Overview of the self-sustaining mechanisms of wall turbulence

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Abstract

This paper reviews the ideas about why wall turbulence is self-sustaining. The paper is to distribute this information to fluid dynamists who are nonspecialist in turbulence. The production of turbulence and Reynolds stress centers on vortices and "streaks" of low speed fluid near the wall. There are two main categories of self-sustaining mechanisms. In one category, parent vortices interact with the wall and produce offspring vortices. In the second category, workers view the mechanisms as instabilities. Streak velocity profiles, profiles where low velocity fluid has accumulated in long streamwise regions of small spanwise extent, are unstable. These regions are caused by streamwise vortices in the near-wall region. The most dangerous perturbation is a sinuous streamwise mode. The mode may be a normal mode or a transient-growth mode. Ultimately the nonlinear development produces a streamwise vortex. In turn, the vortex can reproduce the streak profile. A third category is based on a common mathematical approach. The goal is to construct a low-order dynamical system of differential equations that display the elemental processes of turbulence.

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Nomenclature

A\^{(m,n,q)}(t) \quad \text{amplitude coefficient for basis function } n, m, q

c^+ = c/u_* \quad \text{phase velocity in inner variables}

C_i \quad \text{inner constant}

D \quad \text{drag force in Theodorsen’s figure}

d = \alpha \cdot S \quad d = d(\text{strain})/ds \quad \text{strain vector}

f_0(y^+) \quad \text{inner region stress function}

G(Y) \quad \text{outer region stress function}

h \quad \text{channel half-height}

k_x^+ = k_x u_* / u_0 \quad \text{wave number in inner variables}

L \quad \text{lift force in Theodorsen’s figure}

L_x^+ \quad \text{dimensions of minimal channel}

M(t) \quad \text{mean profile component in Waleffe model}

P = \frac{\langle u w \rangle d U / u_*}{u_*^2} \quad \text{turbulence production}

q \quad \text{velocity in Theodorsen’s figure}

R_{ij} \quad \text{two-point velocity correlation tensor } \langle u_i(y)u_j(y') \rangle

R \quad \text{correlation coefficient in outer variables}

R_{ij} \quad \text{correlation coefficient in inner variables}

Re_* = u_* h / v \quad \text{or } Re = U_0 h / v \quad \text{Reynolds number}

S \quad \text{or } S_{ij} = \frac{1}{2} \left( \frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) \quad \text{strain rate tensor}

t^+ = u_*^2 / v \quad \text{inner time scale}

u_* = (\tau / \rho)^{1/2} \quad \text{friction velocity}

u_i = u_i^A + u_i^f \quad \text{active and inactive components}

\langle uw \rangle \quad \text{Reynolds stress}

u \quad \text{\(x\)-velocity fluctuation}

U_0 \quad \text{centerline or free-stream velocity}

U(y) \quad \text{mean velocity}

U_b(y) \quad \text{base flow profile}

U(t) \quad \text{mean velocity}

\langle uw \rangle dU / dy \quad \text{turbulent kinetic energy}

v \quad \text{\(y\)-velocity fluctuation}

v_i(y) \quad \text{perturbation shape in } y \text{-direction}

V(t) \quad \text{roll profile component in Waleffe model}

w \quad \text{\(z\)-velocity fluctuation}

W(t) \quad \text{instability mode component in Waleffe model}

W(Y) \quad \text{law of the wake}

x \quad \text{flow direction (streamwise)}

x^+ = u_* x \quad \text{inner distance variable}

x_{in}(y^+) \quad \text{typical inner function}

X(y^+, Re_*) \quad \text{typical composite function}

X_{out}(Y = y^+/Re_*) \quad \text{typical outer function}

X_{com}(Y = y^+/Re_*) \quad \text{typical common part}

X_{wake} = X_{out} - X_{com} \quad \text{typical wake function}

y \quad \text{normal from the wall}

y^+ = u_* y / v = Y Re_* \quad \text{inner distance variable}

Y = Y \quad \text{outer distance variable}

\tilde{Y} = \frac{Y}{h} \quad \text{transverse direction}

z \quad \text{transverse direction}

z^+ = u_* z / v \quad \text{inner distance variable}
1. Introduction

This article has a little history. Sometime ago, I was asked to edit a monograph [1] and I chose as the topic; “Self-Sustaining Mechanisms of Wall Turbulence.” The monograph contained the thoughts of 16 author groups on the subject. Subsequently, a NSF-AFOSR sponsored workshop was held in conjunction with the 29th AIAA Fluid Dynamics Conference in 1998. For the workshop all authors in the monograph were invited to update
their work. In addition, four noted researchers were invited to critique the papers (Bogard [2], Smits [3], Moser [4] and Jimenez [5]). Workshop presentations are available in AIAA papers 98-2959 through 98-3002 and a summary was given as AIAA 99-0552. This article relies on that background, but includes some very significant recent developments.

Turbulent wall layers have several characteristics. The fluctuations are three-dimensional, undergo diffusion, have a continuous spectrum, and are self-sustaining. The term “turbulent structure”, “coherent structure”, or “eddy” is used to indicate a fluid motion that has coherence over a spatial region and lasting for a reasonable period of time. The study of these structures, their relationships and interactions is essential in order to define simplified mechanisms that give turbulence its self-sustaining characteristic. Because the product of Reynolds stress and mean shear give the average production of turbulent kinetic energy, \( \langle uw \rangle dU/dy \), there is a significant relation between the production of Reynolds stress and the self-sustaining nature of turbulence. However, this relation is “on the average” and one must look at instantaneous events to give a complete story. In a certain sense, the Reynolds stress is a byproduct of the self-sustaining mechanisms.

There are usually two reasons cited for studying the details of turbulence; as a basis for making engineering models for calculations (although this has not really happened) and to devise strategies to control turbulence. Turbulence control includes both the reduction to wall shear and the enhancement to delay separation.

The paper is organized into sections as follows. Sections 2, Background, and 3, Coherent structures, are for nonspecialists in turbulence. The properties of Self-sustaining Mechanisms are discussed in Section 4. Two broad classifications of mechanisms are, Section 5, Parent-Offspring Mechanisms, and Section 6, Instability-based Mechanisms. These are not all-inclusive groups. It was deemed appropriate to make a separate Section 7, Dynamic Systems. The common mathematical nature of dynamic systems makes it convenient to discuss them as a group. A summary section terminates the article.

### 2. Background: two-layer nature of wall turbulence

Turbulent flows in pipes, channels, and boundary layers have some common features. The flows have two overlapping regions where different physical processes dominate. In each instance, the inner region immediately next to the wall is the same, while the outer regions are different, but nevertheless qualitatively similar. Viscous effects exist in the inner region, while the outer region, with regard to the Reynolds stress and mean velocity profiles, is inviscid. The extent of the overlap regions increases with Reynolds number. The overlap region is called the log region because the mean velocity profile is logarithmic there.

#### 2.1. Velocity profiles

A common coordinate system has the origin on the wall with \( x \) in the flow direction (streamwise), \( y \) measured normal from the wall, and \( z \) in the transverse direction (spanwise or cross-stream). The mean flow is \( U \) and the fluctuations are \( u, v, w \). Parameters that describe the flow are the centerline or free-stream velocity \( U_0 \), the boundary layer thickness or channel half-height \( h \), the fluid viscosity \( \nu \), and the friction velocity \( u_* = (\tau/\rho)^{1/2} \) where \( \rho \) = fluid density, and \( \tau \) = wall shear stress. The important Reynolds number is \( Re_* = u_* h/\nu \) or \( Re = U_0 h/\nu \). Re is a factor of 20–30 larger than \( Re_* \). The outer region distance variable is

\[
Y = \frac{y}{h}
\]

while the inner distance scale, \( v/u_* \) (called a viscous unit), yields variables

\[
y^+ = \frac{u_* y}{v} = Y Re_*, \quad x^+ = \frac{u_* x}{y}, \quad z^+ = \frac{u_* z}{v}.
\]

The associated inner time scale is

\[
t^+ = \frac{t}{u_*/v^+}
\]

Ultimately, one is interested in the distribution of mean velocity \( U(y) \) and Reynolds stress \( \langle uw \rangle \). Theory indicates that for high Reynolds numbers the mean velocity can be represented by a composite function

\[
\frac{U(y)}{u_*} = f_0(y^+) + W(Y).
\]

The effect of Reynolds number is completely absorbed in the variable scale change \( Y = y^+/Re_* \). In the inner region, \( W(Y) \) is negligible and the law-of-the-wall \( f_0(y^+) \) describes the velocity. Numbers that define the various regions are somewhat arbitrary. The inner region consists of a viscous sublayer, \( 0 < y^+ < 7 \), where \( f_0 = y^+ \). This is followed by a buffer region, \( 7 < y^+ < 50–200 \) (some recent evidence (Österlund et al. [6], Zagarola and Smits [7]) suggests that this region should be the larger number), where the production of turbulent energy reaches a maximum. Next comes an overlap or log region, \( 50 < y^+ \) and \( Y < 0.15 \) where the velocity profile is

\[
\frac{U}{u_*} = f_0(y^+) \Rightarrow \infty = \frac{1}{k} \ln(y^+) + C_f
\]

Finally, there is the far outer region, where the law-of-the-wake \( W(Y) \) is qualitatively the same for pipes, channels, and boundary layers. For engineering
purposes, a power law can approximate a large central part of the velocity profile.

