Determining the Optimal Transmission System Usage Contracts for a Distribution Company

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Abstract—Improvements in transmission and distribution networks can be noticed in most countries that had their system architecture changed by the deregulation process. In this new environment one of the biggest challenges is the transmission and distribution open access. In Brazil, the National Electricity Regulatory Agency has established that the monthly amount of transmission system usage contracted by a distribution company (DISCO) should be informed per connection point (one value for each point) between transmission and distribution network. The usage of the transmission assets are represented by the power flowing from the transmission system to the DISCO network. This implies in monthly charges at each border transformer (which represents a connection point) that the DISCO must pay to honor the contracts. If the DISCO exceeds the contract values by certain percentage, monetary penalties are incurred. The penalty costs can lead the DISCO to a more conservative behavior at the time that the usage contracts are settled. On the other hand if the DISCO expects a low trend of its demand it has the possibility to contract less and save money. Determining the optimum amount to contract is a stochastic optimization problem because of future load uncertainties. This paper provides model formulations for the Transmission System Usage Problem with a real case study.

Index Terms—Transmission System Usage, Cluster Analysis, Mixed Integer Programming, Stochastic Programming.

I. NOMENCLATURE

ANEEL: Brazilian Electricity Regulatory Agency
ISO: Independent System Operator
DISCO: Distribution Company
TRANSCO: Transmission Company
CP: Connection Point between TRANSCO and DISCO
TSU: Transmission System Usage

\[ T_G_i: \text{ Transmission Charges at CP i [R$/month]} \]
\[ x_i: \text{ TSU Contract Amount at CP i [MW]} \]
\[ d_i: \text{ Real Usage (Demand) at CP i [MW]} \]
\[ \rho: \text{ Penalty Multiplier Term with value equal to 3} \]
\[ y: \% \text{ allowed to extrapolate at each CP [5%]} \]
\[ P_i: \text{ Exceeded demand at CP i if there is penalty [MW]} \]
\[ y_i: \text{ Binary variable to capture if there is penalty at a CP} \]
\[ \Phi(d): \text{ Probability Density Function of the Demand at a CP} \]
\[ f(x,d): \text{ Cost function at a CP with random demand} \]
\[ g'(\cdot): \text{ First Derivative (with respect to the demand) of the Expected Cost Function} \]
\[ g''(\cdot): \text{ Second Derivative (with respect to the demand) of the Expected Cost Function} \]

II. INTRODUCTION

The restructuring process in the Electricity Power Sector aimed to introduce a competitive market environment for the system participants. Most of the companies were segregated into four segments: generation, transmission, distribution and commercialization. One of the biggest challenges of this new configuration is the transmission and distribution open access [1]. In order to be supplied with electricity, most of the time a DISCO has to use the TRANSCO network. Consequently, the DISCO must pay for the use of the TRANSCO assets. In Brazil, ANEEL has established that the DISCO has to settle a contract with the TRANSCO with respect to the amount of transmission system usage. The contract values have to be informed by the DISCO for each year and for each connection point (CP) between the two networks.

The values of transmission system usage (TSU) must be contracted before hand and the amount established will be valid for every month over the next year. The usage of the system implies in monthly charges at each CP that the DISCO must pay to honor the contract. Once the values are established, the Investment Cost Related Price [2]-[3] methodology is used to determine the transmission wheeling charges for the DISCO at each individual CP.

The contracted amount is based on each day period (peak and off-peak). It is associated with the maximum verified demand at the CP. Besides the usage costs, if in a particular month the DISCO exceeds 5% of the contracted values penalties are incurred. The penalty costs will be evaluated as
three times the value of the tariff applied to the difference between the contracted amount and the verified usage [4].

The penalty costs can lead the DISCO to a more conservative behavior at the time that the usage contract is settled. In order to avoid penalties the company can decide to contract more than it should. Unfortunately, this procedure generates unnecessary expenses most of the time. In other words, the DISCO contract is usually based on the maximum demand over the 12 months, but this amount is not totally used during the remaining of the year. From an economic point of view another scenario must be analyzed where the DISCO contracts less and pay penalty in some months of high demand but save money in others.

The optimal TSU values can be assessed by solving optimization problem. The uncertainty of the future demand and generators dispatches makes this problem stochastic. One approach is to represent the maximum flow at each connection point as a random variable in order to obtain the distribution probability of the transmission charges as a function of the contracted values [5].

