OPTIMIZING MILITARY AIRLIFT

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We describe a large-scale linear programming model for optimizing strategic (intercontinental) airlift capability. The model routes cargo and passengers through a specified transportation network with a given fleet of aircraft subject to many physical and policy constraints. The time-dynamic model captures a significant number of the important aspects of an airlift system in a large-scale military deployment, including aerial refueling, tactical (intracontinental) aircraft shuttles, and constraints based on crew availability. The model is designed to provide insight into issues associated with designing and operating an airlift system. We describe analyses for the U.S. Air Force system concerning fleet modernization and concerning the allocation of resources that affect the processing capacity of airfields.

I n a large-scale military deployment, massive amounts of equipment and large numbers of personnel must be transported over long distances in a short amount of time. Airlift, sealift, and ground transportation assets are all used to execute such a deployment. In the Persian Gulf War, sealift moved 85% of the dry cargo, but the first ships did not arrive for weeks. Strategic (intercontinental) airlift played the dominant role in rapidly moving troops and cargo in the important weeks leading up to the war. (See Lund 1993 for more details of strategic airlift in the Persian Gulf War.) In this paper, we describe a linear programming model that is primarily focused on the airlift system, and we indicate some of the insights it has provided U.S. Air Force (USAF) planners.

Application of optimization modeling to problems in transportation has a rich history. Ferguson and Dantzig (1955) describe a linear programming model for assigning aircraft to routes. In a linear programming model for tanker routing, Dantzig and Fulkerson (1954) introduce the notion of a time-space network. This time-space construct plays a key role in many time-dynamic optimization models (see, for example, Potts and Oliver 1972) including the one developed in this paper. Optimization methods have been successfully applied to a variety of problems in the commercial airline industry; see, for example, Teodorovic (1988) and Yu and Thengvall (1999). The nature of the planning problems faced by decision makers in the civilian airlines and military airlift differ substantially. For example, military airlift requirements are largely driven by infrequent events that can be of enormous magnitude, while the airlines face demands over time that are considerably less variable. Airlines can choose which markets to serve and have the freedom to determine flight frequency while airlift planners do not have this type of control, but in many instances, military planners have greater control over the transportation network infrastructure. A number of simulation-based military mobility models are reviewed in Schank et al. (1991), yet there is a dearth of literature on military airlift optimization. The optimization model we describe in this paper routes cargo and troops through a

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transportation network with a given aircraft fleet, subject to numerous physical and policy constraints. It is not the purpose of the model to provide operational flight schedule recommendations. Instead the purpose is to provide insight into tactical and strategic issues concerning the airlift system. Some of the issues that our model has been used to examine include allocating resources that govern the processing capacity of airfields, examining which route options are best for specific aircraft and war scenarios, assessing the relative performance of different mixes of aircraft types, evaluating investment (or divestment) decisions in airfields, recommending fleet modernization strategies, and studying roles and concepts of operations for aerial refueling aircraft.

Our model is the result of a joint effort between research teams (then) at the Naval Postgraduate School (NPS) and the RAND Corporation; it is called NPS/RAND Mobility Optimizer (NRMO). As implied by its name, NRMO has a distinct lineage from each of its namesakes. NRMO was developed from two previous models at NPS and a model at RAND, drawing on their best features and learning from their shortcomings. Figure 1 overviews the NRMO predecessors.

Optimization of airlift mobility at NPS began with the Mobility Optimization Model (MOM), a project for the Joint Staff’s Force Structure Resource and Assessment Directorate (see Wing et al. 1991). MOM is a time-dynamic model that includes both airlift and sealift assets, but has a single-channel topology, and hence is not designed to capture the airlift system’s transportation network. THRUPUT is a time-static strategic airlift model on a general routing network that was developed by Yost (1994) at the U.S. Air Force Studies and Analyses Agency (AFSAA) in the Pentagon. Then, AFSAA desired a time-dynamic model with the ability to route aircraft through a general network and asked NPS to combine the features of MOM and THRUPUT in one model. This request led to THRUPUT II, first described in an NPS master’s thesis (Lim 1994), and then extended in Morton et al. (1996). An ongoing relationship between AFSAA and NPS led to several M.S. theses that examined stochastic airlift models (Goggins 1995), route generation techniques (Turker 1995), route prioritization (Toy 1996), and aggregation schemes (Fuller 1996). THRUPUT II also served as a real-world test problem for the development of a solution methodology for large-scale staircase linear programs, described in Baker (1997) and Baker and Rosenthal (1998).

In 1995, a team of students and faculty at NPS, guided by military analysts at AFSAA, used THRUPUT II to help decide whether to buy the (then) McDonnell-Douglas C-17, or a freighter version of the Boeing 747 as the next-generation USAF airlifter. Ultimately, the C-17 was chosen. Our analysis supported this conclusion: The modified 747 fleet performed slightly better than the C-17 fleets, because the 747s carry more bulk and oversized cargo at a longer range, freeing the C-5s to deliver almost exclusively outsized cargo. However, this improvement was deemed insufficient to overcome the C-17’s advantage with respect to combat flexibility. Furthermore, the analysis demonstrated a need for modeling in-theater and air-refueling operations, as well as a need for a more parsimonious model formulation that could do similar analyses with fewer variables and constraints (Rosenthal et al. 1997).

In parallel with the THRUPUT II modeling efforts at NPS, a group at RAND developed a similar model called Concept of Operations (CONOP) with RAND’s Project Air Force funding. The CONOP model captures many details not incorporated in THRUPUT II: aerial refueling, flow balance and utilization constraints for air crews, options for direct delivery versus delivering cargo that is subsequently transshipped by in-theater aircraft, and optional in-theater recovery bases, where aircraft may receive services and crew changes. On the other hand, CONOP does not offer sufficient resolution with respect to ownership (the associated military unit) of the cargo being delivered.

Killingsworth et al. (1994) used CONOP to conduct an investigation of the utility of aerial refueling tanker aircraft within the strategic air mobility system. The study found that using air refueling to support a large deployment results in some additional costs, but often substantially shortens the overall time to complete the delivery. Aerial refueling increases cargo throughput most during the earliest stages of the deployment, before the capacity of the enroute system has been expanded. Finally, the study concluded that the difference in marginal costs between aerial-refueling and traditional cargo-hauling concepts of operation is greatest for smaller cargo movements. However, these types of movements are often characterized by urgency and the desire for quick delivery, and also have lower costs that may make them affordable under the circumstances.

**Figure 1.** This figure summarizes the genealogy of the models that led to NRMO. For each model we indicate the developers and the relative merits labeled as “+” (advantages) and “−” (disadvantages).
CONOP was also used to support the C-17 Tactical Utility Analysis study, conducted by the Office of the Secretary of Defense, Program Analysis and Evaluation (Killingsworth and Melody 1995). Unlike older strategic airlift aircraft, the C-17 can operate out of smaller and more austere airfields, permitting their intra-theater use. The C-17 can also be used to deliver strategic loads closer to the battlefield. CONOP found that some level of C-17 presence in-theater was usually recommended, and that direct delivery to small airfields was generally preferred. As with the THRUPUT II analysis, the robust capability of the C-17 justified its procurement as the next-generation USAF airlifter.

CONOP’s ability to examine alternative delivery strategies and THRUPUT II's ability to track cargo ownership were merged with the development of NRMO. In addition to incorporating all the important capabilities of these previous models, NRMO provides for input commonality with existing models used by USAF analysts. Legacy optimizations of airlift were either limited to modeling airlift without real-world complexities such as aerial refueling and transshipment, or did not capture the unit level of detail required by senior decision makers. Previous models were also hampered by the data-gathering morass associated with any large model; NRMO interfaces very easily with the 15,000-line input files of existing models at the USAF’s Air Mobility Command.

In this paper, we give an overview of NRMO in §1, and then give a detailed mathematical description of the linear programming model in §2. NRMO has been designed to provide insight on several types of mobility questions concerning investment (or divestment) in airfield infrastructure, selection of airlift aircraft for acquisition, and the best use of dual-role aircraft. Several studies have been performed and some of the most important of these are discussed in §3.

### 1. PROBLEM STATEMENT

AND MODEL OVERVIEW

Our goal is to move equipment and personnel, in a timely fashion, from a number of origin bases through a transportation network to destination bases using a fleet of aircraft with differing characteristics. Such a deployment is driven by the movement requirements specified in the Time-Phased Force Deployment Data (TPFDD). The TPFDD contains a highly detailed list of cargo and troops that are required by contingency plans for a given theater of operation (Armed Forces Staff College 1993). For our purposes, the movement requirements may be viewed as a list of requirement line identifications, or *line ids*, each of which specifies the associated cargo's onload and offload bases, the day it is first available to move, the day by which it must be delivered, the number of short tons (stons) in each of three cargo classes called bulk, oversized, and outsized, and the number of passengers. Bulk cargo is palletized on 88 × 108 inch platforms. Oversized cargo is non-palletized rolling stock, and is larger than bulk. Outsized cargo is also nonpalletized and represents the largest cargo class. The TPFDD also specifies the cargo’s organizational unit type (helicopter squadron, tank company, etc.) from which we derive an approximate loading efficiency for each aircraft type. For example, ammunition is dense cargo that can be loaded with little wasted space, but helicopters are large, light, and irregularly shaped, thus they use cargo space less efficiently. In addition, some types of cargo can be loaded more efficiently on certain aircraft. Finally, after arriving at the offload base some line ids require subsequent delivery to a forward operating location, and in such cases, the TPFDD also lists the associated forward operating base. (Some aircraft can bypass the offload base and deliver cargo and troops directly to a forward location.)

Table 1 contains a small portion of a representative TPFDD that has been modified for use in NRMO.

<table>
<thead>
<tr>
<th>Line ID</th>
<th>Onload</th>
<th>Offload</th>
<th>Forward</th>
<th>Load Required</th>
<th>Bulk</th>
<th>Oversized</th>
<th>Outsized</th>
<th>Passengers</th>
<th>Owner Type</th>
</tr>
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<tbody>
<tr>
<td>UNIT1486</td>
<td>KBLV</td>
<td>RKPS</td>
<td>10</td>
<td>24</td>
<td>292</td>
<td>1,009</td>
<td>59</td>
<td>0</td>
<td>9</td>
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<td>UNIT1487</td>
<td>KWRI</td>
<td>RKPS</td>
<td>10</td>
<td>24</td>
<td>116</td>
<td>349</td>
<td>35</td>
<td>0</td>
<td>23</td>
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<td>UNIT1488</td>
<td>KTIK</td>
<td>RKPS</td>
<td>10</td>
<td>24</td>
<td>6</td>
<td>9</td>
<td>2</td>
<td>0</td>
<td>2</td>
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<tr>
<td>UNIT1489</td>
<td>KTCM</td>
<td>RKPK</td>
<td>RKTD</td>
<td>10</td>
<td>24</td>
<td>0</td>
<td>15</td>
<td>0</td>
<td>5</td>
</tr>
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<td>RKPS</td>
<td>10</td>
<td>24</td>
<td>0</td>
<td>41</td>
<td>85</td>
<td>0</td>
<td>5</td>
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<tr>
<td>UNIT1491</td>
<td>PAEI</td>
<td>RKPS</td>
<td>11</td>
<td>24</td>
<td>0</td>
<td>110</td>
<td>5</td>
<td>19</td>
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</tr>
<tr>
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<td>KSUU</td>
<td>RKPK</td>
<td>RKPP</td>
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<td>24</td>
<td>133</td>
<td>32</td>
<td>7</td>
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</tr>
<tr>
<td>UNIT1493</td>
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<td>RKPS</td>
<td>11</td>
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<td>29</td>
<td>764</td>
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<td>18</td>
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<td>UNIT1494</td>
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<td>RKPS</td>
<td>11</td>
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<td>63</td>
<td>182</td>
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<td>0</td>
<td>18</td>
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<tr>
<td>UNIT1495</td>
<td>KSUU</td>
<td>RKTY</td>
<td>RKPP</td>
<td>11</td>
<td>25</td>
<td>634</td>
<td>562</td>
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<tr>
<td>UNIT1496</td>
<td>KDOV</td>
<td>RKJK</td>
<td>RKPP</td>
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<td>25</td>
<td>220</td>
<td>208</td>
<td>212</td>
<td>0</td>
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<tr>
<td>UNIT1497</td>
<td>KTIK</td>
<td>RKSO</td>
<td>RKSG</td>
<td>11</td>
<td>25</td>
<td>0</td>
<td>190</td>
<td>83</td>
<td>402</td>
</tr>
<tr>
<td>UNIT1498</td>
<td>KHOP</td>
<td>RODN</td>
<td>PHIK</td>
<td>11</td>
<td>25</td>
<td>47</td>
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<td>PHIK</td>
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<td>UNIT1500</td>
<td>KOFF</td>
<td>PHIK</td>
<td>11</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>814</td>
<td>2</td>
</tr>
</tbody>
</table>
Aircraft used or considered by the USAF for strategic airlift have widely differing capabilities. Some are variants of civilian aircraft; others are of pure military design. Note that aircraft ranges are stated for the given capacity, but the model actually uses a more detailed range-payload curve (Figure 2). The table’s data is from Air Mobility Command (1997).

