \), increases with impact velocity. The ratio of the traveling time to the location of deep closure, \( t_d \), to the time of deep closure, \( T_d \), is almost constant.

In Fig. 12, the location of deep closure is plotted against the impact velocity of a sphere. This graph contrasts analytical predictions with results from hydrocode numerical simulations. The depth of deep closure, \( z_d \), is found to be a strong function of sphere diameter and a weak function of impact velocity. This trend was observed in the low-speed experiments by Gilbarg and Anderson and May. The values of \( z_d \) predicted by the hydrocode numerical simulations are less than those predicted from the analytical model. Some of this error can be attributed to the fact that the depth values were estimated from the undisturbed free-surface in the hydrocode results, thereby ignoring the splash. An alternative method for estimating these values from these numerical results would reduce this discrepancy.

B. Scaling phenomena

Approximate scaling laws for high-speed water entry problems can be obtained by considering a constant velocity decay coefficient, \( \beta = \rho_s A_d C_d / 2 m_p \). Using this approximation, closed form expressions are derived for the time and
location of deep closure. Using the results for cavity dynamics with constant \( \beta \), and then rewriting Eq. (17) as a function of penetration depth we find
\[
T_i = \frac{\alpha^2 - 1}{\beta V_i} + \frac{2}{P_g} \sqrt{\frac{\rho_w N}{\pi}} \frac{m \beta \cdot V_i e^{-\beta z}}. \tag{18}
\]

The location where the deep closure occurs can be estimated from Eq. (18) by minimizing \( T_i \) with respect to \( z \) such that
\[
z_d = \frac{1}{2 \beta} \ln \left[ \frac{2}{P_g} \beta \sqrt{\frac{\rho_w N}{\pi}} m \beta \cdot V_i^2 \right]. \tag{19}
\]

Now the time of deep closure is determined by substituting Eq. (19) into Eq. (18) to yield
\[
T_d = \frac{\alpha V_i - 1}{\beta V_i} + \frac{\alpha^2 / \beta}{\alpha} = \frac{1}{\beta} \left[ 2 \alpha - 1 \right], \tag{20}
\]
where
\[
\alpha^2 = \frac{2 \beta}{P_g} \sqrt{\frac{\rho_w N}{\pi}} m \beta.
\]

Through further reduction, Eq. (20) becomes
\[
T_d = C_0 \left( \frac{P_0 - P_{atm}}{P_0 - P_c} \right)^{1/2} \frac{D}{Z_d} \frac{1}{\bar{C}_1 \bar{F}^{1/2}}, \tag{21}
\]
where \( C_0 = (96N/27C_\rho)^{1/3}, \bar{\rho} = \rho_w / \rho_p \), where \( \rho_p \) is the projectile density, and \( P_g = P_0 - P_c \), where \( P_0 = P_{atm} + \rho_w g z_d \). This equation can be rewritten in nondimensional form as
\[
\left( \frac{g}{D} \right) \frac{1}{T_d} = C_0 \left( \frac{P_0 - P_{atm}}{P_0 - P_c} \right)^{1/2} \left( \frac{D}{z_d} \right)^{1/2} - \frac{C_1}{\bar{F}^{1/2}}, \tag{22}
\]
with the Froude number, \( \bar{F} = V_i / \sqrt{gD} \), and \( C_1 = 1/3 \bar{\rho} C_d \).

For very low-speed impacts, \( P_c \approx P_{atm} \) and we find
\[
\left( \frac{g}{D} \right) \frac{1}{T_d} = C_0 \left( \frac{D}{z_d} \right)^{1/2} - \frac{C_1}{\bar{F}^{1/2}}. \tag{23}
\]

On the other hand, for very high-speed impacts \( P_c \approx 0 \), \( \bar{F} \gg 1 \), and we obtain
\[
\left( \frac{g}{D} \right) \frac{1}{T_d} = C_0 \left( \frac{P_0 - P_{atm}}{P_0} \right)^{1/2} \left( \frac{D}{z_d} \right)^{1/2}, \tag{24}
\]
If the impact kinetic energy is relatively small such that the closure event occurs near the surface (as in the problems studied here), then
\[
\left( \frac{g}{D} \right) \frac{1}{T_d} = C_0 \left( \frac{\rho_w g z_d}{P_{atm}} \right)^{1/2} \left( \frac{D}{z_d} \right)^{1/2}, \tag{25}
\]
or
\[
T_d \frac{P_{atm}}{\rho_w}^{1/2} = C_0 = \text{const.}
\]

The results given in Fig. 13 were determined from the analytical model (with velocity-dependent \( \beta \)) and indicate good scaling of \( T_d \) with \( D \). A similar scaling of \( z_d \) from Eq. (19) (not shown here) yields the results plotted in Fig. 14, indicating a weak dependence on impact velocity.

VII. CONCLUSIONS

An analytical model for the cavity dynamics has been developed by using empirical velocity-dependent drag coefficients to describe the projectile dynamics. The energy dissipated in drag was used to approximate the energy expended in cavity production. Although we have restricted our discussion to the case of high-speed impact by a sphere, the model presented here can be extended for any body velocity or shape. The applicability of these results to quantifying the collapse pressure waves generated by high-speed water entry is discussed in a companion paper currently under preparation.\(^{17}\) Based on the results of the study reported in this paper, the following conclusions can be drawn

1) The time of deep closure is almost constant and independent of the impact velocity for a given sphere diameter, a result reflected in scaling laws developed in this paper.

![FIG. 13. Ratio of the time of deep closure (T_d) to sphere diameter (D) versus impact velocity (V_i). Results shown are for a sphere with diameter (a) 1.0 cm, (b) 1.5 cm, and (c) 2.0 cm.](image)
(2) The location of deep closure is a weak function of impact velocity.
(3) Froude scaling is not valid for high-speed impact. On the other hand, \( T_d/D \) and \( z_d/D \) are found to be nearly constant and independent of the impact velocity.
(4) The analytical predictions compare well with experimental measurements reported in the literature and with numerical simulations reported in this paper.

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