This paper presents two methods of computing the optimal contracts. The problem can be defined as a Non-linear optimization problem over an unconstrained set. Some modeling techniques will be used in this work to create a Mixed Integer Linear [6] version of this problem. Basically, this method is more appropriate when the DISCO has a forecast of the future demand for the next year or when it has a small number of scenarios for the demand behavior.

Under demand uncertainties stochastic optimization will be used to formulate the problem. Initially, based on past demand data the DISCO substations are segregated in groups with high correlation using clustering analysis [7]. Then each group is modeled as a Normal random variable and Monte Carlo technique is applied to generate scenarios for each month [8]. Using DC load flow the DISCO usage at each CP is determined for each scenario. Then it is possible to compute the probability density function for the power flows at each CP. This stochastic version of the problem uses the idea of the newsvendor problem [9] to model the cost function and bisection line search [10] to find the optimal amount to contract.

It is important to notice that the amount contracted at each CP will change the power flow and consequently the tariff at its neighborhood since it is based on the ICRP method. However it will be considered as a constant parameter for simplification. The dispatch of generators inside the DISCO network will also be considered as a constant for the purpose of this paper.

The next sections of this paper are divided as follows: Section III provides the problem characteristics and the motivations for use of mathematical programming models. Section IV presents the modeling section where it is described the problem formulations. Section V provides the modeling part of the load flows that will be used as input of the model. Section VI presents a study case of a DISCO network that has 8 CPs with the TRCO system. Finally Section VII concludes the paper.

III. PROBLEM CHARACTERISTICS AND MOTIVATION

Ideally, the optimal contract for a CP would be the one which is exactly the demand value at the connection point, i.e. it is neither short nor long than the demand. If the contract is long, the company would waste money because the TSU is less than the usage amount settled. If the contract is short at a CP, the company would have to pay penalty at that particular point. Therefore, the best solution would be the one in which the real flows, observed during the month at each CP, corresponds to the exactly contracted usage.

The energy flow at each CP with the transmission grid varies according to the many factors: consumer demand, generators dispatches and power grid configurations. Changes on the grid configuration (for the transmission and distribution systems) are produced from the forced outage or scheduled maintenance of the grid elements (transmission lines, transformers, circuit breakers, etc.). Since nowadays there is no penalty if the DISCO excesses the contract values due to outage of a distribution or transmission line the grid configuration will not be modeled here.

The dispatch of generators connected to the DISCO network can influence in amount the energy that goes from the TRANSCO to the DISCO in the following sense, if a generator on the DISCO network produces less than expected it may increase power flow in the CP. The same is true for consumer demand but in the opposite direction, i.e. the higher the consumer demand the higher is the energy flow required at the CP.

A. Usage Costs Evaluation

In order to evaluate the costs related to TSU of the DISCO only the usage amounts contracted for the demand peak period are considered by ANEEL. It is important to mention that the DISCO has to declare a period of 3 hours of the day that it is most likely to have its highest load. Generally this peak period is declared by the DICOs from 18:00 P.M. to 21:00 P.M. In order to calculate the monthly transmission charges related to any CP it is used (1).

\[ T C_i = t_i \cdot x_i \quad \forall i = 1, ..., n \]  \hspace{1cm} (1)

If there is some violation greater than 5% of the contracted value at a particular connection point we need to add the penalty term. It is important to emphasize that if there is penalty it will be charged on the whole extrapolated amount, not just for what exceeds the contract values plus 5%. If the extrapolated amount is within 5% there is no penalty. Therefore, the real monthly charge is presented in (2).

\[ T C_i = \begin{cases} t_i \cdot x_i & \text{if } d_i \leq (1 + \gamma) x_i \\ t_i \cdot x_i + \rho \cdot t_i \cdot (d_i - x_i) & \text{if } d_i > (1 + \gamma) x_i \end{cases} \]  \hspace{1cm} (2)

It is shown in Fig. 1 a graph based on (2) for any particular CP between the TRANSCO and the DISCO.
The tariff \( t_i \) was set aside just to illustrate the cost function since it is constant and appears in all terms. There is a discontinuity point at the cost function exactly where the real demand is equal to the TSU contract value multiplied by \((1 + \gamma)\).