<table>
<thead>
<tr>
<th>Name</th>
<th>Manufacturer</th>
<th>Capacity (stons)</th>
<th>Range (nm)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-5A/B</td>
<td>Lockheed</td>
<td>110</td>
<td>2,400</td>
<td>Outsized Cargo</td>
</tr>
<tr>
<td>C-17A</td>
<td>Boeing and McDonnell-Douglas</td>
<td>86</td>
<td>2,250</td>
<td>Outsized Cargo</td>
</tr>
<tr>
<td>C-141B</td>
<td>Lockheed</td>
<td>37</td>
<td>2,500</td>
<td>Oversized Cargo</td>
</tr>
<tr>
<td>747F</td>
<td>Boeing</td>
<td>122</td>
<td>4,500</td>
<td>Oversized Cargo</td>
</tr>
<tr>
<td>KC-10</td>
<td>McDonnell-Douglas</td>
<td>45</td>
<td>5,000</td>
<td>Military DC-10 Air Refueler</td>
</tr>
<tr>
<td>KC-135</td>
<td>Boeing</td>
<td>28</td>
<td>5,500</td>
<td>Various 707 modifications</td>
</tr>
</tbody>
</table>

The fleet of aircraft consists of several types of planes, and the fleet mix varies depending on the type of analysis being done. A brief summary of some of the fleet’s aircraft is provided in Table 2. Aircraft differ in what they can carry, where they can go, and what sort of functions they can perform. The C-5 and C-17 can carry all cargo types as well as troops, while the C-141 can carry troops, bulk, and oversized cargo. Tanker aircraft such as the KC-10 and KC-135 serve as aerial refuelers, but can also function as strategic lifters, carrying bulk cargo and troops. Variants of the Boeing 747 can carry bulk and oversized cargo, or passengers. The C-17 can fly into austere environments that are inaccessible by other strategic airlifters. Each aircraft is also defined by a number of characteristics, including airspeed, average (maximum) flying hours per day, cargo- and passenger-carrying capacity, a range-payload curve, service times and airfield capacity consumed at onload, enroute, and offload bases, and aerial refueling capability. A range-payload curve specifies the maximum payload that an aircraft can carry given the desired range. Range-payload curves for four representative airlift aircraft are shown in Figure 2.

Figure 2. These range-payload curves for four airlift aircraft indicate the maximum payload (stons) that an aircraft can carry when flying a given number of nautical miles. The piecewise linear curves are constructed by using linear interpolation between specified points.
Figure 3. This figure depicts a number of the key components of the airlift transportation network including aerial ports of embarkation (APOEs), enroute bases, aerial ports of debarkation (APODs), forward operating bases (FOBs), in-theater shuttle beddown operations, and aerial refueling operations.

Aircraft, such as a KC-10, may begin the deployment as an aerial refueler based at a tanker beddown airfield. After the initial days of the deployment, the KC-10 may then fly to an APOE, change roles, and begin delivering cargo and troops to the theater. Role changes of this type may occur throughout the deployment both between the strategic airlifter fleet and the tanker fleet and between the strategic airlifter fleet and the tactical in-theater shuttle fleet. This type of flexibility is attractive because the goal of some studies is to investigate the value of dual-role aircraft, i.e., aircraft that can serve as aerial refuelers and strategic lifters or aircraft that can serve as both tactical in-theater shuttles and strategic lifters.

Each type of aircraft has its own set of crews, and crews are not interchangeable between different aircraft fleets. Flow balance constraints are maintained for crews at a subset of the airfields called crew stage bases. Stage bases are placed along routes so as not to violate crew duty days; typically, crews are assured 12 hours rest after at most 16 hours on duty. When performing tanker or shuttle missions the crews stay with their planes. Crews within the strategic fleet are allowed to “deadhead,” i.e., they can move from one crew-stage base to another with an appropriate time delay before they become available to fly. Crews need rest and so there are more crews than aircraft; a typical ratio is 3 to 1.

The primary decision variables in NRMO specify the number of aircraft missions for each line id, for each aircraft type, via each eligible route, in each time period. These include direct delivery and transshipment missions on standard and quickturn routes. Separate decision variables track the time-dynamic delivery of the number of short tons of each line id’s equipment in each cargo class. Additional variables account for backchannel missions, in-theater shuttle flights, and the use of tankers to perform aerial refueling. A set of inventory variables specify the number of aircraft of each type at each base acting in the strategic role as well as the shuttle and tanker roles, and another group of variables allow appropriate aircraft to move between these roles. Finally, decision variables also account for the number of rested crews for each aircraft type at each stage base and the number of crews deadheading between stage bases in each period. Based on information from the TPFDD, each line id has a time window in which delivery is permitted. The objective is to minimize a weighted sum of penalties for late and nondelivery plus secondary terms that measure system performance. The model’s constraints can be grouped into seven categories that govern demand satisfaction, flow balance of aircraft, cargo, and crews, aircraft delivery capacity for cargo and passengers, the number of shuttle and tanker missions per period, initial allocations of aircraft and crews, the usage of aircraft of each type, and aircraft handling capacity at airfields.

To better facilitate understanding of the mathematical model presented in the next section, we give additional...
details on some modeling techniques we use. When tanker aircraft change roles they do so between an APOE and a tanker beddown base. We do not model each possible movement because there are a large number of such pairs. Instead all tanker reassignments are made through a fictitious central tanker control point that we refer to as the tanker “cloud.” Approximate time delays for changing roles (and geographical locale) are incorporated in travel times to the cloud. Reassignments as a tanker aircraft from one tanker beddown base to another are also modeled by traveling through the tanker cloud. Not all aerial refueling attempts are successful, and in NRMO, a specified fraction of aerial refueling missions are penalized with a time delay and consume airfield capacity at a so-called divert base.

In order to reduce the number of flow balance constraints at airfields in the theaters, we perform an aggregation. Each theater has one fictitious centrally-located “super node.” Flow balance of strategic airlifters arriving to a theater is maintained at its super node and not at each individual APOE and FOB. Surprisingly little resolution is lost by using this modeling construct. While routes terminate in the network at a super node, decision variables for delivering cargo are indexed by the line id the aircraft is delivering and each line id has a known destination. As a result, we can enforce airfield capacity constraints at each destination and transshipment airbase in the theater, even though aircraft balances are maintained only at the aggregate super nodes. In practice, the airfields in a theater tend to be in close proximity so little is lost by not maintaining precise travel times. This aggregation is performed only for APOEs and FOBs in the theater. Flow balance is maintained at each APOE and enroute base.

2. MATHEMATICAL FORMULATION

The following sections describe the sets, data, decision variables, and finally the mathematical formulation for NRMO. The mathematical model is relatively complex, and as a result there is substantial detail in the presentation. However, a basic understanding is very accessible by first examining the decision variables and then the constraints, referring to the data definitions as needed. That said, much of the effort in formulating such a model concerns the elimination of inadmissible combinations of indices. For example, using \( i \) for line id, \( a \) for aircraft type, \( r \) for route, and \( t \) for time, \( X_{a i r} \) denotes the number of aircraft of type \( a \) directly delivering line id \( i \) on route \( r \) departing at time \( t \). In order for the \((i, a, r, t)\)-tuple to exist for \( X_{a i r} \),

- Route \( r \) must be a direct delivery route with the correct origin and destination for line id \( i \).
- Aircraft of type \( a \) must be able to fly the critical (longest) leg on route \( r \) with a certain minimal payload.
- The start time \( t \) of the mission must be after line id \( i \)’s available-to-load date.
- The delivery time, given that the mission starts at \( t \), must be on or before the required delivery date.

- Aircraft \( a \) must be capable of carrying some cargo type (bulk, oversized, outsized, or passengers) that line id \( i \) contains.
- Aircraft of type \( a \) must be available by time \( t \) at \( r \)’s origin.

This is just one example of the restrictions on allowable combinations of the indices. In the mathematical formulation, such restrictions are captured by defining appropriate subsets for indices. For example, we use \( R_{a i r} \) to denote the subset of direct delivery routes that can be flown by aircraft of type \( a \) carrying cargo and/or passengers for line id \( i \). Because correctly restricting such combinations of indices is so essential for formulating a correct and computationally tractable model, we have decided to present such index restrictions via subsets in some detail. NRMO is implemented with the algebraic modeling language GAMS (Brooke et al. 1992) and solved with the CPLEX software (CPLEX 1993). The screening of index combinations in GAMS is accomplished with restriction operations that correspond closely with the sets defined below.

Like most mathematical programming formulations of continuous-time systems, NRMO uses an approximation based on discrete time periods. The length of a period in this discretization is typically 12 or 24 hours. Decision variables governing aircraft missions, such as \( X_{a i r} \), discussed above, are defined using these discrete time periods. However, parts of the model incorporate time at a finer level of detail. For example, \( r_{i r} \) denotes the time (in units of periods) that it takes an aircraft of type \( a \) to travel from the origin to the destination of route \( r \). And, \( r_{i r} \) is simply \( r_{i r} \), rounded to the nearest integer. In the model, if an aircraft of type \( a \) begins flying route \( r \) in period \( t \) then it arrives at its destination in the integer period \( t + r_{i r} \). However, NRMO also contains a constraint that keeps track of “aircraft-hours” consumed by using the real-valued travel time, \( r_{i r} \). Similarly, within the discrete-time model aircraft utilize airfield capacity for fractions of a period, and the required delivery dates for cargo and troops need not be in integer periods.

2.1. Sets

Sets of Time Periods

\[ T = \text{all time periods } \{1, 2, \ldots, |T|\}. \]

\[ TW_i = \text{delivery time window for line id } i; \text{ a subset of contiguous periods from } T. \]

\[ T_u = \text{set of time periods associated with a utilization rate enforcement block, each block is typically 20 days, with a 10 day overlap between adjacent blocks.} \]

\[ U = \text{utilization rate enforcement blocks.} \]

\[ FT = \text{flow time periods } \{1, \ldots, \text{maximum mission time}\}. \]

Sets of Line Ids

\[ I = \text{all line ids for delivery (from the TPFDD).} \]

\[ I_{i, b} = \text{subset of line ids whose destination is an FOB.} \]

\[ I_{b, i, r} = \text{subset of line ids that have base } b \text{ (FOB or APOD) as a destination.} \]
\(I_{b,rrn}\) = subset of line ids that have APOD \(b\) as a transshipment node.
\(I_{b,imp}\) = subset of line ids that are to be delivered to theater (super node) \(b\).