Fig. 1 is a plot of the mean velocity profiles in a pipe for \( \text{Re}_* \) from 200 to 100,000. Empirical functions were used for \( f_0(y^+) \) and \( W(Y) \). It is somewhat amazing that these functions fit the experiments from \( \text{Re}_* = 100,000 \) down to Reynolds numbers as low as say \( \text{Re}_* = 800 \). Some adjustments in the coefficients are needed to fit at \( \text{Re}_* = 150 \), the lowest value for which turbulence exists.

It is useful to compare the thickness of the regions. Consider a boundary layer flow with \( \text{Re} = 186,000 \) (\( \text{Re}_0 \approx 27,000; \text{Re}_* \approx 6100 \)). These numbers come from tests of Österlund et al. [6]. For purposes of example assume the viscous region as \( 0 < y^+ < 50 \), the overlap (log) region as \( 50 < y^+ < 0.15 \), and the remainder, \( Y > 0.15 \), is the far outer region (outer without overlap). Some recent data indicates that the log region does not start this soon. With these numbers, the viscous region (in terms of \( Y \) is \( 50/\text{Re}_* = 50/10,000 = 1/200 \) of the boundary layer thickness. As a contrast, consider that the first DNS of channel flow had \( \text{Re}_* = 180 \). The viscous region is \( 50/180 \sim 28\% \) of the channel half-height and inner and outer regions are no longer clearly distinguishable.

2.2. Reynolds stress profiles and quadrant decomposition

Because the mean velocity profile and Reynolds stress are mathematically related, the inner–outer structure of one implies an inner–outer structure for the other. Thus, the Reynolds stress is also given by a composite expansion [8].

Here \( g_0(y^+) \) is the inner region stress distribution, \( G(Y) \) that for the outer region, and \( g_0(y \to \infty) = G(y \to 0) = 1 \) is the common part. Fig. 2 is a plot of the composite expansion for the Reynolds stress with \( \text{Re}_* \) values from 200 to 100,000. The empirical relation used in Fig. 2 has been fitted to pipe and channel flow data. These data seem to fit very nicely for all Reynolds numbers of known experiments and DNS (\( \text{Re}_* \) values from 170 to 1650). The location of the maximum of \( \langle uu \rangle \) moves with \( \text{Re}_* \). For high \( \text{Re}_* \) it moves outward in \( y^+ \) units as \( \sim \sqrt{\text{Re}_*} \), however, in \( Y \) units it moves closer to the wall as \( \sim 1/\sqrt{\text{Re}_*} \).

The Reynolds stress is caused by \( x \)-direction velocity fluctuation \( u \) and \( y \)-direction fluctuations \( v \) having a
nonzero correlation $\langle u\nu \rangle$. Wallace et al. [9] provided a major conceptual tool when they divided the contributions to $\langle u\nu \rangle$ into quadrants according to the signs of $u$ and $v$. Events where $u$ and $v$ are $+$ are quadrant one, Q1 events. Similarly $u^-$ and $v^+$ are Q2 events, $u^-$ and $v^-$ are Q3 events, and $u^+$ and $v^-$ are Q4 events. The experimentally determined distribution of events that compose the Reynolds stress is displayed in Fig. 3. Although there is significant cancellation, the results have caused workers to concentrate on Q2 and Q4 events as the major turbulence producing motions. Also shown in Fig. 3 are the quadrant contributions from DNS channel flow at two Reynolds numbers (Moser, unpublished).

2.3. Kinetic energy production

It is of interest to look at the production of turbulent kinetic energy. The product of mean shear and Reynolds stress gives turbulence production

$$P = -\frac{\langle u\nu \rangle \, \delta \langle U \rangle}{u*} \frac{\delta U}{\delta y^+}.$$  \hspace{1cm} (7)

The production curves in Fig. 4 show very little variation with Reynolds number for $Re_*$ values from 200 to 100,000. The maximum production is well within the inner region at $y^+ = 12$. One can appreciate that modeling this region for LES calculations is an extremely difficult problem.

A very encouraging aspect is the fact that the measurements of the mean velocity and the Reynolds stress show the expected theoretical Re trends. One could infer that physical processes producing $\langle u\nu \rangle$ discovered at low Reynolds numbers are likely to also be relevant at higher values. This is not to say there will not be modifications or new events, however, it is likely that the dominant processes are roughly similar.

2.4. Active and inactive motions

Townsend [10] introduced the concept of active and inactive turbulent motions. Active motions make essential contributions to the Reynolds (shear) stress. On the other hand, inactive motions are random in the sense that there is no correlation between $u$ and $v$. Consider a decomposition of the velocity fluctuations.
For a moment let us use the index notation.

\[ u_i = u_i^A + u_i^I, \]  

(8)

It is reasonable to assume that the active and inactive motions are uncorrelated. The Reynolds stress tensor is then,

\[ \langle u_i u_j \rangle = \langle u_i^A u_j^A \rangle + \langle u_i^I u_j^I \rangle. \]  

(9)

By definition when \( i = 1, j = 2 \), i.e. \( \langle w \rangle \), the inactive motion is zero.

\[ \langle u_1 u_2 \rangle = \langle u_1^A u_2^A \rangle. \]  

(10)

The composite expression for \( \langle u_1 u_2 \rangle \) was given above, Eq. (6). Do the components \( \langle u_i u_j \rangle(y) \) with \( i, j \neq 1, 2 \) scale in two overlapping regions? With complete generality, one can express the relations in inner, \( y^+ \), or outer, \( Y \), variables.

\[ \frac{\langle u_i^A u_j^A \rangle}{u^+_*} = R_{ij}^A(y^+, \text{Re}_*) \quad \text{or} \quad \mathcal{A}_{ij}^A(Y, \text{Re}_*) \]  

(11)

\[ \frac{\langle u_i^I u_j^I \rangle}{u^+_*} = R_{ij}^I(y^+, \text{Re}_*) \quad \text{or} \quad \mathcal{A}_{ij}^I(Y, \text{Re}_*). \]  

(11)

Fig. 3. Quadrant contributions to Reynolds stress. Experimental data from Wallace et al. [9]: Q1 squares, Q2 circles, Q3 triangles, and Q4 diamonds. Corresponding lines are Moser’s unpublished calculations for the DNS databases at \( \text{Re}_* = 180 \) and 520.

Fig. 4. Turbulent energy production for \( \text{Re}_* \) values from 200 to 100,000.
If the two-layer viewpoint is valid, the limit $Re_\ast \rightarrow \infty$ yields bounded functions

$$R^A_{ij}(y^+, Re_\ast \rightarrow \infty) = \bar{R}^A_{ij}(y^+)$$ and

$$\bar{R}^A_{ij}(y, Re_\ast \rightarrow \infty) = \bar{R}^A_{ij}(Y),$$

$$R^I_{ij}(y^+, Re_\ast \rightarrow \infty) = \bar{R}^I_{ij}(y^+)$$ and

$$\bar{R}^I_{ij}(y, Re_\ast \rightarrow \infty) = \bar{R}^I_{ij}(Y).$$ (12)

There is some opinion that for $ij = 11$ these relations are not satisfied. On the other hand, if the two-layer structure is valid for an active or inactive component, denoted by $X$, then the Reynolds number variation may be represented by a composite expansion of the form

$$X(y^+, Re_\ast) = X_{in}(y^+) + X_{out}(Y = y^+/Re_\ast)$$

$$- X_{com}(Y = y^+/Re_\ast).$$ (13)

This sum of an inner law and a “defect law” is uniformly valid. A defect law is the outer law minus the common part: $X_{wake} = X_{out} - X_{com}$.

2.5. Wall pressure

One way that the outer flow influences the inner region is through the fluctuating pressures. When one considers that the viscous region may be only 1/200 of the outer layer, it is easy to imagine that eddies in the outer region have associated pressures that are impressed through the viscous region onto the wall. Of course, this is not true when the viscous region is 28% of the outer layer (i.e. $Re_\ast = 180$). A spectral density function $\Phi$ can represent the space–time characteristics of the wall-pressure field and is a function of frequency $\omega$ and $x$-direction wavenumber $k_x$.

Another important characteristic of turbulence is expressed in Taylor’s hypothesis. Physically, the hypothesis is that turbulent eddies are carried past a point with a convective velocity in an essentially frozen state. The development of eddies is slow compared to the speed at which the mean flow carries them along. With these considerations a more natural set of variables for the wall pressure spectrum is wavenumber $k_x$ and a phase velocity $c$ defined by $\omega = k_x/c$. In these variables, Panton and Robert [11] found that there is an inner–outer structure to the spectrum. Fig. 5 shows a schematic of the spectrum in a contour plot using outer variables and also in inner variables. Experimental results at several Reynolds numbers are shown in Fig. 6. As the wavenumber, in outer variables, becomes large, the contours enter an overlap region of parallel levels. All similar cuts across the overlap region, a plane of constant $k_x$ or constant $c$, give the same curve when measured from the convective ridge. The spectrum is normalized to have a constant level in the overlap region. The inner variables are $\Phi(k_x, \omega)k_x^2/(\rho^2, v^2, u_\ast)$ as a function of, $k_x^2 = k_x^2 v/u_\ast$, and $c^+ = c/u_\ast$. At high wavenumbers, those approaching the Kolmogorov length, viscous effects prohibit fluctuations and the spectrum dies. Measurements in this region are difficult because finite size microphones prohibit fine spatial resolution. As the wavenumber, in inner variables, becomes small the contours enter the overlap region. Increasing the Reynolds number, expands the overlap region, and the distance between the ends of the spectrum increases.