The cost function is divided into two pieces. The piece at the left in Fig. 1 represents the case where \( d > (1 + \gamma) x \), the maximum value of this line is represented at the upper left point where the DISCO does not contract anything at a particular CP. In this case the \( TC_i \) for that CP is equal to \( \rho * t * d \). This line decreases until the point where the demand is almost \((1 + \gamma) x \). When the demand is exactly equal to \((1 + \gamma) x \) the discontinuity point is reached. The line at the right represents the case where the DISCO contracts more than the demand or the value of the demand is within \((1 + \gamma) x \). Note that this line increases linearly with the amount contracted by the DISCO.

B. Contract Strategy

Nowadays in Brazil, a lot of discussions about this problem have been raised. It is an important issue because the DISCO TSU contracts are used to simulate future expansions of the Interconnected Power System. As a result of this simulations the ISO obtain information about necessary future improvements for the system as building new transmission lines and power generators. So besides the monetary values that the DISCO has to pay if contract more or less than its demand it is also not beneficial for the whole system if the DISCO contract erroneously its TSU.

In order to get more insight of this problem let’s assume that the future demand is deterministically known. If this is the case, the optimal amount to be contracted by considering just one month should be the one that minimizes the TC function for each connection point. This value is exactly the one where \( d = (1 + \gamma) x \). But, as mentioned before the DISCO has to inform only one value for the whole year for each CP (not one value for each month) so the optimal amount is the one that minimizes the sum of the TC over the twelve months of the year. For this problem, by considering the future demand known it is possible to create an optimization model where the optimal solution will give exactly the \( x^* \) that minimizes the cost function.

However the future demand is uncertain and also other factors may affect the flow at the CP. So it is almost impossible to get the exactly \( x^* \) that minimizes TC. But it is possible to optimize the TC function for a given collection of scenarios based on the future demand distribution function and get a better prediction of \( x \) to be used in the TSU contract. To address this version of the problem stochastic programming techniques will be used.

IV. PROBLEM FORMULATIONS

A. Mixed Integer Linear Programming Formulation

A MILP formulation of the TSU problem for one month can be defined as follows:

\[
\begin{align*}
\text{minimize} & \quad \sum_{i \in I} t_i * x_i + \rho * t_i * P_i \\
\text{s.t.} & \quad P_i \geq d_i - (1 + \gamma)x_i & \forall i \in I \\
& \quad P_i \geq d_i - x_i - C(1 - y_i) & \forall i \in I \\
& \quad P_i \leq C * y_i & \forall i \in I \\
& \quad y_i \in \{0,1\} & \forall i \in I \\
& \quad P_i \geq 0, x_i \geq 0 & \forall i \in I
\end{align*}
\]

Where \( i \in I \) is the set of CP between the TRANSCO and the DISCO; \( C \) is a big number; \( y_i \) is a decision variable that represents the TSU amount contracted by the DISCO at CP \( i \); \( y_i \) is a binary decision variable that is responsible to identify the existence of penalty; \( P_i \) is a decision variable that is responsible to calculate the amount of TSU that the DISCO exceeded from \((1 + \gamma) x_i \) at CP \( i \) and it is used in the objective function to compute the penalty value; \( d_i \) is just a parameter that represents the realized demand at CP \( i \). Eq. (3) corresponds to the objective function of this problem where the goal is to minimize the cost function over all CP. The constraints (4) through (6) allow two cases for the model:

a) \( P_i = 0 \) at a particular CP when its demand is smaller than \((1 + \gamma) x_i \)

b) \( P_i = d_i - x_i \) in the opposite case.

At case (a), constraint (4) is redundant because the right-hand-side will be negative and according to (8) \( P_i \) must be greater than or equal to zero. In this case, since it is a minimization problem the variable \( y_i \) will be equal to zero. This will imply that (5) is also redundant because \( C \) is a big number and constraints (6) and (8) will make sure that \( P_i = 0 \). At case b, constraint (4) will be active because \( d_i - (1 + \gamma) x_i \) is greater than zero. In order to satisfy constraint (6), \( y_i \) must be equal to one. This will activate constraint (5). Again because it is a minimization problem \( P_i \) must be the smaller possible value provided that constraints (4) and (5) are satisfied. In this case, \( P_i = d_i - x_i \).