**Sets of Cargo Types**

\(C = \) all cargo types \{bulk, oversized, outsized, pax (troops)\}.  
\(CC = \) cargo types excluding passengers \{bulk, oversized, outsized\}.  
\(C_a = \) subset of cargo types that can be carried by aircraft \(a\).

**Sets of Aircraft Types**

\(A = \) all aircraft types.  
\(A_c = \) subset of aircraft types that can carry cargo type \(c\).  
\(A_{pas} = \) subset of aircraft types that can carry troops.  
\(A_{mix} = \) subset of aircraft types that can carry troops and at least one other cargo type \{bulk, oversized, or outsized\}.  
\(A_{kb} = \) subset of tanker aircraft types.  
\(A_{rv} = \) subset of aircraft types that can be refueled by a tanker.  
\(A_{chp} = \) subset of aircraft types that can serve as shuttles and hence be “chopped” to the theater.

**Sets of Bases**

\(B = \) all real and virtual bases (APOEs, APODs, FOBs, super nodes, enroute bases, beddown bases, and aerial refueling points).  
\(B_{sup} = \) subset of bases that are super nodes.  
\(B_r = \) subset of bases that are embarkation nodes.  
\(B_{arp} = \) subset of bases that are aerial refueling points.  
\(B_{kb} = \) subset of bases that are beddown bases for tankers.  
\(BS_{rec} = \) subset of super nodes that have at least one recovery base.  
\(BS_{b,den} = \) subset of super nodes that have \(b\) as the shuttle beddown node.  
\(BA_{b,tkr} = \) subset of \(B_{arp}\) that are served by \(b \in B_{tkr}\).  
\(BT_{b,arp} = \) subset of \(B_{tkr}\) that serve \(b \in B_{arp}\).  
\(B_{crw} = \) subset of bases that serve as crew stage bases.

**Sets of Routes**

\(R = \) routes.  
\(RD = \) delivery routes.  
\(RB = \) backchannel routes.  
\(RB_{rec} = \) subset of backchannel routes that include a recovery base.  
\(RD_{b} = \) delivery routes that use base \(b\).  
\(R_{b,ori} = \) routes whose origin is base \(b\).  
\(R_{b,dst} = \) routes whose destination is base \(b\).  
\(RD_{ia,div} = \) subset of routes that can be flown by \(a\) and carry \(i\) for direct delivery.  
\(RD_{ia,rrn} = \) subset of routes that can be flown by \(a\) and carry \(i\) for transshipment.  
\(RB_{ab} = \) subset of backchannel routes that use \(b\) and can be flown by \(a\).  
\(RD_{b,div} = \) set of delivery routes that have \(b\) as a divert base for a failed aerial refueling attempt.  
\(RB_{b,div} = \) set of backchannel routes that have \(b\) as a divert base for a failed aerial refueling attempt.

While there are a large number of sets and subsets, we try to use revealing naming conventions. For example, \(I_{b,dst}\) is the subset of line ids that have base \(b\) as their destination, and \(RB_{ab}\) is the set of backchannel routes that can be flown by aircraft of type \(a\) and use base \(b\).

### 2.2. Data

**Travel Time Data**

\(hrsper = \) number of hours per period.  
\(rtrv_{ar} = \) actual travel time (ground times included) for aircraft \(a\) to traverse route \(r\) (periods).  
\(trv_{ar} = \) travel time (ground times included) for aircraft \(a\) to traverse route \(r\) (integer periods).  
\(etrv_{ab} = \) travel time for aircraft \(a\) to reach base \(b\) when flying route \(r\) (integer periods).  
\(rtrv_{ab} = \) tanker \(a\)’s real travel time from base \(b\) (either embarkation or tanker beddown) to the tanker cloud (periods).  
\(trv_{ab} = \) rounded \(rtrv_{ab}\) (integer periods).  
\(etrv_{ab} = \) \(trv_{ar}\) plus crew rest (integer periods).  
\(ctrv_{ab} = \) \(etrv_{ab}\) plus crew rest (integer periods).  
\(dhtrv_{arb} = \) travel time for crew deadheading from \(b'\) to \(b\) (integer periods).  
\(grv_{ab} = \) in-theater ground travel time for \(i\) (periods).  
\(msntime_{arf} = \) time flown \(f\) periods into a mission (hours).  
\(rtrv_{ab} > f\) (mission continues throughout its \(f\)th period).  
\(0 \) if \(rtrv_{ab} < f - 1\) (mission terminates before its \(f\)th period).  
\(hrsper \cdot (rtrv_{ab} - (f - 1))\) if \(f - 1 \leq rtrv_{ab} \leq f\) (mission terminates during its \(f\)th period).

**Demand-Related Data**

\(rdl = \) required delivery date (periods) for line id \(i\).  
\(dem_{ic} = \) demand for line id \(i\) of type \(c\) (stons for bulk, oversized, and outsized cargo; number of passengers for pax).  
\(latelypen = \) late delivery penalty for \(i\) per day per ston.  
\(nogopen = \) non-delivery penalty for \(i\) per ston.  

**Data Related to Airfield Capacity and Its Consumption**

\(gtime_{ab} = \) ground time for aircraft \(a\) at base \(b\) when flying route \(r\) (hours).  
\(otime_{ab} = \) offload time only for aircraft of type \(a\) at base \(b\) when flying quickturn route \(r\) (hours).  
\(sotime_{ab} = \) ground time for shuttle aircraft \(a\) at base \(b\) (hours).  
\(acpkg_{ab} = \) airfield capacity service slots consumed by aircraft \(a\) at base \(b\).
that can be loaded with troops. The smallest airfield capacity at the bases along a route limit the aircraft flow along that route. (This value is more restrictive than the capacity of the route’s airspace separation requirements.)
nog = airfield capacity service slot hours per period at b. The smallest airfield capacity at the bases along a route limit the aircraft flow along that route. (This value is more restrictive than the capacity of the route’s airspace separation requirements.)

data = airfield capacity efficiency at b.

Data Related to Aircraft Capacity

\( \text{purecap}_{ab} = \) number of stons of line id i’s cargo of type c that can be loaded on plane a for a flight of approximately 3,200nm (i.e., a loading efficiency).

maxpax = maximum number of troops that can be loaded on an aircraft of type a.

rangefac = proportion of a type a aircraft available for loading when flying route r for line id i. This parameter is derived from the longest leg of route r, combined with the range-payload characteristics of aircraft a, and the loading efficiency of line id i. If the payload of aircraft a on route r is less than purecap (using a weighted average of cargo types c for the specified i) then rangefac < 1.

paxfrac = proportion of a type a aircraft’s capacity that can be loaded with troops.

fuelgals = gallons of fuel available per period at base b.

develops = fuel required by aircraft a at base b when flying route r.

daysfuel = daily fuel required by shuttle or tanker aircraft at base b.

Data Associated with Aerial Refueling

\( \text{tkreqs}_{ab} = \) proportion of a full tanker’s fuel consumed by aircraft a refueling at aerial refueling point b on route r (KC-10 equivalent).

\( \text{tkrprop}_{ab} = \) proportion of a full tanker (KC-10 equivalent) available when a is a refueling at aerial refueling point b’ and is bedded at base b.

\( \text{tkrrate}_{ab} = \) maximum number of tanker shuttles to aerial refueling point b’ per period for a tanker of type a when it is bedded at b.

dept = proportion of aerial refueling attempts by aircraft a (the one getting the fuel) that fail.

Intratheater Shuttle Data

\( \text{intchop}_{ab} = \) initial number of aircraft of type a assigned to shuttle duty in theater (super node) b.

\( \text{shutrate}_{ab} = \) maximum number of shuttles per aircraft of type a per period when carrying line id i’s cargo.

Aircraft Utilization Data

\( \text{urate}_{ab} = \) number of hours per day that aircraft a can fly.

fittime = in-flight time only for aircraft type a on route r, f periods into a mission (hours).

tkrtime = in-flight time for tanker a flying from b to b’ and back (hours).

shuttime = in-flight shuttle time for aircraft type a carrying line id i (hours).

Other Data and Notation

\( \text{restrew}_{ab} = \) small unit reward for resting aircraft a at base b ∈ B.

\( \text{usepen}_{ab} = \) usage penalty for theater aircraft and tanker reassignments.

\( \text{newac}_{at} = \) number of new aircraft of type a available in period t.

\( \text{cumac}_{at} = \) cumulative aircraft available of type a by period t (\( \sum_{t < t} \text{newac}_{at} \)).

\( \text{dhpen}_{ab} = \) penalty for deadheading crews.

\( \text{crewrat}_{ab} = \) ratio of available crews to aircraft a.

\( \text{J}(\cdot) = \) indicator function; 1 if argument is true and 0 otherwise.

\( R^{+} = \) positive part operator; = max[0, x].

\( \overline{S} = \) complement of set S.

\( R \setminus S = \) set difference; = \( R \cap \overline{S} \).

2.3. Decision Variables

Aircraft Mission Variables

\( X_{iab} = \) number of aircraft a direct delivering i on standard (nonquickturn) route r departing at time t.

\( X_{T_{iab}} = \) number of aircraft a delivering a transshipment load of i on standard (nonquickturn) route r departing at time t.

\( X_{DR_{iab}} = \) number of aircraft a direct delivering i on quickturn route r departing at time t.

\( X_{TR_{iab}} = \) number of aircraft a delivering a transshipment load of i on quickturn route r departing at time t.

\( X_{S_{iab}} = \) number of (roundtrip) shuttle missions of aircraft a delivering i in t.

\( Y_{abt} = \) number of aircraft a departing at t on backchannel route r.

\( TKRA_{ab} = \) number of (roundtrip) tanker missions of type a flown between b ∈ B_{skr} and b’ ∈ B_{arp} in t.

Aircraft Inventory Variables

\( R_{ab} = \) number of aircraft of type a in inventory at base b ∈ B in period t.

\( RONT_{ab} = \) number of aircraft of type a in inventory at b ∈ B without recovery in t.

\( RONR_{ab} = \) number of aircraft of type a in inventory at b ∈ B with recovery in t.

\( IRONT_{ab} = \) number of aircraft of type a initially assigned to b (nonrecovery).

\( IRONR_{ab} = \) number of aircraft of type a initially assigned to b (recovery).

\( THCHOP_{ab} = \) number of aircraft assigned to super node b’s shuttle fleet from nonrecovery routes in t.

\( THCHOPR_{ab} = \) number of aircraft assigned to super node b’s shuttle fleet from recovery routes in t.