3. Simple coherent structures and events

It is natural in the quest to understand turbulence that we use elemental flow patterns as models. The purpose of this section is to define and describe these elements in a historical context.
3.1. Horseshoe eddy structures

The aerodynamicist Theodore Theodorsen [12] proposed one of the first turbulent elements. He imagined that horseshoe (hairpin) vortices arose from the wall to transport fluid and produce Reynolds stress. The ends are located near to the wall. Fig. 7 was drawn by his daughter and appears in Theodorsen [13]. Jim Wallace has the original drawing in his laboratory. As an addition to the hairpin, the picture shows secondary and tertiary vortices that spring from the main structure. In keeping with aerodynamics custom, the velocity is denoted by \( q \). Also notice that lifting line concepts are used to imagine that the vortex has a lift force \( L \) and a drag force \( D \).

Townsend [10], without any reference to Theodorsen, proposed an attached eddy concept. Fig. 8 is taken from the 1976 version of Townsend’s book. The double cone vortices were envisioned to be “attached” to the wall along a line and extend into the log layer. Townsend [14] states “Our concept of an attached eddy is of a flow pattern which is of finite size, mechanically coherent and resistant to disintegration. If energy is supplied to or removed in one part of the pattern, energy flows to or from that part to preserve the structure”. Peter Bradshaw gave me the following characterization of Townsend’s concept: “The double-cone “attached eddy” Townsend deduced from correlation measurements is best described as a Headless Horseshoe—In other words I think it is the same thing as the (symmetrical or asymmetrical) horseshoes everybody else talks about today. Like all concentrations of vorticity it moves with (the mean motion of) the fluid. One can imagine the horseshoe arising from a small, local, upward perturbation of a spanwise vortex line in a shear flow: the head of the incipient horseshoe is dragged downstream, the legs are stretched and rotate faster, and the velocity induced by the legs is in a sense that pushes the head upward (into a higher velocity region). With an eye of faith one can regard the elongated legs as streaks—though in pairs rather than a continuous distribution. I think that Townsend used the word “Attached” to distinguish between a large eddy that was influenced by the impermeability condition and the smaller eddies in a wall flow which are not.” Thus, the current view is that the Horseshoe (Hairpin) and Attached Eddy are essentially the same.

A considerable amount of detail has been added to our knowledge by examining DNS data. Robinson [15, 16], examining the boundary layer DNS results of Spalart [17], found it convenient to divide the horseshoe vortex into three parts; legs, neck, and arch, as shown in Fig. 9. He also found that the majority of horseshoes were incomplete or one sided. They are then referred to as arches. As the horseshoe is convected by a shear flow, the legs become elongated and become streamwise vortices. Streamwise, quasi-streamwise vortices, (also called rolls) are a very important element in turbulence. Although this is one mechanism to produce streamwise vortices, there are also other mechanisms.
3.2. Low speed streaks and the bursting process

Streaks or low-speed streaks were also one of the first, and, as it turns out, most important elements identified. They are observed in the inner region near the wall. In a review article Corrsin [18] discusses experiments of Beatty, Ferrell, and Richardson (unpublished?). They watched dyed fluid being replaced by clear fluid in a tube flow. Corrsin states:

The significant property seems to be the strong orientation into streamwise filaments of the residual dye.

For the same article Francis Hama supplied Corrsin with a photograph of dye seeping from a flush, cross-stream slit in the wall into a turbulent boundary layer. The dye collects into streamwise streaks in the flow as seen in Fig. 10. To continue quoting Corrsin:

Presumably this indicates a predominance of axial vorticity near the wall, "sweeping" the (dyed) wall fluid into these long narrow stripes.

The interpretation is that there are many streamwise vortices near the wall collecting fluid from very near the wall, fluid that has a very low velocity, into filaments. Since this fluid has a low speed characteristic of its former location, the term low-speed streak is often used. On the other side of the vortex, fluid from a higher speed region is being brought closer to the wall. The high-speed regions are devoid of dye because the fluid comes from further distances from the wall. In this paper, we have the first observation of streaks and speculation as to their cause.

The importance of streaks was identified and emphasized by Kline et al. [19]. Kline and co-workers went on to determine that streaks are about \( \Delta z^+ = 80–100 \) wide and can be \( \Delta x^+ = 1000 \) in length. They are not observed above \( y^+ > 40–50 \). The current concept is that relatively short, \( \Delta x^+ = 100 \), streamwise vortices are convected over the wall, bring up the low speed fluid and leave it behind in the long trails. Streaks are thus somewhat distinct from their method of formation.

The term streak originated from the behavior of flow visualization markers. Now it is also used to mean a \( U(y, z) \) velocity profile that has an oscillation in the \( z \) direction as well as a profile in \( y \). This profile has no roll or streamwise vortex component. The streak velocity profile will turn out to be important in stability considerations.

Kline's group observed that the dye would gradually lift up, oscillate and then abruptly "burst". Fig. 11 shows the process as they envisioned it. Later Bogard and Tiederman [20] showed that one streak had several ejections, which were grouped together to be called a burst. Some estimates are that bursting comprises 80% of the Reynolds stress. Burst as a name for a single event has been dropped from the vocabulary in favor of the term bursting process (or simply the old term burst) to cover the complete sequence of events. Streaks from a hydrogen bubble wire are shown in Fig. 12 (courtesy of David Bogard and Steven Trujillo; University of Texas, unpublished).

About the same time that Kline was looking at the near wall region, Brodkey was using seeded particles in the bulk flow [21]. Among the events which he added to the picture is a sweep; a packet of fluid from the outer region comes close to the wall. The bursting process is ended by a sweep. This possible sequence brings forth the question of an inner–outer interaction.

The ejection event is a Q2 Reynolds stress contribution. As a result people sometimes call any event that produces a Q2 contribution an ejection. Similarly, people frequently call any coherent event that produces a Q4 contribution a sweep.
These activities are also interpreted in terms of shear layer generation and breakdown; Johansson et al. [22]. Other events that occur in the outer region include the growth and bundling of horseshoe vortices. Head and Bandyopadhyay [23] observed that the vortices extended from the wall to the edge of the boundary layer. Although the vortices were δ units long, they maintained a constant width of 100 viscous units. They envision large scale eddies as bundles of these vortices.

Another viewpoint is due to Falco [24]. In flow visualization of the wall layer, he observed pockets that are clear of marking fluid. Pockets are formed by outer fluid moving to the wall and the edges between inner and outer fluid have eddies that scale on viscous units.

3.3. Distribution of coherent structures

Experimental researchers have measured many characteristics of the turbulent events by conditional averaging; for instance, Wark and Nagib [25]. A trigger event at a single-point starts the averaging period. When databases from DNS became available, much more information was at hand. The problem of identifying spatially coherent vortices from field variables, rather than from flow visualization markers or single point trigger schemes, became important. The first attempts at defining a vortex as a bundle of vortex lines was unsuccessful. Likewise, looking for instantaneous circular streamlines in a transverse plane proved inadequate. Robinson [16] made a detailed analysis of a DNS boundary layer. He identified vortices by an elongated low-pressure region that coincides roughly with the vortex core. Kline and Portela [26] do not think that Robinson's low-pressure criteria will identify all vortices. They propose a vortex identification method that requires a full turn in the streamlines in an x–y or y–z plane. They comment that the Robinson method of identifying vortices finds only the 50% strongest vortices. Nevertheless, they do not disagree with Robinson's conclusions.

One of the many general results of Robinson's work is the conclusion that streamwise vortices populate the inner region and transverse vortices populate the outer region. The overlap layer contains a mixture. This is depicted in Fig. 13. The concept of a hairpin vortex generally implies fluid particles in a symmetric pattern. However, Robinson [16] found that strong asymmetries are the rule, rather than the exception.

Cantwell et al. [27,28] have started to reexamine DNS data using the vortex definition of Chong et al. [29,30]. For every point in the flow they compute the discriminate of the strain rate tensor $S$:

$$\frac{\partial \omega}{\partial t} = \omega \cdot S + v \nabla^2 \omega, \quad (14)$$

3.4. Remark on vorticity

Two comments on vorticity will conclude this section. The vorticity transport equation for incompressible flow is

- Fig. 12. Low-speed streaks courtesy of David Bogard and Steven Trujillo; University of Texas.
- Fig. 13. Types of coherent structures populating different regions of the wall layer. From Robinson [16].
where $S$ is the strain rate tensor; $S_{ij} = \frac{1}{2}(\partial v_j/\partial x_i + \partial v_i/\partial x_j)$. Researchers frequently use the concept that vortex lines follow the material particles in discussing the physics of turbulence. This is valid in inviscid situations or in viscous flows with time scales that are short compared to the viscous diffusion time. Bursting is usually so fast that viscous effects are secondary. However, when one talks about a cycle, longer times are involved. If one assumes the Rayleigh diffusion distance scale, $\delta = 3.6\sqrt{(vt)}$, then the viscous diffusion time to go to $y^+ = 10$ is only $t^+ = 7.7$. To cross the inner layer, $y^+ = 50$, requires a time $t^+ = 190$. Vortex lines have only an instantaneous identity when viscous effects are important. The history of a viscous vortex line is indeterminate.

Another issue involves the intensification of vorticity by straining and turning embodied in the term, $\omega \cdot S$. Neglecting viscosity the $x$-component of the vorticity equation is

$$\frac{D\omega_x}{Dt} = \omega_x S_{xx} + \omega_y S_{xy} + \omega_z S_{xz}.$$  \hspace{1cm} (15)

It is common to interpret the term $\omega_x S_{xx}$ as vortex line stretching and the second and third terms as vortex line turning. This is actually incorrect.