Note that the sets of constraints (4) and (6) can be joined to create only one constraint set as (9). The number of variables for this problem is \( 3|I| \) and the number of structural constraints using (9) instead of (4) and (6) is \( 2|I| \).

\[
C * y_i \geq d_i - (1 + \gamma) x_i & \forall i \in I
\]

It is possible to state the TSU formulation for one year as...
follows:

\[
\text{minimize } |M| \sum_{it} t_i x_i + \rho * t_i \sum_{m \in M} P_{im} \quad (10)
\]

s.t. \[ C * y_{im} \geq d_{im} - (1 + \gamma) x_i \quad \forall \, i, \forall \, m \in M \]

\[ P_{im} \geq d_{im} - x_i - C(1 - y_{im}) \quad \forall \, i, \forall \, m \in M \]

\[ y_{im} \in \{0, 1\} \quad \forall \, i, \forall \, m \in M \]

\[ P_{im} \geq 0 \land x_i \geq 0 \quad \forall \, i, \forall \, m \in M \]

Where M is the set of months of the period analyzed (in this case 12 that corresponds to one year). Notice that now instead of \( d_i \), \( y_i \), and \( P_i \), the model has \( d_{im} \), \( y_{im} \), and \( P_{im} \). These variables and parameters have the same concept as before except that now they are defined for each month. This model has \(|I| + 2|I||M|\) decision variables and \(2|I||M|\) structural constraints.

Now suppose a set of scenarios \( \Omega \) generated by Monte Carlo. The variables \( x_i \) will not change but there will be third index for \( d_i \), \( d_{im} \), \( P_i \), \( P_{im} \), and \( y_i \). \( y_{im} \) increasing the number of variables to \(|I| + 2|I||M||\Omega|\). The constraint (11)-(14) will be defined for each scenario leading to a total of \(2|I||M||\Omega|\) structural constraints.

These formulations are nice because allow the DISCO to perform scenario analysis where they model the behavior of the system as they expect to be. Although they work for the stochastic case the computational effort to solve this problem increases rapidly with the number of scenarios.

**B. Stochastic Linear Programming Formulation**

Another possibility that can be used to model the TSU problem is Stochastic Programming. Recall the Transmission Charges Cost function (2) and Fig. 1. Because of the discontinuity point between the two lines a possible approach is to consider the demand as a random variable that is governed by a particular probability density function represented by \( \phi(d) \) and a cumulative distribution function denoted by \( \Phi(d) \). Consider a single CP and let the cost function for that particular CP be denoted as \( f(x, d) \), where \( x \) is the contract amount and \( d \) is the random demand vector. Now it is possible to create a new optimization problem (15).

\[
\text{minimize } E_d f(x, d) \quad (15)
\]

s.t. \[ x \geq 0 \]

The objective of this problem is to minimize the expected value of the function \( f(x, d) \) given by (2).

Now, taking the expected value with respect to \( d \) (17) is obtained.

\[
E_d f(x, d) = E_d [tx] + E_d [\rho t (d - x)]
\]

\[
E_d [tx] = tx + \int_{(1+y)x}^{\infty} \rho u (1+y) \Phi(u) \, du = tx - \rho tx - \rho tx y \Phi(1+y) - \rho \int_{(1+y)x}^{\infty} \Phi(u) \, du
\]

Note that \( t \) in this model is just a constant, therefore it will be omitted during the analysis of (17). Assume that \( g(\cdot) \) is a function that represents (17) without considering \( t \). It is possible to check that this function is continuous twice differentiable function for many types of distributions that may model \( d \). However it is not the purpose of this paper to show such result, instead we will consider that the first \( (g'(\cdot)) \) and second derivatives \( (g''(\cdot)) \) with respect to the demand in (17) exist and are given by (18) and (19) respectively.

\[
g'(\cdot) = 1 - \rho - \rho y (1+y) x \Phi(1+y) + \rho \Phi(1+y) x \]

\[
g''(\cdot) = \rho (1+y)^2 (1+y) x \]

The idea here is to set (18) equal to zero and use the bisection line search method to find the optimal point that minimizes the function. If (17) is a convex function it is possible to apply the bisection procedure without worrying about convergence of the algorithm. To check convexity of the function one can check if \( g''(\cdot) \geq 0 \). If that is the case (17) is a convex function otherwise it is not. If (17) is not convex means that it is possible to have more than one minimum point (local minimum) and one global minimum, in this case the bisection procedure may fail to achieve the optimal point.