\( TKRB_{ab} = \) number of tankers a whose beddown base is b ∈ B_{skr} in t.
Aircraft Changing Roles

\[ \text{ALLOC}_{abt} = \text{number of new aircraft } a \text{ allocated to } b \in B_i \text{ in } t. \]

\[ \text{TKREC}_{abt} = \text{number of tankers } a \text{ leaving } b \in B_e \text{ in } t \text{ for service as a refueler (to cloud).} \]

\[ \text{TKRCE}_{abt} = \text{number of tankers } a \text{ leaving tanker fleet (from cloud) in } t \text{ for } b \in B_e \text{ for cargo hauling.} \]

\[ \text{TKRBC}_{abt} = \text{number of tankers } a \text{ leaving } b \in B_{tkr} \text{ in } t \text{ for reassignment or service as a cargo hauler (to cloud).} \]

\[ \text{TKRC}_{abt} = \text{number of tankers } a \text{ being reassigned (from cloud) in } t \text{ to } b \in B_{tkr} \text{ for refueling.} \]

Cargo

\[ \text{DTONS}_{iact} = \text{stons of } i \text{'s cargo of type } c \text{ direct delivered by } a \text{ that will arrive in } t. \]

\[ \text{TTONS}_{iact} = \text{stons of } i \text{'s cargo of type } c \text{ for transshipment by } a \text{ arriving (at the transshipment node) in } t. \]

\[ \text{STONS}_{iact} = \text{stons of } i \text{'s cargo of type } c \text{ shuttled by } a \text{ in } t. \]

\[ \text{GTONS}_{iact} = \text{stons of } i \text{'s cargo of type } c \text{ that will arrive by ground at the FOB in } t. \]

\[ \text{NOGO}_{ic} = \text{stons of } i \text{'s cargo of type } c \text{ not delivered.} \]

Crews

\[ \text{SCREWS}_{abt} = \text{number of rested strategic airlift crews available for aircraft } a \text{ at base } b \in B_{crew} \text{ at the beginning of time } t. \]

\[ \text{DHCRES}_{abt} = \text{number of deadheading crews for aircraft } a \text{ at base } b' \text{ at time } t \text{ for reassignment to } b. \]

Each of the above decision variables is constrained to be nonnegative. In NRMO, all of the decisions are modeled using continuous decision variables. Ideally, many of the decisions (e.g., those involving numbers of aircraft missions) would be constrained to take integer values, but such a model would be computationally intractable. It is possible to construct a feasible integer solution by “rounding down” a fractional solution, since fewer missions use fewer resources and it is not necessary to deliver all cargo and troops.

2.4. Objective Function

minimize

\[ \sum_{a \in A} \sum_{c \in C} \sum_{t \in TW} \text{latepen}_{ic} \cdot (t + \text{rdd})^+ \cdot \text{DTONS}_{iact} \] (1a)

\[ + \sum_{i \in I} \sum_{a \in A} \sum_{c \in C} (t + \text{rdd})^+ \cdot \text{TTONS}_{iact} \] (1b)

\[ + \sum_{i \in I} \sum_{a \in A} \sum_{c \in C} (t + \text{rdd})^+ \cdot \text{STONS}_{iact} \] (1c)

\[ + \sum_{i \in I} \sum_{c \in C} \text{nogopen}_{ic} \cdot \text{NOGO}_{ic} \] (1d)

\[ + \sum_{a \in A} \sum_{b \in B} \sum_{c \in C} \text{usepen}_{ca} \cdot [\text{THCHOP}_{abt} + \text{THCHOPR}_{abt}] \] (1e)

\[ + \sum_{a \in A} \sum_{b \in B} \sum_{t \in T} \text{usepen}_{ca} \cdot \text{TKREC}_{abt} \] (1f)

\[ + \sum_{a \in A} \sum_{b \in B} \sum_{t \in T} \text{usepen}_{ca} \cdot \text{TKRBC}_{abt} \] (1g)

\[ + \sum_{a \in A} \sum_{b \in B} \sum_{t \in T} \sum_{c \in C} \text{dhpen}_{ca} \cdot \text{DHCRES}_{abt} \] (1h)

The first three terms of the objective penalize deliveries that arrive after the required delivery date for cargo arriving directly (1a), by shuttle (1b), and by ground (1c). The unit penalty increases in proportion to the number of days late, provided the arrival date is within the delivery window. Term (1d) penalizes nondelivered cargo. The objective also includes secondary terms that discourage: planes being left in the theater in shuttle fleets (1e), reassigning planes between delivery and tanker fleets (1f), and deadheading crews (1g). A small reward (1h) encourages planes to remain at an APOE, as this is often in the continental US and near their home station. The penalty (and reward) structure is such that late delivery is preferred to nondelivery but neither will occur if there are sufficient resources to achieve on-time delivery. In practice, delivery “requirements” are typically aggressive and difficult to fully achieve. An elastic formulation with opportunities for late and nondelivery allows analysts to discover when, and which, resources are being fully utilized. While NRMO does not explicitly model uncertainty, the idea behind providing a small reward for keeping planes in inventory at an APOE is that they are then well-positioned to respond to unforeseen contingencies, as well as undergo unforeseen repairs.

2.5. Demand Satisfaction Constraints

\[ \sum_{a \in A} \sum_{c \in C} \text{DTONS}_{iact} + \text{NOGO}_{ic} \]

\[ + \sum_{a \in A} \sum_{t \in TW} \text{STONS}_{iact} + \sum_{c \in C} \text{GTONS}_{iact} \]

\[ = \text{dem}_{ic} \quad \forall i \in I, \quad c \in C. \] (2)

For each line id and cargo class (bulk, oversized, outsized, and troops) deliveries that arrive directly, and if the destination is a forward operating base, by shuttle and ground must equal the demand or be counted as nondelivered cargo. It may be desirable to prevent “early” deliveries because, for example, it is anticipated that the associated cargo will not yet be ready for loading. This information is provided in the TPFDD via each line id’s available-to-load date. Similarly, deliveries past a certain point in time (possibly later than the “required” delivery date, rdd) may be useless. Such situations are captured by appropriately defining the time window TW, since cargo is counted as being delivered in (2) only if it arrives at a period in the delivery time window.
2.6. Flow Balance Constraints

2.6.1. Aircraft Balance Constraints at Embarkation Nodes

\[
\sum_{i \in I_f, b \in B_{trv}} XT_{iab} + \sum_{a \in A} \sum_{b \in B_a} XD_{iab} + \sum_{i \in I_f, b \in B_{trv}} XTR_{iab} + \sum_{a \in A} \sum_{b \in B_a} XDR_{iab} + \mathcal{J}(a \in A_{ikr}) \cdot [TKREC_{ab}] + RON_{ab} \\
= \text{RON}_{ab, t-1} + \sum_{a \in A, b \in B_a} Y_{a,t-1-trv_{ab}} + \text{ALLOC}_{ab} + \mathcal{J}(a \in A_{ikr}) \cdot [TKRCE_{ab}]
\]

\forall a \in A, b \in B_a, t \in T. (3)

The terms on the left-hand side of the equation represent all the ways that an aircraft of type \( a \) can depart an embarkation base \( b \) in period \( t \): planes leave the base for transshipment and direct deliveries on both standard and quickturn routes, dual-role tanker aircraft can depart the base in order to join the aerial refueling fleet, and aircraft can remain on the ground until the next period. The right-hand side of the equation represents available aircraft including inventoried aircraft from the previous period, aircraft that depart from theater at time \( t-trv_{ab} \) and hence arrive at base \( b \) at time \( t \).

2.6.2. Aircraft Balance Constraints at Super Debarkation Nodes

\[
\sum_{b \in B_{trv}} Y_{b} + \text{RON}_{ab} + \text{THCHOP}_{ab}
\]

\[
= \sum_{i \in I_f, b \in B_{trv}} XT_{iab} + \sum_{a \in A} \sum_{b \in B_a} XD_{iab} + \sum_{i \in I_f, b \in B_{trv}} XTR_{iab} + \sum_{a \in A} \sum_{b \in B_a} XDR_{iab} + \mathcal{J}(t=1) \cdot \text{IRON}_{ab} \forall a \in A, b \in B_{sup}, t \in T. (4)
\]

\[
\sum_{b \in B_{trv}} Y_{b} + \text{RONR}_{ab} + \text{THCHOPR}_{ab}
\]

\[
= \sum_{i \in I_f, b \in B_{trv}} XTR_{iab} + \sum_{a \in A} \sum_{b \in B_a} XDR_{iab} + \sum_{i \in I_f, b \in B_{trv}} XD_{iab} + \sum_{a \in A} \sum_{b \in B_a} XDR_{iab} + \mathcal{J}(t=1) \cdot \text{IRONR}_{ab} \forall a \in A, b \in B_{sup}, t \in T. (5)
\]

A super node is a surrogate for all of the bases in a theater. Aircraft flow balance constraints are enforced at super nodes, but other resources, such as airfield capacity, are modeled at individual bases. In constraints (4) and (5), aircraft depart the theater along backchannel routes, are inventoried, or are reassigned to serve as shuttles in the theater. Aircraft arrive at the super node on transshipment or direct delivery routes, because they rested there from last period, are part of the theater’s shuttle fleet, or because at the beginning of the deployment they were assigned to the theater. Constraints (4) and (5) are identical in form, but the former accounts for aircraft on quickturn routes while the latter tracks aircraft that recover at an APOD in the theater. We distinguish between these two types of missions because of the differing ways that the aircraft consume airfield capacity; see the subsequent airfield capacity constraints in §2.12.

2.6.3. Tanker Fleet Balance Constraints

\[
\sum_{b \in B_{trv}} TKREC_{ab,t-trv_{ab}} + \sum_{b \in B_{trv}} TKRBC_{ab,t-trv_{ab}} = \sum_{b \in B_{trv}} TKRCE_{ab} + \sum_{b \in B_{trv}} TKRCB_{ab} \forall a \in A_{ikr}, t \in T. (6)
\]

\[
\text{TKRBC}_{ab,t} + \text{TKR}_{ab,t} = \text{TKR}_{ab,t-1} \forall a \in A_{ikr}, b \in B_{ikr}, t \in T. (7)
\]

Constraint (6) models tanker aircraft reassignments. The constraint may be viewed as an aircraft flow balance constraint at a central control point, called the “tanker cloud.” An aircraft must travel through the cloud in order to change roles between serving as an aerial refueler and a cargo lifter or when changing its beddown location as a refueler. The left-hand side of (6) models aircraft flying to the cloud and the right-hand side models aircraft departing the cloud. We use this modeling construct because it significantly reduces the number of decision variables over allowing all possible point-to-point flights. The number of tankers in the fleet at each tanker beddown base is tracked in constraint (7).

2.6.4. Transshipped Cargo Flow Balance Constraints

\[
\sum_{a \in A} TTONS_{iab} = \sum_{a \in A} \sum_{i \in I_f, c \in C} \sum_{t \in T} STONS_{iab} + \mathcal{J}(t+trv \in T) \cdot GTONS_{ic,t+trv} \forall i \in I_f, c \in C, t \in T. (8)
\]

In each period, flow balance is maintained for transshipped cargo in each class for every line id by constraint (8). The left-hand side represents the amount flown into the transshipment point by strategic lifters while the right-hand side captures flow to the final destination by shuttle aircraft or by ground transportation. Note that no explicit geography is needed in this constraint because it is implicit within the line id index \( i \) and enforced by the restriction of allowable index combinations.
2.6.5. Strategic Crew Flow Balance

\[ \text{SCREWS}_{ab,t+1} = \text{SCREWS}_{ab,t} + \sum_{i \in I_{\text{job}}} \sum_{r \in RD_{a,i} \cap RD_{b,i} \cap RD_{b,i}} [XD_{ia,t} \cdot \text{ctrv}_{ab} + XDR_{ia,t} \cdot \text{ctrv}_{ab}] \]
\[ + \sum_{i \in I_{\text{job}}} \sum_{r \in RD_{a,i} \cap RD_{b,i} \cap RD_{b,i}} [XT_{ia,t} \cdot \text{ctrv}_{ab} + XTR_{ia,t} \cdot \text{ctrv}_{ab}] \]
\[ + \sum_{r \in RD_{a,i} \cap RD_{b,i} \cap RD_{b,i}} Y_{ar,t} \cdot \text{ctrv}_{ab}, \quad \forall a, b \in B_{\text{crew}}, \ t \in T \setminus \{T\}. \] (9)

Each aircraft fleet of type \( a \) has its own set of crews and flow balance of crews is maintained at each crew stage base in constraint (9). The number of available crews in period \( t+1 \) at \( b \) is the number available in \( t \), plus crews made available because they have rested sufficiently from previous direct, transshipment, and backchannel missions, less crews that depart in \( t \) on direct, transshipment, and backchannel missions, plus the net number of crews made available from tanker deployments and returns, plus the net number of crews made available from shuttle deployments and returns, plus additional crews that enter an APOE with newly available aircraft, plus the net change in crews arriving and departing on deadhead missions.