Stretching a vortex line is given by the projection of $\omega \cdot S$ in the $o$-direction. If the vorticity direction is given by the unit vector $\mathbf{z}$, i.e. $\mathbf{z} = \omega/|\omega|$, the strain term is

$$\omega \cdot S = |\omega| \mathbf{z} \cdot S = |\omega| d$$

where $d \equiv \mathbf{z} \cdot S$. \hspace{1cm} (16)

The strain vector $d = \mathbf{z} \cdot S$ (Panton [33]) is the strain rate between two particles separated a unit distance in the direction $\mathbf{z}$; that is $d = dV_{strain}/ds$. In the case of Eq. (16) $\mathbf{z}$ is along a vortex line. Thus, the strain vector gives the straining motion between two particles on a vortex line. The strain vector multiplied by $|\omega|$ gives the vorticity intensification. The stretching (extensional) component is $\mathbf{z} (\mathbf{z} \cdot d) |\omega|$ and the turning (shear) component is $(\mathbf{z} \cdot d) \cdot \mathbf{z} |\omega|$.

The $x$-component of the stretching is actually the complicated expression

$$\text{Stretching} = \omega_x \{ \omega_x [\omega_x S_{xx} + \omega_y S_{xy} + \omega_z S_{xz}] + \omega_y [\omega_x S_{xy} + \omega_y S_{yy} + \omega_z S_{yz}] + \omega_z [\omega_x S_{xz} + \omega_y S_{yz} + \omega_z S_{zz}] \}.$$  \hspace{1cm} (17)

Hence, there is more to stretching than $\omega_x S_{xx}$. Vortex line turning has a similarly complicated expression and is not simply $\omega_x S_{xx} + \omega_y S_{xy}$. Fig. 15 depicts the vectors and their components.

4. Characteristics of self-sustaining mechanisms

Turbulent flows are distinct from laminar flows because of the Reynolds stresses. Researchers agree that in the near wall region, $0 < y^+ < 50–200$, important events occur. This is where the Reynolds stress is first produced and the kinetic energy production term reaches a maximum. The evidence also points to the bursting process as a major Reynolds stress-producing element. What keeps this process going? The processes of self-sustenance and of production of Reynolds stress are associated, but they are really separate issues.

4.1. Mechanism characteristics

Self-sustaining mechanisms have several characteristics. The first might be the time scale of the repetition of events. Some mechanisms propose a cycle with a specific feedback process. Other mechanisms are intermittent with a random trigger mechanism that is undefined. The turbulence is in a neutral configuration waiting for a disturbance to set the subsequent events in progress. Whatever the sequence, the time duration should be comparable to the time between bursts.

Another characteristic of importance is the location of the mechanism. Is the mechanism confined to the inner region? Some schemes propose an outer layer event...
interacting with the inner layer. Nomenclature is important here. Many talk of the inner region as (say $y^+ < 50–150$) and regard the outer region as anything above this. This is particularly true when discussing outer–inner interactions. However, from another viewpoint the inner region is where Reynolds stress scales with $y^+$. Hence, that region includes the overlap region (log region). Any event in the log region that interacts with the near wall region, $0 < y^+ < 50–150$, will scale on inner variables.

The magnitude of the Reynolds stress scales with $u_*$, in both the inner and outer layers. Moreover, the time scale of the bursting events scales on $u_*^2/v$ as shown by Luchik and Tiederman [34]. Thus it is suspected that self-sustaining mechanisms scale on inner variables.

It has long been proposed that pressure is an important mechanism of communication between outer and inner layers. As $Re_*$ increases, the inner region becomes very thin compared to the outer. Thus, wall-pressure fluctuations contain influences of the outer layer. The long wave length fluctuations are associated with eddies in the outer layer. They display a convection velocity that is a function of the eddy position in the layer. On the other hand, the length scale of structures in the Reynolds stress-producing region is $v/u_*$. In stability theory we are accustomed to seeing flow patterns becoming unstable to disturbances of certain wavenumbers traveling at certain speeds. If a self-sustaining turbulence structure follows this instability pattern, then the pressure perturbation would scale on $v/u_*$ and travel with a velocity that scales on $u_*$. Thus, in order to maintain Reynolds number similarity, the perturbation would be in the overlap region of the pressure spectrum (Fig. 5). These pressure events come from the log region. Recall that the overlap region may be considered as the “outer” part of the inner layer.

An alternate viewpoint is that the magnitude of the pressure trigger is important, but not its wavelength or phase velocity. In this case any strong event would suffice.

4.2. Categories of mechanisms

Another characteristic concerns the nature of the process. Does the mechanism include an instability? Does the flow evolve into a configuration that only a small local perturbation will set off a sequence of events? An alternate view uses the fact that, in certain conditions, a vortex can generate another vortex. This parent–offspring event is integral to many proposed mechanisms.

In discussing self-sustaining mechanisms we will use two main categories. The “Parent–Offspring” group is defined by having a flow structure that develops in time to replicate itself without involving an instability. The “Instability” group has, during part of the cycle, a velocity profile that is unstable to a small (perhaps mathematically finite) disturbance. The base flow profile must exist for sufficient time for the instability to develop. To a certain extent these classifications are just different ways to analyze the events. A third category, low dimensional dynamic systems, is characterized by having a common mathematical viewpoint. Sometimes an arbitrary distinction must be made to place a mechanism in one or the other category.

5. Vortex-wall interactions

The physical processes that occur in the interaction of a vortex and a solid wall are important to the parent–offspring mechanisms. These processes constitute flow elements with which one can interpret the more complex behavior of real turbulence. The first part of this section reviews calculations of line vortices in the presence of walls and vortex sheets in a wall shear layer. Next, viscous effects caused by a boundary layer under a vortex are given. The section continues with a review of simplified experiments that illustrate the processes. Finally, DNS studies that show hairpin vortex regeneration are discussed.

5.1. Inviscid calculations of a spanwise line vortex in a wall shear layer

One of the first investigations of relevance was by Hama [35] who computed the self-induction of a perturbed vortex filament. Of direct importance to the turbulent wall layer is the work of Hon and Walker [36]. They consider a weak spanwise vortex filament of $\omega_z$ that is convected in a base flow shear layer. The base layer is a linear profile $U_b(y)$ followed by a uniform flow or, in a second set of calculations, a profile resembling the mean profile of a turbulent wall layer. The motion of the vortex is computed by an approximation to the Biot-Savart law. In a detailed derivation, Aref and Flincham [37] show that this approach is valid for small vortex core sizes and time scales short enough to neglect viscous diffusion.

Calculations were done for initial conditions corresponding to several vortex configurations. Fig. 16 shows the history of a vortex filament in three planes. The elemental processes are adequately demonstrated by the case of two straight-line vortex segments connected by a short curved segment. The straight segments are at slightly different $y^+$, $x^+$ positions. Because of their different $y^+$ positions, the segments have different convection velocities and move apart. The connecting vortex section develops a streamwise component that must lengthen. In addition, the strongly curved portions migrate by self-induction. Trailing portions tend to be
propelled toward the wall and the head-like portions rise above the wall. Hence, the differing convection velocities are enhanced by the migration. At the end of the calculation a sharp loop exists near the wall and has fallen behind in the $x$-direction. A gentler higher loop has moved ahead. The conclusion is that any non-uniformity in a vortex filament will develop into a more contorted shape, in many instances a one or two-legged hairpin.

From the figure one can see that the vortex has convected about $1100x^+$ units. The velocity at $y^+ = 50$ is about $u^+ = 15$ so the time for the calculation is $t^+ = 73$. Now, the viscous diffusion distance for vortex decay is about $\delta^+ = 4.3\sqrt{r^+}$. From this one can estimate that the diffusion distance for $t^+ = 73$ would be $\delta^+ = 36$. Although this is a significant number compared to 50, we can take these calculations as suggestive processes needing modification for viscous effects.

5.2. Roll up of a streamwise vortex layer

It is observed in DNS of minimal channel flow that streamwise vorticity tends to roll up into vortices. In a frequently referenced paper Jimenez and Orlandi [38] consider a layer of streamwise vorticity above a wall. The layer contains a uniform $\omega_z$ in a region of variable thickness $h(z)$ in the spanwise direction. Above $h(z)$ the vorticity is zero. The authors essentially calculate the time evolution of the layer thickness with inviscid equations and slip at the wall. They conclude that a nonuniform initial $h(z)$ rolls up into compact streamwise...
vortices. They also confirm that a periodic $h(z)$ does not roll up because the induced velocities are symmetric.

5.3. Boundary layer under an ideal vortex

Let us now consider viscous effects near a wall and under an inviscid vortex. Assume a line vortex of strength $\kappa$ ($\Gamma = 2\pi \kappa$) a distance $h$ from the wall. The vortex is propelled in the direction parallel to the wall by its image. It moves at a speed of $\kappa/2h$. The situation is depicted in Fig. 17 in a coordinate system moving with the vortex. Doligalski and Walker [39] review similar vortex–wall interaction under a great variety of conditions.

The inviscid flow is steady and the wall moves to the right at the propulsion velocity $\kappa/2h$. Fig. 18 shows the inviscid velocity and pressure at the wall. From point A to point B the flow decelerates to a stagnation point. Under the vortex, the flow accelerates from point C to a maximum velocity of three times the propulsion velocity at the mid-point, and then decelerates to stagnate at point B. Hence, the stagnation point at B has flow decelerating toward it from two directions. From the stagnation point at C the flow accelerates back to the propulsion velocity at point D. Although the inviscid flow above the wall is steady, there is no boundary layer solution for this situation. The flow at A comes from infinity over a wall and one cannot provide an initial

Fig. 17. Inviscid vortex above a wall; streamlines in a coordinate system moving with the vortex.