Another way to check the convexity of the function is to analyze (18). If (18) is an increasing function in \( x \) it means that (17) will be convex and will have just one minimum point. When presenting the numerical example it will be shown that this is indeed true. For now let’s just assume it is convex. Therefore the minimum will be achieved at \( g(\cdot) = 0 \) and corresponds to the solution of (20).

\[
\Phi(1+y)x = 1 - \frac{1}{\rho} + xy(1+y) \Phi(1+y) x \]

The solution to (20) is not straightforward so it necessary to use the bisection method. Let \( h(x) \) be the difference between the left hand side and the right hand side of (20).

\[
h(x) = \Phi(1+y)x - 1 + \frac{1}{\rho} - xy(1+y) \Phi(1+y) x \]

It is important to notice that (21) represents the equation to find the minimum \( x \) for just one CP and one month. The same idea can be extended for the other CPs and 12 months. Let’s assume that \( d_{im} \) is a random variable corresponding to the demand at CP \( i \) in month \( m \) with cumulative density function \( \Phi_{im} \) and probability density function \( \phi_{im} \). Assuming independence among the months and the CPs it is possible to define (22) for all \( i, m \).

\[
h(x_k) = \sum_{i, m} \left( \phi_{im} (1+y) x_i - 1 + \frac{1}{\rho} - xy(1+y) \phi_{im} (1+y) x_i \right)
\]

So the bisection procedure utilizes (22) to find the optimal \( x \) values for each \( i, m \). The algorithm can be described as follows:

1. Consider the interval \([a_k, b_k]\) with \( h(a_k) < 0 \) and \( h(b_k) > 0 \), set a tolerance factor \( \delta > 0 \) and a counter \( k = 1 \).
2. Set \( x_k = 0.5 (a_k + b_k) \) and compute \( h(x_k) \). If \( h(x_k) = 0 \) stop and output \( x_k \) as the optimal solution, otherwise go to step 3.
3. If \( h(x_k) > 0 \) then set \( a_{k+1} = a_k \) and \( b_{k+1} = x_k \). Otherwise, \( h(x_k) < 0 \), set \( a_{k+1} = x_k \) and \( b_{k+1} = b_k \).
4. If \( b_k + 1 - a_{k+1} \leq \delta \) stop and output \( x_k \) that is in the interval \([a_{k+1}, b_{k+1}]\). Otherwise set \( k = k + 1 \) and go to step 2.

**V. DISCO DEMAND MODELING**

One of the most important parameters that have direct influence on the load flows at the CP is the DISCO consumers’ demands. As mentioned before the load flows at
the CPs are measured at the border transformers that connect the TRCO network and the DISCO network. The idea is to model the DISCO substations loads that represent the consumers’ loads inside the distribution concession area. After modeling these loads the goal is to simulate the Optimal Power Flow to obtain the flows at the CPs and then use these flows as input for the models described in the previous section. For the MILP problem the values are simply plugged in the model. But for the stochastic problem the scenarios are used to model the probability density function for the demands at each CP.

Generally the DISCOs have operational database for bookkeeping of the utilization of their assets. In this database the parameters of interest here are the maximum demands for each substation, for each hour and day of the year.

A. One Case Scenario Modeling

For the one case scenario using the Mixed Integer Linear Model there are two options described next.

a. The company has a prediction of the future maximum demand of the DISCO for each month of the next year separated for each substation. This data will be used to simulate Optimal Power Flow for each month;

b. The company has a prediction of how much its demand will increase next year and uses this percentage together with the past data to calculate the future demand that will be used to simulate the Optimal Power Flow for each month;

At this situation the DISCO may want to perform some different analysis as varying the dispatch of some power generator that may affect the flows at the CPs; deactivate some interconnection line of the network. In this approach there is the possibility of getting some insight into how much the flows would change (and consequently the optimal value) by using some specific modifications.

B. Multi Scenario Modeling

For the multi scenario the goal is to create the probability density function of the flows at each CP that will be used as input of the stochastic optimization model described in Section IV. In order to perform such accomplishment it is necessary to use the data provided by the company database. Now instead of just the maximum demand of each substation for each month it is necessary to have the maximum demand for each day at a specific time slot. Because of the magnitudes of the substations’ demands are very different in some cases, it is used a procedure to normalize the data. The normalized data is then used as input of the clustering analysis that is performed to reduce the size of the problem.