2.7. Aircraft Delivery Capacity

Constraints (10), (11), and (12) are of identical form and differ only in that they account for delivery capacity for direct, transshipment, and shuttle flights respectively. \( \text{purecap}_{\text{lac}} \) represents aircraft capacities if solely loaded with cargo of type \( c \) from line id \( i \); these figures are for a 3,200 nm flight. The effective capacity of the aircraft is scaled by \( \text{rangefac}_{ar} \) depending on the length of the longest (critical) flight leg on route \( r \). These three sets of constraints are supplemented by (13), (14), and (15) which restrict the number of troops based on the aircraft’s seating configurations. \( \text{paxfrac}_c \) in (10)–(12) denotes the fraction of the plane filled when its seats are filled with troops. Note that \( \text{DTONS}_{ia,\text{pax},t} \) for \( c \in \{ \text{bulk, oversized, outsized} \} \) has units of short tons.

2.7.1. Direct Delivery Capacity

\[ \sum_{c \in C_C \cap \text{CC} \cap \text{purecap}_{\text{lac}}} \frac{\text{DTONS}_{\text{lac}}}{\text{paxfrac}_a} \cdot DTONS_{ia,\text{pax},t} \cdot f(a \in A_{\text{pax}}) \leq \sum_{r \in RD_{a,\text{pax}}} \text{rangefac}_{ar} \cdot [XD_{ia,t} \cdot \text{ctrv}_{ar} + XDR_{ia,t} \cdot \text{ctrv}_{ar}] \quad \forall i \in I, \ a \in A, \ t \in TW_t. \] (10)

2.7.2. Transshipment Delivery Capacity

\[ \sum_{c \in C_C \cap \text{CC} \cap \text{purecap}_{\text{lac}}} \frac{\text{TTONS}_{\text{lac}}}{\text{paxfrac}_a} \cdot STONS_{ia,\text{pax},t} \cdot f(a \in A_{\text{pax}}) \leq \sum_{r \in RD_{a,\text{pax}}} \text{rangefac}_{ar} \cdot [XT_{ia,t} \cdot \text{ctrv}_{ar} + XTR_{ia,t} \cdot \text{ctrv}_{ar}] \quad \forall i \in I_{\text{job}}, \ a \in A, \ t \in TW_t. \] (11)

2.7.3. Shuttle Delivery Capacity

\[ \sum_{c \in C_C \cap \text{CC} \cap \text{purecap}_{\text{lac}}} \frac{\text{STONS}_{\text{lac}}}{\text{paxfrac}_a} \cdot SHTONS_{ia,\text{pax},t} \cdot f(a \in A_{\text{pax}}) \leq \sum_{r \in RD_{a,\text{pax}}} \text{rangefac}_{ar} \cdot [XT_{ia,t} \cdot \text{ctrv}_{ar} + XTR_{ia,t} \cdot \text{ctrv}_{ar}] \quad \forall i \in I_{\text{job}}, \ a \in A, \ t \in TW_t. \] (12)

2.7.4. Direct Delivery Troop Capacity

\[ \text{DTONS}_{ia,\text{pax},t} \leq \sum_{r \in RD_{a,\text{pax}}} \text{maxpax}_a \cdot [XD_{ia,t} \cdot \text{ctrv}_{ar} + XDR_{ia,t} \cdot \text{ctrv}_{ar}] \quad \forall i \in I_{\text{job}}, \ a \in A_{\text{mix}}, \ t \in TW_t. \] (13)

2.7.5. Transshipment Delivery Troop Capacity

\[ \text{TTONS}_{ia,\text{pax},t} \leq \sum_{r \in RD_{a,\text{pax}}} \text{maxpax}_a \cdot [XT_{ia,t} \cdot \text{ctrv}_{ar} + XTR_{ia,t} \cdot \text{ctrv}_{ar}] \quad \forall i \in I_{\text{job}}, \ a \in A_{\text{mix}}, \ t \in TW_t. \] (14)

2.7.6. Shuttle Delivery Troop Capacity

\[ \text{STONS}_{ia,\text{pax},t} \leq \text{maxpax}_a \cdot X_{\text{pax}} \quad \forall i \in I_{\text{job}}, \ a \in A_{\text{mix}}, \ t \in TW_t. \] (15)
2.8. Shuttle and Tanker Capacity Constraints

\[
\sum_{i \in I, a \in A, r \in RD_{a}, b \in RB_{a}} \frac{XS_{ab} \cdot shutterate_{ab}}{X_{shuttle}} \leq \text{THCHOP}_{a,b} + \text{THCHOPR}_{a,b} \\
\forall \ a \in A, \ b \in B_{a}, \ t \in T.
\]

(16)

\[
\sum_{b \in RB_{a}, t \in T} TKRA_{a,b} + \frac{TKRerate_{ab}}{TKR_{a,b}} \leq \text{TKR}_{a,b} \\
\forall \ a \in A_{b}, \ b \in B_{b}, \ t \in T.
\]

(17)

\[
\sum_{i \in I, a \in A, r \in RD_{a}, b \in RB_{a}} \text{tkreqvs}_{ab} \cdot X_{D_{a,b}, t \in t-trv_{ab}} \\
+ \sum_{i \in I, a \in A, r \in RD_{a}, b \in RB_{a}} \text{tkreqvs}_{ab} \cdot X_{D_{a,b}, t \in t-trv_{ab}} \\
+ \sum_{i \in I, a \in A, r \in RD_{a}, b \in RB_{a}} \text{tkreqvs}_{ab} \cdot X_{D_{a,b}, t \in t-trv_{ab}} \\
+ \sum_{a \in A, r \in RB_{a}} \text{tkreqvs}_{ab} \cdot Y_{a,b, t \in t-trv_{ab}} \\
\leq \sum_{b \in B_{a}, t \in T} \sum_{a \in A} \sum_{i \in I} \text{tkrprop}_{ab} \cdot TKRA_{a,b} \cdot t \\
\forall \ b \in B_{a}, \ t \in T.
\]

(18)

Constraint (16) restricts the number of shuttle missions based on the size of the shuttle fleet of type \( a \) aircraft at time \( t \) in the theater associated with super node \( b \). \( shutterate_{ab} \) is the maximum number of roundtrip missions per period that can be performed by aircraft \( a \) carrying line id \( i \). The number of tanker missions per period in each theater is similarly constrained by (17). The number of aircraft flying on routes that use aerial refueling point \( b \) is constrained by (18), based on the tankers serving \( b \).

2.9. Initial Allocation of Aircraft and Crews

\[
\sum_{b \in R_{a}} \text{ALLOC}_{ab} = \text{newac}_{ab} \quad \forall \ a \in A, \ t \in T.
\]

(19)

\[
\sum_{b \in R_{a}} \text{SCREWS}_{ab} + \text{crewrat}_{ab} \cdot \sum_{b \in R_{a}} \text{TKRB}_{ab} = \text{crewrat}_{ab} \cdot \text{newac}_{ab} \quad \forall \ a \in A, \ t = 1.
\]

(20)

\[
\text{IRONT}_{ab} + \text{IRONR}_{ab} = \text{initchop}_{ab} \\
\forall \ a \in A_{b}, \ b \in B_{a}.
\]

(21)

Constraint (19) governs the allocation of newly available aircraft to embarkation bases. A similar function is performed by (20) for allocating the crews to stage bases for \( t = 1 \). In future time periods, new crews are allocated with their aircraft; see constraint (9). For each super base, constraint (21) divides the initial shuttle fleet among recovery and nonrecovery bases in the theater.

2.10. Aircraft Utilization Constraints

\[
\sum_{t \in T} \sum_{i \in I, r \in RD_{a}} \text{fltime}_{a,f} \cdot X_{D_{a,b}, t \in t-trv_{ab}} \\
+ \sum_{t \in T} \sum_{i \in I, r \in RD_{a}} \text{fltime}_{a,f} \cdot X_{D_{a,b}, t \in t-trv_{ab}} \\
+ \sum_{t \in T} \sum_{i \in I, r \in RD_{a}} \text{fltime}_{a,f} \cdot X_{D_{a,b}, t \in t-trv_{ab}} \\
+ \sum_{t \in T} \sum_{i \in I, r \in RD_{a}} \text{fltime}_{a,f} \cdot X_{X_{D_{a,b}, t \in t-trv_{ab}}} \\
+ \sum_{t \in T} \sum_{i \in I, r \in RD_{a}} \text{fltime}_{a,f} \cdot X_{X_{D_{a,b}, t \in t-trv_{ab}}} \\
+ \sum_{t \in T} \sum_{i \in I, r \in RD_{a}} \text{fltime}_{a,f} \cdot X_{X_{D_{a,b}, t \in t-trv_{ab}}}
\leq \sum_{t \in T} \sum_{i \in I} \sum_{r \in RD_{a}} \text{cumac}_{a,f} \cdot \text{urate}_{a} \\
\forall \ a \in A, \ u \in U.
\]

(22)

Based on historical data, an aircraft of type \( a \) can average only a certain number of flight hours per day. These averages capture a number of factors, such as aircraft reliability, that are not directly modeled. See Wilson (1985) and Gearing et al. (1988) for more detailed discussions of utilization rates. We do not enforce the utilization limit on a daily basis because that would be overconstraining. Rather, we enforce it over blocks of time, denoted \( T_{a} \) for \( u \in U \). We use blocks with 20 days, and adjacent blocks have 10 days of overlap. \( \text{fltime}_{a,f} \) specifies the number of hours that aircraft \( a \) is airborne in the \( f \)th period of its mission when flying route \( r \). The left-hand side of (22) sums the flight times for all of the aircraft of type \( a \) within the block of time periods specified by \( T_{a} \). The sum over \( f \in FT \) captures all missions initiated in periods prior to \( t \) but not yet complete by \( t \).

2.11. Aircraft-Hours Consumption Constraints

\[
\sum_{i \in I, r \in RD_{a}} \sum_{t \in T} \sum_{f \in FT} \text{msntime}_{a,f} \cdot X_{D_{a,b}, t \in t-trv_{ab}} \\
+ \sum_{i \in I, r \in RD_{a}} \sum_{t \in T} \sum_{f \in FT} \text{msntime}_{a,f} \cdot X_{D_{a,b}, t \in t-trv_{ab}} \\
+ \sum_{i \in I, r \in RD_{a}} \sum_{t \in T} \sum_{f \in FT} \text{msntime}_{a,f} \cdot X_{D_{a,b}, t \in t-trv_{ab}} \\
+ \sum_{i \in I, r \in RD_{a}} \sum_{t \in T} \sum_{f \in FT} \text{msntime}_{a,f} \cdot X_{D_{a,b}, t \in t-trv_{ab}} \\
+ \sum_{i \in I, r \in RD_{a}} \sum_{t \in T} \sum_{f \in FT} \text{msntime}_{a,f} \cdot X_{D_{a,b}, t \in t-trv_{ab}}
\]

\[
\leq \sum_{i \in I, r \in RD_{a}} \sum_{t \in T} \sum_{f \in FT} \text{msntime}_{a,f} \cdot X_{D_{a,b}, t \in t-trv_{ab}} \\
\forall \ a \in A, \ u \in U.
\]

(22)
Constraint (23) has a similar mathematical structure to the aircraft utilization constraint (22), but the multiplying coefficients in (23) are designed to capture all the time consumed in period \( t \) by each aircraft activity, not just the flying time. The right-hand side of (23) is the total number of hours available for aircraft of type \( a \) through period \( t \), and the left-hand side accounts for all possible aircraft activities through \( t \) (delivery flights, shuttle flights, backchannel flights, tanker missions, and aircraft in inventory). Formulating the aircraft balance constraints with travel times rounded down to an integer number of periods can lead to overly optimistic results, and (23) helps remedy this. See Morton et al. (1996) for supporting experimental results. We note that if all of the travel times are integer-valued, then (23) is redundant.

