

PhD Qualifier - Optimization Syllabus

- Unconstrained Optimization
 - Optimality Conditions
 - first-order, second-order necessary
 - sufficiency under convexity
 - Algorithms for Univariate Optimization
 - bisection, Newton, safeguarded Newton, golden section search, Fibonacci
 - rates of convergence
 - Algorithms for Multivariate Optimization
 - gradient methods: $x^{k+1} = x^k + \alpha^k d^k$
 - important special case: $x^{k+1} = x^k - \alpha^k D^k \nabla^T f(x^k)$
 - choosing direction: steepest descent, Newton, quasi-Newton, conjugate gradient
 - choosing the steplength: Armijo, linesearch
 - the gradient-related condition and the general convergence theorem
 - rates of convergence
- Constrained Optimization Over a “Simple” Convex Set
 - Optimality Conditions
 - QP with Equality Constraints
 - Feasible Direction Methods
 - Frank-Wolfe Algorithm
 - Gradient Projection
 - Active Set Methods ($Ax \geq b$)
- Lagrange Multiplier Theory and Duality
 - Lagrange Multipliers
 - equality constraints and need for regularity condition
 - inequality constraints: KKT conditions and sufficiency for convex programs
 - Lagrangian Duality
 - weak duality and its implications
 - strong duality for convex programs
 - connections with LP duality
 - Saddle Point Conditions
- Barrier Methods and Penalty/Augmented Lagrangian Methods
 - Barrier Methods
 - barrier functions
 - convergence
 - log-barrier for linear programming
 - Penalty/Augmented Lagrangian Methods

penalty weight and multipliers
convergence (as penalty grows)
inequality constraints
method of multipliers

- Special Topics
 - Cutting-Plane Algorithms
 - convergence of Kelly's cutting-plane method
 - Nonsmooth Optimization
 - optimality conditions
 - subgradient and bundle methods
- Integer Programming
 - Modeling and Formulation
 - Good (Tight) Formulations
 - Optimality, Relaxation, and Bounds
 - Duality, Weak Duality, Strong Duality
 - LP Relaxation, Total Unimodularity
 - Branch and Bound
 - Implicit Enumeration
 - 0-1 Enumeration
 - LP Relaxation Based Branch and Bound
 - Cutting Plane Algorithms
 - Valid Inequalities
 - Gomory's Fractional Cutting Plane Algorithm
 - Mixed Integer Cuts
 - Polyhedral Theory and Strong Valid Inequalities
 - Strong Valid Inequalities
 - Polyhedra, Faces, and Facets
 - Convex Hulls
 - Mixed 0-1 Inequalities
 - Branch and Cut Algorithm
 - Lagrangian Duality
 - Theory of Lagrangian Relaxation
 - Lagrangian Relaxation, Integrality Property, and LP Bounds
 - Polyhedral Interpretation of Lagrangian Relaxation
 - Strength of Lagrangian Relaxation Bound
 - Choosing the Right Relaxation
 - Solving the Lagrangian Dual
 - Subgradient Optimization
 - Lagrangian Heuristics
 - Feasibility Restoration and Variable Fixing

References

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