Fig. 18. Velocity and pressure at the wall under an inviscid vortex.
profile. Another, more important difficulty is that the fluid in the region from points B to C is moving in opposite directions within the boundary layer. Walker [40] proposed that the problem be considered as a time-dependent situation. He imagined that for time \( t < 0 \) the vortex exists over a wall that allows slip. Then, the no-slip condition is applied for \( t > 0 \). The solution shows the generation of a secondary vortex near point B at \( t^* = t/(2h^2/\kappa) = 0.281 \). Fig. 19 gives the streamline pattern at \( t^* = t/(2h^2/\kappa) = 0.45 \) as computed by Peridier et al. [41]. Observe that the direction of rotation of the primary vortex in these calculations was taken to be opposite that in Figs. 17 and 18. The first important fact is that an inviscid vortex near a wall produces a secondary vortex from the underlying boundary layer. The secondary vortex has the opposite direction of rotation from the parent.

Further developments center on the flow at point C. The adverse pressure gradient operates on fluid of nearly zero vorticity (i.e. zero wall shear). These two conditions, zero wall shear and adverse pressure gradient, produce an unsteady separation and an eruption of fluid away from the wall. The later stages must be calculated with a Lagrangian analysis of the Navier–Stokes equations in order to follow the rapid development of intense regions of shear. The eruption process is an unsteady boundary layer phenomenon and conditions under which it occurs are not well defined. Similar events are noted when a circular cylinder is impulsively set into translation motion. The mechanism of pressure gradient focusing and fluid eruption is a generic fluid mechanics event. Van Dommelen and Cowley [42] have given a general theory. Some turbulence researchers refer to the eruption phenomenon by the term “viscous–inviscid interaction”.

The problem above is valid for a high vortex Reynolds number (intense vortex) where viscous effects within the vortex are neglected. One can roughly relate the results of Peridier et al. [41] to a streak vortex in the turbulent wall layer. In making this analogy one should keep in mind that a wall streak vortex does not have the velocity profile of an ideal vortex. Consider a vortex \( \Delta y^+ = 100 \) (the streak spacing) with center at \( \Delta y^+ = 30 \). The circulation \( \Gamma = 2\pi k \) is the integral of distance times tangential velocity around a cross-section of the vortex. If \( v^+ \) is the average vertical velocity on the sides of the streak, the contribution from two sides is \( 2 \times 2\Delta y^+ v^+ \). The wall and the region above the vortex have no spanwise velocity, and hence no contribution to \( \Gamma \). In terms of wall units the time \( t^* \equiv t/(v/u^*) = 2\pi t^+ \Delta y^+ v^+ \). Assuming that \( v^+ \sim 1 \), \( \Delta y \sim 30 \), then the calculated time to form the secondary vortex, \( t^* = 0.45 \), corresponds to a wall-unit time \( t^+ \sim 30 \). This is a reasonable value.

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**Fig. 19.** Birth of secondary vortex. Streamline pattern at \( t^* = t/(2h^2/\kappa) = 0.45 \) as computed by Peridier et al. [41]. Coordinates are not physical and vortex direction is reversed from Figs. 17 and 18.
5.4. A ring vortex interacting with a moving wall

The secondary vortex in Section 5.3 contains vorticity that is generated by the flow over the wall caused by the primary vortex. One can also imagine that a vortex approaches a wall that already has a shear layer. Fig. 20 (courtesy of M. Stanislas and P. DuPont, Ecole Centrale de Lille) shows the process of a ring vortex interacting with a wall. A vorticity layer exists near the wall because the wall is a moving belt. To begin the experiment, the wall, with a dye layer applied to it, is started moving to the left. This develops a shear layer just above the wall as in the Rayleigh problem. A short time later, a ring vortex is produced and is directed at a shallow angle toward the wall. Fluorescent dye has also been placed in the ring vortex core. The dye is illuminated by a vertical slit of light. One can observe that the action of the vortex ring rolls up boundary layer vorticity and ejects an offspring vortex as shown. The same process would occur if a line vortex were used. Details of the experiments are given in DuPont et al. [43] and LeMironet et al. [44]. This experiment does not display the violent pressure gradient eruption process.

5.5. Hairpin vortex development and regeneration: experiments

Flow visualization experiments have also been done to track and verify the development and regeneration of hairpin vortices. Bradshaw took photographs 99 and 100 of Van Dyke [45]. They show a train of horseshoe vortices in a laminar boundary layer. The vortices are generated by gently sucking fluid at a constant rate through holes in the wall. Gad-el-Hak and Hussain [46] reported on an experiment to produce artificially streaks and bursts. In another effort, Acarlar and Smith [47] placed a hemispherical bump on a wall in a laminar flow. A water channel allowed flow visualization by dye and Hydrogen bubbles. The results show a train of intertwined hairpin vortices moving downstream. The question is whether the vortices are regenerating themselves or are they associated with periodic shedding from the bump? As an answer Haidari and Smith [48] produced a single hairpin vortex by ejecting a blob of fluid into the flow through a thin slot in the wall. The injection velocity and injection time duration were varied. Below a certain ejection velocity, the fluid diffuses without producing a hairpin. For a higher level of injection velocity and duration, a single hairpin vortex is formed. For yet higher velocities, the vortex is strong enough to eventually produce three hairpin vortices.

Fig. 21 shows the events for an initial boundary layer Reynolds number of $Re_x = 440$. Each picture is a combination of a plan view and an end view of the visualization markers from a hydrogen bubble wire. In pictures (a)–(d) the wire is located at $y/\delta = 0.4$ and various $x/\delta$ locations; (a) $x/\delta = 1$, (b) $x/\delta = 5$, (c) $x/\delta = 10$, (d) $x/\delta = 30$. For each picture, HBW denotes the location of the Hydrogen bubble wire. As the flow pattern converges downstream, it grows slightly and the camera view angle is increased. A 1 cm scale is shown near each picture. The situation for pictures (a’)–(d’) is similar to (a)–(d) except that the bubble wire is located at a low value of $y/\delta = 0.1$. The hairpin vortex head is above the bubble wire, in picture (a). The vortex legs are marked “L” as they penetrate the sheet of bubbles and trail to the left. The bubble wire in picture (a’) is below the legs which are mainly streamwise vortices. The white, streak-like concentration of bubbles is caused by an upwelling of low-speed fluid as the vortices pass over the region. This is clearly shown in the end view. In picture (b) a secondary hairpin, marked $S_1$, is beginning to form upstream of the original. The head of $S_1$ is above the bubble sheet and cannot be seen. The original vortex has passed to the right and out of picture in (c). In the plan and end views of (c) and (c’) another set of secondary hairpin vortices, $S_2$, have developed outboard of the $S_1$ legs. At the furthest downstream location, pictures (d) and (d’), four low speed streaks are observed. Subsequent development, as described by the authors, is much like that in a turbulent spot.

5.6. Hairpin vortex development and regeneration: DNS

Direct numerical simulation of a flow very similar to the experiments of Haidari and Smith [48] was performed by Singer and Joslin [49]. They simulate the injection of fluid from the wall and followed the subsequent development of the flow. The initial vortex events are directly connected to the injection processes. As the calculations continued, they conclude that the final events were developing much like a growing
turbulent spot. They observe displaced secondary hairpin vortices that are consistent with the flow visualization experiments. It should be noted that the calculations of Singer and Joslin [49] include the pressure field and they remark that high-pressure regions often appeared between the vortices. Some vortex regeneration events are associated with the pressure gradient and fluid eruption process. However, in other instances vortices form under conditions dissimilar to that process.

They also observed the formation of quasi-streamwise vortices. As more vortices are born, many first appear under the elongated streamwise legs of other vortices. The appearance of a new vortex under the parent vortex is a little different. It is more like the first appearance of the secondary vortex in the viscous calculations of Walker [40] and Peridier et al. [41].

5.7. Vortex generation in a flow with a Q2 ejection structure

In a series of papers Adrian, Zhou, and coworkers (Adrian and Moin [50], Zhou et al. [51–53]) have numerically investigated the development of a typical Q2 event. They use a mean velocity profile that mimics a turbulent flow. Into this flow a vortical Q2 structure is inserted and its development in time is computed using the Navier–Stokes equations. The characteristic Q2 structure was found from the channel flow DNS of Kim et al. [54]. It is much like two counter-rotating, streamwise vortices that have an upward flow between them. They are about 200 wall units long extending from $y^+ = 12$ at the upstream ends to $y^+ = 65$ at the downstream ends. Near the wall, the vortices are about 100 wall units apart (the typical streak spacing).
narrowing to about 40 units at the downstream ends. A bridge of weak spanwise vorticity connects the vortices about 2/3 of the way to the upstream ends. This structure is found by a linear stochastic estimate, Adrian et al. [55]. It is the flow field that produces the maximum contribution to the average $\langle w \rangle$ Q2 Reynolds stress. The authors use $u_m v_m$ as the strength vector of nominal intensity. The magnitude of the flow structure is $a u_m, a v_m$. All velocities in the structure scale with $a$. For $a < 1$ the initial peak $w$ is about five times the mean $\langle w \rangle$ and never exceeds this value during the calculation. Effects of both the location in $y^+$ and the intensity, designated by $a$, of the initial Q2 structure are investigated.

Fig. 22 shows the initial Q2 vortical structure for $a = 2$ and $y^+ = 30.3$, and its early development. The bridge of spanwise vorticity always intensifies and grows upward to produce a hairpin vortex with the addition of two fingers pointing downstream. The fingers are a direct result of the character of the initial structure and the bridge. Subsequent development of the hairpin and fingers depends on the intensity and location. For low intensity $a \leq 1$, this structure decays by viscous action. For $a > 2$ and moderate values of $y^+$, the calculations develop new hairpins and also new streamwise vortices. However, at $a = 2$ and $y^+ = 30$ and 105 the structure decayed. The reasons for the decay are as follows. Near the wall, $y^+ = 20$, viscous effects are strong, and far from the wall, $y^+ = 105$, the mean shear decreases so that vortex stretching effect is ameliorated. One should note that the calculations and the data base from which the initial Q2 event is derived have a Reynolds number of $Re_* = 180$. Thus, there is considerable intermeshing between inner and outer layers.