2.12. Airfield Capacity Constraints

2.12.1. Airfield Parking and Servicing

Capacity Constraints

\[
\sum \sum \sum \sum gtime_{abr} \cdot acpkg_{ab} 
+ \sum_{i \in \mathcal{I}_{loc}} \sum_{a \in \mathcal{A}_{loc}} gtime_{abr} \cdot acpkg_{ab} \cdot XS_{iat}
\]

\[
+ \sum_{i \in \mathcal{I}_{loc}} \sum_{r \in \mathcal{R}_{loc}} gtime_{abr} \cdot acpkg_{ab} \cdot \frac{\text{hrsper}}{\text{shuttlerate}_{ia}}
\]

\[
+ \sum_{i \in \mathcal{I}_{loc}} \sum_{r \in \mathcal{R}_{loc}} msntime_{arf} \cdot Y_{ar, t-(f-1)}
\]

\[
+ \mathcal{F}(a \in \mathcal{A}_{loc}) \left[ \sum_{b \in \mathcal{B}_{loc}} \sum_{e \in \mathcal{E}_{loc}} \text{hrsper} \cdot TKRA_{abr} \cdot \text{trv}_{abr} \right]
\]

\[
+ \sum_{b \in \mathcal{B}_{loc}} \sum_{e \in \mathcal{E}_{loc}} \text{hrsper} \cdot TKREC_{abr}
\]

\[
\leq \text{hrsper} \cdot \text{cumac}_{at} \quad \forall a \in \mathcal{A}, \ t \in \mathcal{T}.
\] (23)

The ability of airfields to handle aircraft is captured in two sets of constraints: (24) and (25). The first set captures parking ramp space and nonfuel related services, while the second constrains throughput based on fuel limitations. Even though flow balance is only maintained at the theater’s super node, these capacity constraints are enforced at all “real” bases in the model, i.e., all modeled bases except aerial refueling points, whose capacity is captured in (18), and super nodes, which need no capacity constraints. The amount of ground time, and hence airfield capacity, that an aircraft consumes depends on whether the base serves as an onload, enroute, or offload base for a strategic lifter.

Term (24a) captures ramp and service consumption at enroute bases for standard and quickturn direct delivery routes. Quickturn routes (24c) require minimal servicing at the onload base, and hence spend less time there than standard routes (24b). Terms (24d), (24e), and (24f) serve the same purpose as (24a), (24b), and (24c) except that they are for transshipment routes. Capacity is consumed by in-theater shuttles at their FOBs and transshipment APODs in (24g) and at their beddown base in (24h). Consumption of ramp and service capacity for backchannel routes is captured in (24i), and tanker aircraft consume capacity at their beddown bases in (24j). Finally, in the model a certain fraction of aerial refueling attempts fail and,
as a result, associated cargo lifters are diverted to their tanker’s beddown base and consume airfield capacity there, (24k)–(24o).

### 2.12.2. Airfield Fuel Capacity Constraints

\[
\sum_{i \in I} \sum_{a \in A} \sum_{t \in T} \text{fuel}_{ab} \cdot \begin{bmatrix} XD_{iar}, t-\text{enr}_{ab} + XDR_{iar}, t-\text{enr}_{ab} \end{bmatrix} \\
+ \sum_{i \in I} \sum_{a \in A} \sum_{t \in T} \text{fuel}_{ab} \cdot XD_{iar}, t-\text{trv}_{ab} \\
+ \sum_{i \in I} \sum_{a \in A} \sum_{t \in T} \text{fuel}_{ab} \cdot XDR_{iar}, t-\text{trv}_{ab} \\
+ \sum_{i \in I} \sum_{a \in A} \sum_{t \in T} \text{fuel}_{ab} \cdot [XT_{iar}, t-\text{enr}_{ab} + XTR_{iar}, t-\text{enr}_{ab}] \\
+ \sum_{i \in I} \sum_{a \in A} \sum_{t \in T} \text{fuel}_{ab} \cdot XT_{iar}, t-\text{trv}_{ab} \\
+ \sum_{i \in I} \sum_{a \in A} \sum_{t \in T} \text{fuel}_{ab} \cdot XTR_{iar}, t-\text{trv}_{ab} \\
+ \sum_{b \in R_b, ab} \sum_{a \in A} \text{daysfuel}_{ab} \cdot [THCHOP_{ab} t + THCHOP_{ab}'] \\
+ \sum_{a \in A} \sum_{t \in T} \text{fuel}_{ab} \cdot Y_{ar}, t-\text{enr}_{ab} \\
+ f(b \in B_{ab}) \cdot \begin{bmatrix} \sum_{a \in A_b} \text{daysfuel}_{ab} \cdot TKRB_{ab} \end{bmatrix} \\
+ \sum_{i \in I} \sum_{a \in A} \sum_{t \in T} \text{dpct}_{ab} \cdot \text{fuel}_{ab} \\
\cdot XD_{iar}, t-\text{enr}_{ab} \\
+ \sum_{i \in I} \sum_{a \in A} \sum_{t \in T} \text{dpct}_{ab} \cdot \text{fuel}_{ab} \\
\cdot XDR_{iar}, t-\text{enr}_{ab} \\
+ \sum_{i \in I} \sum_{a \in A} \sum_{t \in T} \text{dpct}_{ab} \cdot \text{fuel}_{ab} \\
\cdot XT_{iar}, t-\text{enr}_{ab} \\
+ \sum_{i \in I} \sum_{a \in A} \sum_{t \in T} \text{dpct}_{ab} \cdot \text{fuel}_{ab} \\
\cdot XTR_{iar}, t-\text{enr}_{ab} \\
+ \sum_{a \in A} \sum_{t \in T} \text{dpct}_{ab} \cdot \text{fuel}_{ab} \\
\cdot Y_{ar}, t-\text{enr}_{ab} \\
\leq \text{fuelgals}_b \quad \forall \ b \in B \setminus B_{top} \setminus B_{arp}, \ t \in T. \tag{25}
\]

Constraint (25) is of very similar form to the ramp and service consumption constraint (24), except here aircraft consume gallons of fuel instead of parking-space hours. Also, note that shuttle aircraft are fueled for an entire day at their beddown base and so there is no analog of (24g) in (25).

### 2.13. Initial Conditions

\[
\begin{align*}
RON_{ab} & \equiv 0 \quad \forall t \leq 0. \\
XD_{iar} & \equiv 0 \quad \forall t \leq 0. \\
XT_{iar} & \equiv 0 \quad \forall t \leq 0. \\
Y_{ar} & \equiv 0 \quad \forall t \leq 0. \\
THCHOP_{ab} & \equiv 0 \quad \forall t \leq 0. \\
TKRB_{ab} & \equiv 0 \quad \forall t \leq 0. \\
TKBC_{ab} & \equiv 0 \quad \forall t \leq 0. \\
TKREC_{ab} & \equiv 0 \quad \forall t \leq 0.
\end{align*}
\]

### 3. ANALYSES

In this section, we describe in some detail two studies in which NRMO helped provide analyses to inform air mobility decisions. The first study focuses on the impact of enroute airfield resources on the performance of the airlift system. The second study concerns fleet modernization, and NRMO was used to assess the system’s ability to deliver cargo and troops under two fleet alternatives. The section concludes with a discussion of ongoing analyses and computational requirements for some of our largest model instances.

#### 3.1. Analyzing the Effects of Airfield Resources on Airlift Capacity

The ability of an airlift system to deliver cargo and passengers depends on a number of factors including the ground resources available at airfields. Stucker and Williams (1998) performed a study of the impact of airfield resources on airlift capacity and we review some of the results of their analysis here. This RAND study utilized NRMO in conjunction with another model called the Airfield Capacity Estimator (ACE). ACE models parking and servicing, loading and unloading, and fueling operations in detail, and is described in Stucker et al. (1998). For our purposes, we may view ACE as taking inputs that describe an airfield and its resources and providing for NRMO estimates of aircraft ground times (gtime_{ab}, and qtime_{ab}) and airfield capacities (mog). We note that ACE allows for several ground-serving roles for an airfield including the notion of quickturn and recovery stops versus a full-service offload stop.

The primary purpose of Stucker and Williams’ study was to investigate how the level of airfield resources affects the quantity of strategic airlift deliveries. The study’s sponsor, the Force Projection Directorate in the Office of the Secretary of Defense (OSD), asked that a previously constructed major regional contingency (MRC)-East deployment scenario be used. In this scenario, cargo and passengers depart the US from Dover, Delaware and are delivered to Dhahran, Saudi Arabia via enroute bases in England, Germany, and Spain. The relevant distances are shown in Table 3. Each of these enroute bases actually has the combined resources of two bases from their country. The English base has the capability of Mildenhall and Fairford, the German base,
Ramstein and Rhein Main, and the Spanish base, Moron and Rota. Table 3 shows that the critical leg in each route is the US-Europe leg. The route via England has the shortest critical leg and the shortest overall distance. Politics clearly plays an important role in such deployments, and while NRMO contains little political modeling, the legs between the English and German bases and the destination base in Dhahran contain detours around Central and Eastern Europe. The study uses the 1996 Air Mobility Command fleet of 95 C-5s, 18 C-17s, 174 C-141s, and 37 KC-10s plus the Civil Reserve Air Fleet (CRAF) of 64 wide-body cargo planes (WBCs), 109 wide-body passenger planes (WBPs), and 51 narrow-body cargo planes (NBCs). Under the CRAF agreement, the Department of Defense leases civilian passenger and cargo aircraft in times of national emergency. The CRAF share resources with the military aircraft at onload and offload bases, but use separate non-military enroute European bases. The focus of the study concerns the role of enroute airfield capacity at the military European bases. The primary measure of effectiveness RAND used in this analysis was average daily throughput of cargo and passengers.

Under the baseline scenario using NRMO, all of the passenger movements are completed within their allotted time windows, with an average of 6,600 passengers delivered per day over the 30 day horizon used in this analysis. All of the passengers are carried by CRAF aircraft. Over 30 days, 135,000 stons of cargo are delivered, an average of 4,500 stons per day. The baseline scenario linear program has 40,702 structural constraints, 112,944 decision variables, and 907,340 nonzero constraint coefficients. The model solves in about 10 minutes on a Pentium III with a 500 MHz processor using GAMS calling CPLEX 6.0’s barrier algorithm.

### Table 3. Distances in nautical miles between the three enroute bases in Western Europe and the destination base in Dhahran, Saudi Arabia (Stucker and Williams 1998).

<table>
<thead>
<tr>
<th>Airfield</th>
<th>Dover (nm)</th>
<th>Dhahran (nm)</th>
<th>Total (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mildenhall</td>
<td>3,123</td>
<td>2,953</td>
<td>6,076</td>
</tr>
<tr>
<td>Ramstein</td>
<td>3,437</td>
<td>2,765</td>
<td>6,202</td>
</tr>
<tr>
<td>Moron</td>
<td>3,213</td>
<td>2,879</td>
<td>6,092</td>
</tr>
</tbody>
</table>

### Table 4. Existing airfield ramp and fuel resources at the three enroute bases. Ramp space is measured in terms of narrow-body-equivalent parking space hours per day. A narrow-body space corresponds to that required by a C-141.