1) Data Normalization

In order to normalize the data it is used (23) and (24) that corresponds to the average and standard deviation for the data set.

\[
\bar{d}^s = \frac{\sum_{j,m} d_{jm}^s}{|M| |J|}
\]

\[
\sigma = \sqrt{\frac{\sum_{j,m} (d_{jm}^s - \bar{d}^s)^2}{|M| |J|}}
\]

Where \( j \in J \) is the set of distribution substations, \( m \in M \) is the set of months, \( d_{jm}^s \) is the demand of substation \( j \) in month \( m \) in [MW], \( \bar{d}^s \) is the average of the data set and \( \sigma \) is the standard deviation. To get the normalized data set (25) is used, where \( d_{jm}^s \) is the normalized demand of substation \( j \) in month \( m \).

\[
d_{jm}^s = \frac{d_{jm}^s - \bar{d}^s}{\sigma}
\]

2) Clustering Analysis

Once the data set is normalized, the clustering process of the substations starts. Because of the large number of substations and possible existence of correlation between the loads of these substations a clustering technique is used for decreasing the size of the problem and keeping the correlated substations together. The K-means method \([7]\) is utilized in order to perform such part of the analysis. The K-means method separates \(|J|\) entities in \( K \) clusters. The proximity measure selected to use with the method is correlation between the substation’s demands. The K-means procedure can be summarized as follows:

a. \( K \) elements are chosen randomly between the \(|J|\) entities of the considered set. These \( K \) elements are considered as the center points of each cluster created.

b. It is calculated the distance measure in the \(|J|\)-dimensional vector. For this case, it is calculated the correlation between the remaining elements of the set \( J \) and the \( K \) center points of each cluster. The next element to be placed in a particular cluster is the one with largest correlation measure with the center point of that cluster.

c. It is recalculated the coordinates of each one of the \( K \) center points. The new coordinate will be the one resulting from a combination of all the elements inside the cluster.

d. Steps b and c are repeated until there are no more alterations of the elements grouped at each cluster.

3) Scenario Generation

In order to generate the scenarios, the average \((\mu_k)\) and standard deviation \((\sigma_k)\) of each cluster \( k \in K \) is computed using (23) and (24) except that the set \( J \) is replaced by the set of substations belonging to the cluster \( k \).

Suppose \( \Omega \) is the set of scenarios. Then, for each \( \omega \in \Omega \) a normal random variable is obtained for each cluster based on its average and standard deviation as in (26).

\[
\text{Normal}(\mu_k, \sigma_k) = \mu_k + \sigma_k * \text{Normal}(0,1)
\]

Using this procedure it is possible to assure that the high correlated demands will have the same behavior at each scenario. Therefore the generated scenarios are more realistic. In order to respect the seasonal variations the same generated normal random variable is be used for all months. Given that \( j \in J \) belong to cluster \( k \), the generated demand \( P_{f_{jm}} \) is
computed using (27) for all $m \in M$.  

$$PL_{jm} = \frac{d^2f_{jm}}{\mu_{CLK}} \cdot \text{Normal}(\mu_{CLK}, \sigma_{CLK})$$  \hspace{1cm} (27)$$

Usually the DISCO has a prediction of how much its demand will increase next year. This percentage growth is applied to all $PL_{jm}$. Based on this generated demands the power flow at each connection point is calculated for each scenario and each month.

VI. STUDY CASE

The DISCO in study has 84 substations inside its concession area that provides energy for all the customers. There are also 6 generators that are DISCO’s property but all of them have centralized dispatch executed by the ISO. The DISCO has 8 connection points with the transmission grid as shown in Fig. 2.

![Fig. 2. DISCO Concession Area](image)

The historical data available corresponds to the maximum demand for each hour of the day at each substation spanning the period from January 2005 to December 2008. The clusters for the 84 substations created by the K-means method are presented in Fig. 3.

![Fig. 3. Clusters Generated by K-means Method](image)

The dispatches of the generators inside the DISCO area were also analyzed for the same period. Just to illustrate Fig. 4 shows the dispatch of the six generators for 2008. It can be observed that the variations are not very expressive. Therefore for simplicity it will be consider constant for all scenarios.