Fig. 23 shows the vortical structures after $t^+ = 288$ for $a = 3$ and $y^+ = 49.6$. The fingers pump fluid upward into the shear layer. This intensifies into another bridge and then evolves into a downstream hairpin vortex. On the upstream side, the legs produce secondary and tertiary hairpins. The secondary and tertiary vortices are formed by a process where an upward kink in the legs pokes upward by self-induction. This again creates a bridge in the shear layer. The shear layer rolls up, the legs pinch off, and the elements connect to form a

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Fig. 22. Formation of a hairpin vortex from initial Q2 event. Isosurfaces indicate vortex at (a) $t^+ = 54$, (b) $t^+ = 72$, (c) $t^+ = 90$, (d) side view $t^+ = 90$. From Zho et al. [52].
Fig. 23. Formation of secondary (SHV) and tertiary (THV) vortices from a primary hairpin vortex (PHV). Isosurfaces indicate vortices at $t^* = 297$. Downstream (DHV) and quasi-streamwise vortices (QSV) are also shown. From Zho et al. [52].
new hairpin detached from the primary. All of these offspring have the same direction of rotation as the parent.

Of further interest is the birth of three sets of streamwise vortices near and slightly below the original streamwise vortices. These new hairpins are relatively weak and decay.

In these calculations one can again observe two important mechanisms: hairpin formation by bridging across streamwise vortices, and streamwise vortex generation from another streamwise vortex. In the bridging process it is immaterial whether the streamwise vortices are isolated or are the legs of a hairpin vortex.

Zhou et al. [53] also investigate the effect of asymmetry in the initial Q2 field. This means that the two streamwise vortex structures have different intensities. They find that the required initial amplitude is somewhat smaller and that the asymmetry produces vortices with one leg considerably longer that the other, including “cane” vortices with only one leg. New hairpins form in more rapid succession and their streamwise separation is smaller, leading to a better comparison with experiments.

6. Parent–offspring mechanisms

The parent–offspring mechanism is described as a flow structure that replicates itself. In this process, the interaction of the flow element with the surrounding flow and the wall proceed as a pattern developing in time.

It is almost universally agreed the streamwise vortices near the wall sweep low speed fluid into the streaks. This produces a characteristic streak velocity profile with a low x-velocity component (perhaps even after the vortex has passed by as noted earlier). Subsequently, the bursting process produces the Reynolds stress. Many view the streamwise vortices as the legs of a hairpin.

From this viewpoint, the self-sustaining mechanism centers on how streamwise vortices are produced given the initial situation as a fully developed turbulent state.

The first part of this section deals with streamwise vortex regeneration. Next, hairpin regeneration from an initial hairpin vortex is discussed. Two final sections deal with related issues; inner/outer layer interactions and hairpin packets.

6.1. Streamwise vortex regeneration

Several researchers view the replication of streamwise vortices as an important process in the self-sustaining mechanisms. Brooke and Hanratty [56] calculated a DNS channel flow (Re* = 150) and examined vortex-like velocity patterns in y-z planes. Viewing y-z planes emphasizes streamwise vortices and de-emphasizes spanwise vortices. Unless the vortex axis is exactly in the y-z plane it leaves a shadow in the y-z plane. They found that the new vortex is born at the downstream end of the parent on the downwash side. This end is lifted from the wall and they refer to it as the detachment point (the end very near the wall loosely termed the attachment point). Vorticity in the child vortex is of the opposite sign than that of the parent. Fig. 24 shows the x-y planes at various streamwise locations at 30τr time units after the beginning of the new vortex. At this time the vortex is about 150 wall units long while at rτ = 50 it is about 250 wall units long. Hanratty and Papavassiliou [57] remark that regeneration is not influenced by outer flow events. Thus, they envision a regeneration process that is entirely within the inner region. The latest contribution and summary is in Heist et al. [58]. Fig. 25 is a plan view of their computer results Vortex A is followed as a parent that produces vortex B. Vortex B grows in the spanwise direction and then elongates in the streamwise direction and intensifies.

In concurrent research, Bernard et al. [59] also tracked streamwise vortices in a DNS channel flow calculation (Re* = 120). They also observe regeneration of streamwise vortices if they were of sufficient strength, and were sufficiently close to the wall. The new vortices generally appear on the sweep side of the parent at the attachment end. Often the offspring move under and sometimes were lifted up the other side of the parent. The new vortex has considerable wall-normal vorticity, ωz, both Hanratty and Bernard note the shearing motion turns the vorticity into the streamwise direction. Fig. 26 from Bernard et al. [59] trace this activity. The time sequence is from bottom pictures to the top in 3.2τr intervals. The figure has a total time for the sequence of 19.2τr units. Side views on the left and plan views on the right display the vorticity vectors in the new vortex as the vortex grows and stretches.

In a more recent work, Bernard and Wallace [60] note that vorticity concentration mechanisms are varied and take on different forms depending on local conditions. They consider that regeneration is opportunistic; occurring when local conditions are correct. In effect, there are a variety of conditions too numerous to define. For these reasons they do not elaborate on the mechanism.

Miyake et al. [61] using essentially a DNS calculation, confirm the existence of the viewpoints of both Hanratty et al. and Bernard et al. They find new vortices are born near both upstream and downstream parts of the parent vortex. This produces a picture that is essentially the same as Fig. 14, but with the interpretation that the center vortex was the parent and that those on either end were the offspring. This is of course only an ensemble average picture.
6.2. Hairpin vortex regeneration

Hairpin vortices have been employed to explain turbulent motions, pressure fluctuations, the bursting process, and as elements in mathematical models (Theodorsen [12], Willmarth and Tu [62], Offen and Kline [63,64], Hinze [65], Head and Bandyopadhyay [23], Kasagi [66]). The concept of a hairpin vortex generally implies fluid particles in a symmetric pattern. However, Robinson [16] found that strong asymmetries are the rule, rather than the exception. This is not an important issue as most of the processes described herein have been validated for asymmetric loops or hook-like vortices.

The first ideas about hairpin structures come from observation of the bursting process. A streamwise vortex collects fluid from near the wall and creates a low speed streak. Next, the low-speed streak region lifts into the faster moving stream. This may be a natural event as the streamwise vortices pump more fluid into the streak area. The lifted region produces a stronger \( U(y) \) shear layer which rolls up, much like a Kelvin–Helmholtz cat’s eye, into a vortex arch or head. The vortex lines in the head must extend down and finally proceed in the \( z \)-direction. As the head is convected downstream, the legs are stretched and the swirling thereby intensified. Thus, the hairpin formation process consists of streak lift-up, shear layer intensification, and hairpin formation.


Smith [70], using vortices as an element in the bursting process, proposes a nested sequence that straddled the low speed streak as it lifted up. Therefore, the streak spacing is the hairpin scale, and a single hairpin spawns others in its wake (see Fig. 27). Thus, Smith explicitly addresses the question of regeneration. Note that the legs of the nested vortices coalesce. The experiments of Haidari and Smith [48] discussed above are used as evidence that this mechanism is viable. In the Spalart [17] DNS data for a boundary layer, Robinson [16] sees mainly one-sided vortices. He proposes that another and more common, mechanism is observed in addition to that described by Smith. New small arches (hairpins), as seen in Fig. 28, form the low speed region that is generated by a single streamwise vortex. The streamwise vortex might be the leg of a large hairpin or may have another origin. It is a variation of the Smith [70] model.
In a somewhat different and even more detailed description, Smith et al. [71] (also Smith and Walker [72,73]) explain how hairpin vortices are formed at the locations where the parent vortex is curved. To be specific, hairpin vortices are formed at the curves, either at the end of a streamwise leg or under the head as shown in Fig. 29. By considering the pressure field of an inviscid vortex above a wall, they envision that adverse pressure gradients exist in these regions. The pressure force is the promoting mechanism. Recall the flow focusing effect and the eruption process discussed earlier (viscous–inviscid interaction). Smith et al. [73] make a point of including this as an important part of the regeneration process. Also note that a one-sided hairpin without a second leg is not excluded from this scenario. Another point is that now the hairpin width scale is not set by the low speed streak spacing, but by the curvature of the primary vortex. Kline and Portola [26] state that they essentially agree with these ideas. However, they talk in terms of a lifted shear layer being unstable.

The flow development starting with a Q2 event as initial conditions was discussed in Section 5.7. Recall that a hairpin vortex is formed from two streamwise vortices by a bridge of spanwise vorticity that is...
intensified by the fluid lifting between the streamwise vortices. Subsequently, more hairpins are formed and pinched off the first hairpin. Although described in different terms, these events are much like the streak lift-up, shear-layer intensification, hairpin formation process envisioned long ago. An essential extension is uncovering the ability for the legs to pinch off and continue to replicate more hairpins.

6.3. An inner/outer interaction

This section is more about an interaction between the outer layer and the inner layer than about a self-sustaining mechanism. It involves some nomenclature introduced by Falco [24] and Klewicki [74,75]. From flow visualization tests Falco observed small mushroom-like eddies along the upstream side of large-scale outer eddies. He termed these “Typical Eddies”. He proposes that they are ring vortices which are formed by either; (1) pinch off of a hairpin vortex or, (2) instability of a region of vorticity. The second method is quite vague. In experiments where the sublayer was filled with smoke upon which a typical eddy interacts, one observes a smoke free region that they term a “pocket”.