<table>
<thead>
<tr>
<th>Airfield</th>
<th>Ramp Space ((mog_b)) (narrow-body-equivalent hours per day)</th>
<th>Fuel ((fuelgals_b)) (million gallons per day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>England</td>
<td>240</td>
<td>1.61</td>
</tr>
<tr>
<td>Germany</td>
<td>336</td>
<td>1.57</td>
</tr>
<tr>
<td>Spain</td>
<td>168</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Three enroute bases, so that particular base will handle two types of aircraft. These aircraft-specific service personnel restrictions are called “birds of a feather flock together” constraints. Ramp constraints impose a maximum number of parking-space hours per day at each airfield \((mog_b)\), and fuel constraints limit the number of gallons of fuel that can be pumped at an airfield each day \((fuelgals_b)\). Table 4 contains these values for the three enroute bases.

In order to assess the impact of each type of airfield capacity constraint, the model was run four times: (i) with no airfield capacity constraints, (ii) with just service center (birds of a feather flock together) restrictions, (iii) with service center and ramp constraints, and (iv) with service center, ramp, and fuel constraints. The service center constraints are implemented using a mixed-integer programming variant of NRMO in which 12 binary decision variables were used to partition the C-5, C-17, and C-141 among the bases in England, Germany, and Spain, and also assign the KC-10 to one of these three enroute bases. The addition of the binary variables increased the run time for this scenario from 10 minutes to two hours and 45 minutes.

Table 5 summarizes the results of these runs. When no airfield constraints are present, all aircraft are routed through England since it has the shortest critical leg and shortest overall length. When service center constraints are enforced and the centers are allocated to the three enroute bases, C-5s fly through Spain, C-17s through Germany and C-141s through England. Daily cargo deliveries decrease by 100 stons, less than two percent, to about 5,600 stons per day. This decrease is primarily due to increased flying times on the longer routes which result in fewer round trips because of binding aircraft utilization constraints (§2.10). Additional runs showed that these choices of the service centers are not critical and similar results can be achieved with other assignments. Imposing restrictions for ramp space has a larger impact and average daily deliveries decrease by 600 stons to 5,000 stons per day. The service center constraints remain in place and C-5s still go through Spain, saturating its ramp space from day 2 to day 30. The C-17 and C-141 service centers swap locations. The C-17s,
Table 5. This table shows the results of four runs with differing types of airfield capacity constraints enforced. Service center (birds of a feather flock together) constraints require that each type of aircraft be routed through its unique enroute maintenance airbase. The location of the service center is shown for each aircraft. Note that the average daily cargo delivered decreases as additional airfield resource constraints are incorporated (Stucker and Williams 1998).

<table>
<thead>
<tr>
<th>Airfield Capacity Constraints Present at Enroute Bases</th>
<th>None</th>
<th>Service</th>
<th>Service/Ramp</th>
<th>Service/Ramp/Fuel</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-5 center</td>
<td>England</td>
<td>Spain</td>
<td>Spain</td>
<td>Germany</td>
</tr>
<tr>
<td>C-17 center</td>
<td>England</td>
<td>Germany</td>
<td>England</td>
<td>Spain</td>
</tr>
<tr>
<td>C-141 center</td>
<td>England</td>
<td>England</td>
<td>Germany</td>
<td>England</td>
</tr>
<tr>
<td>Daily Cargo (stons)</td>
<td>5,700</td>
<td>5,600</td>
<td>5,000</td>
<td>4,500</td>
</tr>
</tbody>
</table>

daily deliveries are shown under three strategies for locating the specialized aircraft service centers at the enroute bases (Stucker and Williams 1998).

<table>
<thead>
<tr>
<th>Strategy</th>
<th>C-5 Center</th>
<th>C-17 Center</th>
<th>C-141 Center</th>
<th>Daily Delivery (stons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Germany</td>
<td>Spain</td>
<td>England</td>
<td>4,500</td>
</tr>
<tr>
<td>B</td>
<td>Spain</td>
<td>England</td>
<td>Germany</td>
<td>4,400</td>
</tr>
<tr>
<td>C</td>
<td>England</td>
<td>Germany</td>
<td>Spain</td>
<td>4,200</td>
</tr>
</tbody>
</table>

along with the small KC-10 fleet, utilize all of the ramp space at the English base on half of the days and utilize it at over a 90% level every day. The C-141s use over 90% of the German base’s ramp space every day. Imposing fuel pumping constraints also has a significant impact, decreasing average daily deliveries by another 500 stons to the baseline result of 4,500 stons per day. In this case, the locations of the service centers have all changed. This is primarily driven by the relatively small fuel capacity of the Spanish base. The smaller C-17 fleet is now routed through Spain and consumes its fuel supply each day. Ramp space constrains C-5s in Germany and C-141s in England.

The results of systematically mislocating the specialized aircraft service centers are shown in Table 6. Strategy A, the optimal solution with all airfield capacity constraints enforced (the right-most column of Table 5) serves as the baseline. Strategy B, with the C-5 center in Spain, the C-17 center in England, and the C-141 center in Germany leads to a 2% reduction in daily deliveries. Strategy C leads to nearly a 7% reduction, primarily because the large C-141 route is forced through the fuel-limited base in Spain.

In contrast to conclusions from some previous studies (see the discussion in Stucker and Williams 1998), these results indicate that airfield capacity constraints at enroute bases can play a critical role in an airlift system’s ability to deliver cargo.

3.1.2. Redistributing Ramp Space and Fuel Pumping Rates. At each of the three enroute bases, the fraction of the airfield as well as the fuel resources, accessible by the US airlift aircraft may be negotiable. For this reason it can be valuable to study optimal allocations of ramp space and fuel pumping equipment. Stucker and Williams do so in the following manner. They assume the ramp space and fuel totals over all three bases remain constant but they allow redistribution among the three bases. The original and the optimally redistributed values are shown in Table 7. The redistribution yields a 12% increase in daily cargo deliveries from 4,500 stons per day to 5,100 stons per day. Recall that the case with no enroute airfield capacity constraints delivers 5,700 stons per day, so this redistribution results in a significant increase.

The C-5 is the largest aircraft, hauls the most cargo, and consumes the most fuel. Its service center is relocated in Spain where the redistribution increased both the ramp space and fuel pumping rate. England now has more ramp space to accommodate the wide-body C-17. Finally, the ramp space and pumping rate in Germany are significantly reduced, but the base still has enough capacity to handle 82 narrow-body C-141s per day.

These results estimate that under this MRC-East scenario if the US were to negotiate with its European allies in order to redistribute existing ramp space and fuel pumping rates in the recommended fashion, deliveries could be increased by 12% over the first 30 days of the deployment.

3.2. Fleet Modernization: Comparison of NRMO with an Airlift Simulation

Concurrent with the RAND/OSD infrastructure analysis described above, the Air Mobility Command’s Studies and Analyses Flight (AMCSAF) at Scott AFB, Illinois, was exploring alternatives for airlift fleet modernization in the next decade. AMCSAF conducted this study, as well as all of its most detailed analyses, with a large simulation model known as the Airlift Flow Module (AFM) of the Mobility Analysis Support System (Air Mobility Command 1996). AFM is an established and respected model,
and so its results provided an opportunity to help validate NRMO, as well as to compare and contrast the two modeling approaches: optimization and simulation. Although it has the capability to be run as a stochastic discrete-event simulation, AFM is most often run as a deterministic simulation because of its computational requirements. In this study AFM was run in its deterministic mode. Thus, the purpose of using NRMO in the fleet modernization study was twofold: (i) to instill confidence in, and familiarity with, an optimization model for airlift mobility, and (ii) to highlight additional insight that can be gleaned from an alternative modeling approach to airlift mobility.

Simulation and optimization provide two of the most frequently used modeling tools in operations research. Simulation models are descriptive, i.e., they characterize or describe complex systems. And, simulation models can incorporate details that are not possible to capture in a mathematical program. On the other hand, optimization models are prescriptive, i.e., they are typically used to recommend a course of action. One important difference between NRMO and AFM concerns the issue of scheduling aircraft. NRMO assigns aircraft to routes each time period in an optimal manner as prescribed by its objective function. In contrast, AFM schedules aircraft in a relatively myopic fashion. For each aircraft and origin-destination pair, route priorities are specified ahead of time by a skilled aviator-analyst using rules-of-thumb. When a mission is to be scheduled, the simulation model chooses the highest priority route that has enough available resources (e.g., enroute airfield capacity).

Any plan to modernize the airlift fleet must consider several wartime scenarios. Foremost among these scenarios are a war near the Arabian peninsula (MRC-East), and a war on the Korean peninsula (MRC-West). Although they are only a small subset of the possible missions, these two contingencies are among the most stressful from an airlift standpoint; both have huge delivery requirements, and both are thousands of miles from the continental United States. In this study, the onload, enroute, and offload airbase capacities were varied to reflect current infrastructure, as well as the forecast infrastructure of the next decade.

Although AMCSAF modeled many fleet alternatives using AFM, we chose two of the leading contenders for the comparison with NRMO. Specifically, we assumed an unmodified C-5 fleet of 95 aircraft, as well as an upgraded C-5 fleet of the same size. The upgraded fleet offered enhanced reliability because of replaced engines, avionics, landing gear, and some structural components. Increased reliability is reflected in NRMO via larger urate values and smaller groundtime (gtimeabr and gtime) values. Both fleets also included 95 C-17 aircraft, as well as 174 contracted CRAF aircraft.

Commonality of input between the optimization and simulation models is very important, not only because this allows direct comparison of the outputs, but because providing input data for large models is labor intensive. We have developed a number of utilities that convert the established AFM input files into NRMO input format. These include the movement requirements, the aircraft capacities for cargo types and classes, the airfield locations and capacities, and the available route structure. NRMO provides a prescriptive solution at the expense of additional detail that the descriptive simulation model (AFM) offers. To accommodate this reduced detail, the conversion utilities allow the aggregation of similar movement requirements and airfields. For example, several small movement requirements that have the same origins, destinations, time windows, and cargo densities can be combined into one larger requirement with minimal loss of model fidelity. Nearby airfields may be similarly aggregated.

Essentially four instances of NRMO were used in this study: models for the Arabian peninsula and the Korean peninsula, each under two fleet configurations. All model instances were built and solved on a Sun Ultra using GAMS and calling CPLEX 3.0’s barrier algorithm. Each model has 30 daily time periods and solves in less than 30 minutes. The dimensions of the linear programs are roughly 30,000 structural constraints, 110,000 decision variables, and 900,000 nonzero constraint coefficients.

### 3.2.1. Korean Peninsula

Airlift operations from the United States to Korea are characterized by long overwater flights, stopovers in congested Japanese bases, and offloads.