![Fig. 4 – Generators dispatch for 2008](image)

A. Results Using the MILP Model

Based on the 10 clusters, 1000 scenarios were generated to model the demand at the CPs with the transmission grid. This model does not consider independence among CPs and months. The solution is presented on TABLE I.

<table>
<thead>
<tr>
<th>$x_i$ (MW)</th>
<th>$I / \text{R$/kW.month}$</th>
<th>$TC / \text{R$/ano}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S. Jose 138kV</td>
<td>1.111,17</td>
<td>4,765</td>
</tr>
<tr>
<td>Adriano 138kV</td>
<td>1,51</td>
<td>4,752</td>
</tr>
<tr>
<td>Grajaú 138kV</td>
<td>1,491,31</td>
<td>4,819</td>
</tr>
<tr>
<td>Jacarepaguá 138kV</td>
<td>658,23</td>
<td>4,838</td>
</tr>
<tr>
<td>C. Paulista 138kV</td>
<td>99,47</td>
<td>4,609</td>
</tr>
<tr>
<td>Sta Cruz 138kV</td>
<td>101,32</td>
<td>4,798</td>
</tr>
<tr>
<td>São Conrado 138kV</td>
<td>35,75</td>
<td>4,802</td>
</tr>
<tr>
<td>Brasmar 138kV</td>
<td>72,81</td>
<td>4,798</td>
</tr>
</tbody>
</table>

B. Results Using the Bisection Method

For this numerical example considering a normal distribution for the demand it is possible to verify that $g(x)$ will be a convex function by checking that the first derivative is an increasing function. In Fig. 5 $g'(x)$ is plotted with respect to $x$ for a specific connection point. The same behavior was observed for the other connection points and other distributions and for many cases simulated.

![Fig. 5 – Function $g'(x)$ for a connection point](image)

For the normal distribution it was also verified that (20) has just one root as depicted in Fig. 6 for a connection point. The green curve corresponds to the right hand side and the blue
The results for all the connection points are shown on TABLE II.

TABLE II. Results Obtained with Bisection Method

<table>
<thead>
<tr>
<th>Connection Point</th>
<th>MILP</th>
<th>Bisection</th>
</tr>
</thead>
<tbody>
<tr>
<td>S. Jose 138kV</td>
<td>1.157,51</td>
<td>4.765</td>
</tr>
<tr>
<td>Adriano 138kV</td>
<td>1.51</td>
<td>4.752</td>
</tr>
<tr>
<td>Grajan 138kV</td>
<td>1.553,58</td>
<td>4.819</td>
</tr>
<tr>
<td>Jacarepaguá 138kV</td>
<td>684,67</td>
<td>4.838</td>
</tr>
<tr>
<td>C. Paulista 138kV</td>
<td>103,06</td>
<td>4.668</td>
</tr>
<tr>
<td>Sta Cruz 138kV</td>
<td>105,49</td>
<td>4.798</td>
</tr>
<tr>
<td>Nilo Peçanha 138kV</td>
<td>37,09</td>
<td>4.802</td>
</tr>
<tr>
<td>Brasam P 138kV</td>
<td>75,49</td>
<td>4.776</td>
</tr>
</tbody>
</table>

The method provides similar solution validating the independence assumption of the bisection method. Also, the computational effort to solve the MILP model is very high. Therefore the model is more suitable for the deterministic case when the DISCO have a demand forecast for next year or a small set of scenarios whereas the bisection can only be applied to the stochastic case.

Considering the current DISCO contract at one basis point, Fig. 7 relates both model results to these values. It is possible to observe that the DISCO is over contracting at most of the CPs.

VII. CONCLUSION

This paper proposed a methodology to set the contracted values at each connection point between Disco and Transco networks. In Brazil, the DISCOs usually uses a heuristic approach that tries to forecast the demand at each connection point based on historical data and their own experience in order to define the values to be contracted. This paper presented two methods to optimize the contracts of transmission system usage. The proposed method can help the decision makers to find the best way to contract based on a risk neutral approach.

A numerical example for a Brazilian distribution company was discussed, provided real data for the last 3 years of its substations loads. Both approaches produced similar results and proved that the current Brazilian DISCO approach based on over contracting is leading to unnecessary expenses.

VIII. ACKNOWLEDGMENT

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IX. REFERENCES

X. BIOGRAPHIES

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