In the sequence of Fig. 30 Falco [24] outlines the events that they envision. In Fig. 30(a) a typical eddy...
moving over the wall organizes flow into two streaks. This is contrary to the view that streamwise vortices produce the streaks. For some unspecified reason, the typical eddy moves closer to the wall, Fig. 30(b). This causes a redistribution of the spanwise vorticity in the wall shear layer forming two vortices. Fig. 30(c) shows that the spanwise vortices develop into a primary hairpin vortex and a “pocket” vortex. The later vortex forms secondary hairpins from the streaks. Events in the final panel, Fig. 30(d), show the pocket vortex is strongly stretched, intensified, and wraps secondary hairpins around it. At the same time the primary vortex lifts from the wall. Asymmetric versions of these events are expected.

6.4. Hairpin packets

The outer region of a wall flow has large eddies that scale with the outer thickness. Head and Bandyopadhyay [23] performed flow visualization experiments that revealed the existence of tilted (45°) hairpin eddies that were grouped together. These were observed for $\text{Re}_\theta < 10,000$ with experiments in the range $500 < \text{Re}_\theta < 17,500$. An interesting result is that, although the length of the vortices scales with $\delta$, the diameters scale with viscous variables, $u_\kappa / \nu$. The idea of packets was also suggested in the work of Smith [70], and Smith et al. [71,72].

The concepts of packets has been emphasized and expanded by Zhou et al. [53] and Adrian et al. [76]. They propose that hairpin vortices, as a result of their regeneration process, form into groups or packets. Fig. 31 displays packets as observed in an experimental channel flow by Adrian et al. [76]. The inner region formation process produces a packet that holds together as the vortices grow and migrate to the outer region. They claim that this helps explain the fact observed by Bogard and Tiederman [20], Luchik and Tiederman [34] and Tardu [77] that a burst is composed of several ejections. The individual vortex causing the ejection, while the packet is associated with a burst that is composed of several ejections. Mienhart and Adrian [78] observe another aspect of the log layer. They find large zones of relatively uniform streamwise velocity exist between internal shear layers. Their current view is that these zones are packets of organized vortices. The number of vortices in a packet increases with Reynolds number. There are smaller packets within larger packets. More than ten hairpins were observed in a packet and a packet extended out to $0.8\delta$ with a streamwise length of $2\delta$. The paper of Adrian et al. [76] is particularly convincing as it has experiments at $\text{Re}_\theta = 6845$. Many workers regard at least $\text{Re}_\theta = 6000$ as the minimum Reynolds number to have fully developed turbulent statistics.
7. Instability mechanisms

Next, the category of self-sustaining mechanisms involving instability mechanisms will be discussed. The instability process has, at least during part of the self-sustaining cycle, a velocity profile that is unstable to small disturbances. This definition is somewhat arbitrary and it is often difficult to class a mechanism as an “instability”. A stability analysis begins with a velocity profile called the base flow. The base flow profile must exist for sufficient time for the instability to develop. A perturbation is added to the base flow and the question is; does the disturbance grow? More specifically, what kinds of perturbations grow and how fast do they grow? In order to form a complete cycle, the instability should ultimately lead back to produce the base flow. An alternate view is that the inner region is in a sort of neutrally stable state where a random disturbance triggers a turbulence-producing event.

A base profile and a disturbance are the only common elements of a stability analysis. In turbulence the choice of base flow is not obvious. Likewise, by its very nature a fully developed turbulent flow has a variety of finite disturbances. The next issue is about the equations that are used to follow the development of the disturbance. Are the full nonlinear Navier–Stokes equations used or are the equations linearized? The typical laminar flow stability analysis uses linearized equations under the assumption that they govern the initial development of an infinitesimal disturbance. Linear equations have a wealth of mathematical properties. Their solution lead to simplified physical processes that are sometimes hidden in complex treatments. Linear equations allow an analysis into normal modes; the eigenfunctions and eigenvalues of the system. One can then identify the characteristics of the most-unstable infinitesimal disturbance with the fastest (exponentially) growing normal mode.

Fig. 29. Generation of hairpin vortices in regions of adverse pressure gradients. After Smith and Walker [71].
In some cases this approach has failed. The reason is subtle. The linearized Navier–Stokes equations are "nonnormal" and their eigenfunctions, often called normal modes, are not orthogonal. This allows certain disturbances that are composed of several normal modes, to grow to large amplitudes (factors of 1000 or more have been found. At large times the disturbance approaches the behavior of the dominant eigenfunction.
This transient growth is algebraic and for that reason this phenomenon is also called algebraic growth. When the transient decays exponentially it joins the dominant eigenfunction that may be stable (decaying exponentially) or unstable (growing exponentially).

Trefethen et al. [79] make an analogy with vectors. Imagine two almost oppositely directed basis vectors that have large amplitudes. These are normal mode components. Their resultant, the perturbation, has only a small magnitude. Now one basis vector magnitude decreases with time while the other is unchanged. This causes the resultant perturbation to increase in amplitude. Ultimately, the behavior follows the dominant eigenvector whose stability characteristics are determined by other factors.

Transient growth was ignored for many years. Ultimately, it was realized that transient growth could lead to profiles that, because of their large amplitude, are subject to secondary instability and or nonlinear effects. Transient growth has a history of development and a nice introductory review is Trefethen et al. [79]. Reddy and Henningson [80] specifically investigated Couette flow. A significant conclusion by Butler and Farrell [81] is that the most unstable modes are three-dimensional, much like the streamwise vortex.

Thus, in the instability mechanism category there are many questions: finite or infinitesimal disturbance, normal mode or transient growth, linear or nonlinear development? Finally, when nonlinear effects must become important, what are the dynamics of the regeneration process that sustains the turbulence?

### 7.1. Inflectional profiles

In the first flow visualization experiments that identified the bursting process, Kline et al. [19] noted an oscillation in the dye streak (see Fig.11) and proposed that an instability existed. The conjecture was that the lifted low-speed-streak produced an instantaneous $U(y)$ profile that had a point of inflection. The reasoning makes an analogy with linear stability of plane two-dimensional profiles. Two-dimensional profiles with a point of inflection are possible unstable by Rayleigh’s criterion. The inflection point is a necessary, but not sufficient condition.

Rayleigh’s criterion has been superseded by the stronger Fjortoft’s theorem. That theorem can be summarized as follows: if $U(y)$ is a base profile with an inflection point at $y_0$ where $U(y_0) = U_0$, a necessary, but not sufficient, condition for inviscid instability is that $d^2U/dy^2 \cdot (U - U_0) < 0$. This rules out certain types of inflection points as depicted in Fig.32. According to Fjortoft’s theorem only Fig. 32(b) is possible unstable by inviscid processes. The inflectional profile shown in the Fig. 32(a) is not unstable.

In Fig. 11, Kline et al. [26] focus on the history of dye elements and the production of ejection motions to create Reynolds stress. As the low speed streak lifts, it creates a shear layer that then develops into a spanwise vortex. From the standpoint of a self-sustaining mechanism the question is; how is the streak produced? Heist et al. [58], in DNS studies observe the formation of streamwise vortices by the rotation of arch vortices. In Fig. 33 they track the birth of vortex B from parent A (see Section 6.1). Subsequently vortex B is turned and stretched into a streamwise vortex. These vortices are located between $y^+ = 30$ and 40.

Blackwelder [82,83] continues the inflectional profile argument by noting that the $U(y,z)$ profile of a streak also has inflections in the $z$-direction. Fig. 33 diagrams the self-sustaining relationships Blackwelder envisions. The figure shows instability mechanisms in both $U(y)$ and $U(z)$ that cause streak lift-up and fluid ejection away from the wall. A question mark indicates an uncertain interaction where the bursting impacts the outer region. In other words; are there large-scale structures that are related to the bursts? The figure then indicates that outer-fluid sweeps toward the wall to form the visual markers called pockets (cf. Fig. 21). Another question mark indicates how the outer events affect the streamwise vortices and low-speed streaks.

In searching for an experiment that would have velocity profile with streamwise vortices and low-speed streaks, but without complicating turbulence, Swearin- gen and Blackwelder [84] chose an array of Görtler vortices. This flow is an example of a streaky flow with inflection points in both $y$- and $z$-directions. A concave wall generates the vortices that continue to grow as the flow proceeds. Fig. 34 is a plan view of the flow made visible by a horizontal smoke wire at $x = 20$ cm from the leading edge. Oscillations and instabilities are evident in the figure. The authors observe that the $z$-direction inflectional instability occurs before the instability in the $y$-direction. They measure the streamwise wavelength of the instability as about $\lambda_z = 175$, and that breakdown occurs in about two wavelengths. Blackwelder concludes that this rapid breakdown is compatible with a $z$-direction (spanwise) shear-layer instability.

A strictly two-dimensional $z$-direction profile $U(z)$ produces vortices with axes in the $y$-direction. An explanation is needed as to how the streamwise vortices are produced. One viewpoint is that the $y$-direction vortices are then tilted by the mean velocity profile to become streamwise vortices much like the mechanism of Heist et al. [58] discussed above. Thus, a regeneration cycle is completed.

### 7.2. Curved streamlines and Görtler vortices

Curved streamlines sometimes lead to a centrifugal instability as in cylindrical Couette flow or the instability
that forms Görtler vortices. Brown and Thomas [85] speculated that sweeping motions of fluid approaching the wall would have curved streamlines and could lead to such an instability. This interpretation has not gained acceptance.