<table>
<thead>
<tr>
<th>Airfield</th>
<th>Existing Ramp Space (hours per day)</th>
<th>Preferred Ramp Space (hours per day)</th>
<th>Existing Fuel Pumping Rate (million gallons per day)</th>
<th>Preferred Fuel Pumping Rate (million gallons per day)</th>
<th>Service Center</th>
</tr>
</thead>
<tbody>
<tr>
<td>England</td>
<td>240</td>
<td>281</td>
<td>1.61</td>
<td>0.99</td>
<td>C-17</td>
</tr>
<tr>
<td>Germany</td>
<td>336</td>
<td>178</td>
<td>1.57</td>
<td>1.01</td>
<td>C-141</td>
</tr>
<tr>
<td>Spain</td>
<td>168</td>
<td>285</td>
<td>0.82</td>
<td>2.00</td>
<td>C-5</td>
</tr>
</tbody>
</table>

This table shows the original and redistributed ramp and fuel resources among the three enroute bases. The resulting locations of the service centers for each aircraft type are also shown. The NRMO model estimates that this redistribution and relocation leads to a 12% increase in deliveries over the first 30 days of the deployment. Such insight is valuable because the US may be able to negotiate the amount of airfield capacity made available by our NATO allies (Stucker and Williams 1998).
In contrast, most human airlift planners routinely use a
AFM did not use wind direction and velocity as an input.
not account for the effects of wind, principally because
situation and simulation. Previous large-scale airlift analyses did
representation of modified C-5s, since neither directly incorpo-
ents; AFM suggests that an additional 500 stons per day can be delivered using a fleet
4,550 stons per day (500 stons per day is roughly equiv-
AFM averaged 5,000 stons per day, while NRMO averaged
was more sensitive to the modified C-5 than the optimiza-
tion that similar amounts of cargo can be delivered to the Korean peninsula during a
wartime contingency. Each bar shows the
delivered stons per day assuming the current
airbase infrastructure, as well as assuming
proposed infrastructure improvements for the
next decade. The two models differ somewhat regarding the effect of aircraft improve-
ments; AFM suggests that an additional 500 stons per day can be delivered using a fleet
of improved C-5 cargo aircraft, while NRMO
suggests a much smaller improvement.

Figure 4. Both the AFM airlift simulation model and
the NRMO airlift optimization model predict that similar amounts of cargo can be
delivered to the Korean peninsula during a
wartime contingency. Each bar shows the
delivered stons per day assuming the current
airbase infrastructure, as well as assuming
proposed infrastructure improvements for the
next decade. The two models differ somewhat regarding the effect of aircraft improve-
ments; AFM suggests that an additional 500 stons per day can be delivered using a fleet
of improved C-5 cargo aircraft, while NRMO
suggests a much smaller improvement.

at small Korean airfields with limited parking and servicing
capability. Most missions fly a northern Pacific route
through Alaska, or a mid-Pacific route through Hawaii, and
sometimes Guam or Okinawa. Missions that originate on
the East coast or in the Midwest will also stop at an Air
Force base near Seattle or San Francisco. Because most
aircraft return to the U.S. empty, many of the eastbound
enroute stops are unnecessary, provided the aircraft is fully
fueled in Japan.

The results of the AFM simulation model and the NRMO
optimization model were similar with respect to average
daily cargo throughput. Figure 4 shows that both models
indicate a cargo throughput of just over 4,000 stons per
day using the current airfield infrastructure, regardless of
whether the C-5 is modified or not. However, the simulation
was more sensitive to the modified C-5 than the optimiza-
tion. With modified C-5s and the proposed infrastructure,
AFM averaged 5,000 stons per day, while NRMO averaged
4,550 stons per day (500 stons per day is roughly equiv-
alent to 5 or 6 fully loaded C-5s, or 7 or 8 fully loaded
C-17s). It is likely that both models underestimate the con-
tribution of modified C-5s, since neither directly incorpo-
rates stochastic ground times due to aircraft reliability.

One of the most significant insights gleaned from the
NRMO runs had nothing to do with comparing optimization and simulation. Previous large-scale airlift analyses did
not account for the effects of wind, principally because
AFM did not use wind direction and velocity as an input.
In contrast, most human airlift planners routinely use a
west wind at 50 knots as a rough approximation of sum-
mertime winds at flight level. In the jet stream, wintertime
winds in Northeast Asia are often 100 knots at cruising altitude. When NRMO was run with 100 knot westerly
winds, daily cargo throughput dropped by an average of
15.6%. Thus, wind can have more effect on throughput than
either the proposed infrastructure or aircraft improvements! Air Mobility Command is now incorporating wind into the
next AFM release. We note that Middleton (1998) develops
a stochastic variant of a deterministic optimization model
developed by Borsi and Whisman (1997). While this model
is simpler than NRMO in many respects, it uses multiple
scenarios to capture uncertainties in weather, as well as
maintenance and damage to airbases or planes by enemy
activities.

3.2.2. Arabian Peninsula. As described in the infra-
structure study of §3.1, airlift deployments to the Arabian
peninsula involve at least one intermediate stop, usually in
England, Germany, or Spain. Fueling and servicing capa-
bility at the offload locations, while generally less restric-
tive than in Korea, is still constrained. Additionally, the
political assumptions are trickier; permission to fly over
any of the patchwork of countries in Europe is not assured,
and may change depending on who is fighting whom. For
instance, the U.S. had very few overflight and enroute refu-
eling options during the resupply of Israel in the Yom
Kippur War of 1973, yet had numerous alternatives during the

Our focus for NRMO in this study did not involve a
direct comparison with AFM. Rather, we wanted to show
how an optimization solution could complement a simula-
tion solution. We looked at route prioritization and offload
airfield congestion.

As indicated earlier, route priorities for each aircraft in
AFM must be specified by the user, whereas NRMO selects
an optimal route for a given time, aircraft, and unit deliv-
ered. As noted in the infrastructure study, making the best
match between routes and aircraft types is a function of the
critical leg as well as the airfield capacities. Making inap-
propriate matches can be rather punitive. Similar to that
seen in the infrastructure study, the optimization preferred
and flew C-17s on the route with the shortest critical leg
72% of the time, and the route with the longest critical leg
only 1% of the time. In order to reserve capacity at
the enroute airbases for C-17s, C-5 route preferences were
reversed; 29% of those missions flew the shortest-critical-
leg route, while 40% flew the longest-critical-leg route.
These frequencies, which are output from NRMO, were
consistent with the priority lists constructed by USAF ana-
lysts for input for AFM. That is, the most frequently used
routes in NRMO corresponded the highest priority routes in
AFM. If NRMO’s output turned out to be inconsistent with
the analyst’s priority list, then the frequencies with which
NRMO chose routes could be used to suggest a priority list
for AFM.

Determining where to spend additional infrastructure
dollars is essentially a byproduct of any NRMO run.
Although not a goal of the fleet modernization analysis, examination of the Arabian peninsula dual solutions showed a large range of airbase congestion, as indicated by the offload bases’ airfield parking capacity constraints. The optimal dual variable levels ranged from an average of only 26.1 stons delivered per parking space per day, up to 104.4 stons delivered per parking space per day. Marginal analysis from NRMO dual solutions provided additional insight that was unavailable from AFM.

3.2.3. Other NRMO-AFM Comparative Analyses.

Since the completion of the analyses described above, the NRMO model has continued to be used in the analysis of infrastructure and fleet issues. Two additional studies, both sponsored by the Force Projection Directorate in OSD are in progress at the time this paper is being written.

The purpose of the first study is to prioritize spending on several proposed fuel system improvements at air mobility bases around the world. Fuel infrastructure in the Pacific and Europe is crucial for the support of deliveries to Northeast Asia and Southwest Asia, two areas of possible future conflict. Much of the available infrastructure in these areas, especially in the Pacific, is aging and in need of repair. This fuels infrastructure study, performed by Thomas et al. (1999) at RAND, is complementary to work done at AMCSAF using AFM. As a part of the analysis, the RAND team performed a detailed comparison of NRMO and AFM results for the baseline versions of two scenarios. The comparison included analyses of deliveries, missions flown, average payloads, and aircraft cycle times. In response to the analyses, NRMO was improved to capture additional details of the air mobility system, such as home station servicing and aircraft load factors, which were represented in the AFM simulation. The results of the comparison are summarized in Table 8.

The purpose of the second study, performed at the Naval Postgraduate School by Baker et al. (2000) is to determine the requirements for USAF tanker aircraft in future years. These requirements have changed markedly in the post-cold war world. Classically used to support nuclear bomber refueling, tanker requirements in the 1990s have focused more on deployment and employment of airlift and fighter assets to other contingencies. Recognizing this evolution of requirements, AMCSAF is currently conducting the Tanker Requirements Study for 2005 (TRS-05) using AFM and other models. Although the simulation and optimization efforts are not completely parallel, the analyses will begin from several common baseline scenarios. To date, only the dual MRC West-to-East scenario has been implemented for NRMO. Preliminary analysis shows that NRMO results are comparable to AFM results for this scenario, as indicated in Table 9.

The differences between the delivery capabilities suggested by NRMO and AFM in these two studies are consistent with those found in the fleet modernization analyses described in §§3.2.1 and 3.2.2. Also, the two studies sketched in this section use large MRC-sized scenarios that have minimal aggregation. Therefore, the model instances and solution times arising from these analyses provide a good indication of the computational effort required to solve some of the largest model instances we have faced. The sizes of these instances, associated run times, and other related data are given in Table 10. The fuels infrastructure study runs were made on a Sun Ultra2 with two 300 MHz

| Table 8. Average daily deliveries of cargo and troops for the NRMO optimization model and the AFM simulation model are compared under two scenarios for the fuels infrastructure study. |
|---------------------------------|------------|------------|----------|------------|------------|
| Fuels Infrastructure Study      | West 2006  | East 2006  | %Diff    | West 2006  | East 2006  |
| Daily cargo (stons)             | 4,859      | 4,750      | 2%       | 5,514      | 5,090      | 8%        |
| Daily troops                    | 7,661      | 6,965      | 10%      | 7,011      | 6,788      | 3%        |

| Table 9. This table compares the output of the NRMO optimization model and the AFM simulation model for one scenario of the tanker requirements study. |
|---------------------------------|------------|------------|
|                               | West-East  |
|                               | NRMO       | AFM        | %Diff    |
| Daily cargo (stons)            | 4,950      | 4,575      | 8%       |
| Daily troops                   | 9,025      | 8,400      | 7%       |

| Table 10. This table shows the numbers of bases, routes, time periods, and the number of demand requirements in the TPFDD for model instances from the fuels infrastructure and tanker requirements studies. “Rows” and “Columns” specify the number of structural constraints and decision variables respectively, while “Nonzeros” shows the number of nonzero constraint coefficients. |
|---------------------------------|------------|------------|----------|------------|
| Fuels Infrastructure            | TRS-05     |            | West/East|
|                                 | West 2006  | East 2006  |          |
| Bases                           | 37         | 41         | 68       |
| TPFDD lines                     | 250        | 191        | 204      |
| Routes                          | 79         | 48         | 289      |
| Time periods                    | 45         | 45         | 90       |
| Rows                            | 71,346     | 63,001     | 108,967  |
| Columns                         | 428,901    | 402,082    | 373,022  |
| Nonzeros                        | 7,663,812  | 6,917,393  | 3,753,174|
| Solve time                      | 2.5 hrs    | 2.0 hrs    | 2.0 hrs  |
RISC processors using GAMS calling the parallel CPLEX 6.0 barrier algorithm. The TRS-05 runs were done on a Pentium III with a 500 MHz processor using GAMS calling the CPLEX 6.0 barrier method.

4. SUMMARY
In recent years, the US military has repostured from a forward-deployed force reacting, relatively few areas of likely conflict, to a largely US-based force reacting to many disparate smaller conflicts. The resulting burdens on the strategic airlift force have become acute, and have been the subject of numerous analyses for updated aircraft, infrastructure, and concepts of operation.

NRMO has played a significant role in the conduct of these analyses, as it was designed to model the delivery of troops and cargo in the early stages of a conflict. NRMO grew out of the combined advantages of several optimization models (MOM, THRUPUT, THRUPUT II and CONOP). NRMO and its progenitors have been used to assist USAF planners in analyzing important issues concerning fleet modernization and aircraft acquisition, investment and divestment in airfield resources, and how to best use aircraft that may be utilized in multiple roles. Because NRMO has been developed over several years and utilized in multiple analyses, we have simultaneously streamlined the model for computational efficiency and expanded it to include many of the key features of the airlift system. We believe it has been instrumental in identifying key areas for improvement of this critical national resource.

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