Görtler vortices have a periodic structure in the $z$-direction with regions of low and high $x$-direction velocities. In the $y$–$z$ plane they have the roll structure of the vortices. Some papers expressly try to explain the observations of the Swearingen Blackwelder [84] experiments as a secondary instability of the vortices. Hall and Horsemann [86] observe that in boundary layers, Görtler vortices continue to grow as the flow develops in the streamwise direction. They calculate a three-dimensional Görtler vortex state, and using this as a base flow, formulate the stability problem. A $x$-dependent perturbation is introduced, and the stability calculated using linearized, inviscid equations. In agreement with the conclusions of Swearingen and Blackwelder [84] they find that the sinuous mode has an amplification rate about twice that of the varicose mode. The $x$-wavelength of the fastest growing mode is predicted to be about 3 cm, while the measured value is about 4.2 cm. Compared to the experimental observation of 130 Hz, the predicted frequency is 110 Hz.

In a different approach, Yu and Liu [87], perform calculations using a base flow that only has the streamwise velocity profile $U(y, z)$ of a Görtler vortex flow. The cross-stream swirling flow is neglected. However, they include viscous terms in the stability equations. Their results also confirm that the sinuous mode prevails over the varicose mode. For a wavelength equal to the observed value, they find that the predicted sinuous frequency is off by 8% from the measured 130 Hz value.

Since these two papers have such different assumption (one with cross-stream mean flow, the other without, one with inviscid equations, the other viscous equations) it is difficult to make conclusions about the detailed physical nature of the instability mechanism.

Görtler vortices occur in many laminar boundary layers and their stability has been investigated extensively (but incompletely) with regard to the transition to turbulence. Most of these papers are not specifically
pointed toward the streak breakup problem of established wall turbulence. An example is the work of Mendonca et al. [88]. They investigate the interaction of Görtler vortices and Tollmien–Schlichting waves. Two types of interactions are found depending on the relative amplitudes of the vortices and the TS wave. In one interaction higher harmonics of the Görtler vortices are destabilized by the TS wave. In a type of second interaction, the Görtler vortices damp the TS wave.

7.3. Minimal flow unit

The minimal flow unit is a concept introduced by Jiménez and Moin [89]. They use the DNS code pioneered by Kim Moin and Moser [54] for a turbulent channel. By reducing the size of the channel width, (the length was 250–600\(\lambda^+\)) they suppressed effects of the outer turbulent region. The idea is to isolate a basic process of turbulence. When the spanwise box width is about 100\(\lambda^+\), turbulence could be maintained on only one wall. The flow has only one streak, consisting of a streamwise vortex that lifts a layer of spanwise vorticity away from the wall. Fig.35 depicts the process. Calculations confirm that the cycle is regenerative, however, it has an unrealistically long time scale, perhaps because of strong viscous effects.

Jiménez and Pinelli [90] continue this approach. A noteworthy difference is that the channel is wider, \(L_z = 300\) wall units. This is wide enough for two or three streak profiles and the flow also contains some large-scale outer structures. By various artificial modifications to the calculations, they test the viability of the streak instability mechanism and the parent–offspring mechanism. They conclude that a streak instability cycle is possible and exists entirely in the inner region below \(y^+ = 60\) and above \(y^+ = 20\). This self-sustaining cycle needs no interaction with the outer region. On the other hand the parent–offspring cycle does not appear viable under these severely viscous conditions. The conclusion about the parent–offspring mechanism is probably the result of the small computation region. As the authors note, these conclusions may be modified at higher \(Re_*\) and/or larger domains. However, they believe that the streak instability mechanism is the dominant self-sustaining mechanism, and because it exists only in the inner region, it could be dominant at all Reynolds numbers.
7.4. Streak velocity profiles

It is universally agreed that streamwise vortices produce the low speed streak velocity profiles. However, streak profiles \( U(y, z) \) are frequently observed without streamwise vortices. In the case of Görtler vortex flow, one has streak profiles along with a strong swirl motion from the vortex. Work discussed in this section has the streak profiles \( U(y, z) \) profiles (without swirl) as the base flow.

One should note that the stability analysis of Yu and Liu [87] actually uses a streak profile. Much laminar stability analysis of shear layers has been done to explain transition. There is always a hope that some mechanisms are also useful to explain the self-sustaining processes of fully developed turbulent flow. Of note is the “direct resonance” idea of Benny (see, for example, Benny and Gustavsson [91]). First, there is transient growth of oblique modes. Then nonlinear interaction (resonance) of left and right modes produces a vertical velocity which according to Jang et al. [92] leads to the streamwise vortices. Some elements or adaptations of the resonance concept could have relevance to a self-sustaining mechanism.

Phillips et al. [93] analyzed the formation of longitudinal vortices in a shear layer over wavy terrain as a Craik–Leibovich instability. The wall waviness is not thought to be an essential part of the mechanism, but allows easier analysis. In Phillips [94] his attention is directed to Stokes drift of nonlinear wave–wave development. These interpretations are essentially sophisticated instability approaches and, although they may have some relevance to initial developments, do not address the self-sustaining mechanisms completely.

Waleffe et al. investigate the stability and regeneration of streak profiles in plane Couette flow. Their major results are summarized in Waleffe et al. [95], Hamilton et al. [96] and Waleffe and Kim [97–99]. They consider Couette flow between two plane walls, \( y/h = \pm 1 \). The walls move in the \( x \)-direction, \( U/U_0 = \pm 1 \). The base flow is produced as follows. Starting with the linear laminar profile, they impose a streamwise roll (vortex) structure that extends across the whole channel. After about a quarter of a revolution the vortices redistribute the mean flow to produce streak profiles \( U(y, z) \) as given in Fig. 36 (a and b). In terms of wall units, the channel is 40\( y^+ \) units high. That is roughly equivalent to the buffer region. Note that the \( U \) velocity is constant on each wall and that the \( z \)-oscillating streak structure has its maximum intensity at the centerline. The streak profile is analyzed with linear, viscous, stability equations. Imposing a perturbation of a form that is sinuous in the \( x \)-direction,

\[
v = \exp(\lambda t) \exp(i\alpha x) \sum v_n(y) \sin(n\pi z)
\]

allows one to calculate the stability diagram. Results for the most-unstable fundamental sinuous mode are shown in Fig. 37. One can observe that the growth rate for a spanwise mode with \( \gamma = 5/3 \) is a maximum of 0.135 for a \( x \)-direction wavenumber of 0.74. Results are relatively insensitive to the intensity of the streak profile. It is also

\[V(y,z), W(y,z)\]

\[U(y,z) \text{ for } V=0.02\]

Fig. 36. (a) Streamwise vortices \((0, V(y,z), W(y,z))\) in Couette flow. (b) Contour plot of the initial streak profile \( U(y,z) \) produced by the vortices. \( V = 0.02 \) is the strength parameter. From Waleffe [95].

\[\text{growth rate} \]

\[\alpha\]

Fig. 37. Growth rate of the fundamental sinuous mode as a function of wavenumber, \( \alpha \). Wall conditions are: solid line, no slip; dashed, line slip. From Waleffe [95].
noteworthy that the authors conclude that transient growth is not an important mechanism in Couette flow turbulence.

The stability calculations above were inspired by DNS calculations of Couette flow. In these calculations the authors reduce the Reynolds number until only the essential features of the turbulence remain. A flow pattern quasi-periodic in time and periodic in space (because of the boundary conditions) results. The initial tendency of the DNS calculations is to produce two sets of vortices, one on either wall. However, because of the strong viscous nature of the flow, the vortices merge into one set that spans the channel.

Three events summarize the self-sustaining process:

1. Streamwise rolls redistribute the mean flow to form streak profiles.
2. A sinuous streamwise disturbance leads to instability of the streak profiles.
3. Nonlinear feedback of the unstable mode injects energy to reform the rolls.

Process (1) is well known and is the same as Corrsin’s conjecture of 1956. Process (2) is observed in the Görtler flows, but is sensitive to the base flow that the authors needed to establish that it occurs in Couette flow. Process (3) is an essentially new contribution.

This cycle is completely within the inner region. The Couette flow where these events are observed did not have two turbulent wall layers, one on either wall, but a turbulent process within the entire channel. However, the distance between the walls is 40 relative units. The flow might be considered as a wall region with a moving lid on it. Nevertheless, as with the minimal channel calculations, the supposition is that the major characteristics of the process are similar to those in a traditional wall layer.

Shoppa and Hussain [100,101] examine turbulent flow in a channel, driven by a pressure gradient, in great detail. They use a minimal channel of \( L_x^+ = 300 \), \( L_y^+ = 200 \), \( L_z^+ = 100 \). In this channel flow a mean \( U_0(y) \) is constructed so that the upper wall is laminar-like and the lower wall turbulent-like. Their first calculation is a linear stability analysis. It identifies a sinuous, linear, inviscid instability that only occurs if the streak-profile strength is sufficiently strong. In this analysis the base-flow streak profile is

\[
U(y, z) = U_0(y) + \frac{\Delta u}{2} \cos(\beta z) \exp(-\eta y),
\]

\[ V = W = 0. \quad (19) \]

Here \( \beta \) is chosen to give a streak spacing of 100 and \( \eta \) chosen to give a plateau in \( y \)-vorticity at \( y^+ = 10-30 \). Instead of using \( \Delta u/2 \) to indicate the strength of the streak profile, a more physical concept was introduced. The streak profile strength is denoted by \( \Theta_20 \); the maximum inclination of a base-flow vortex line at \( y^+ = 20 \). This occurs on the flank of the streak. Fig. 38 shows the base-flow profile and the \( y-z \) plane velocity contours with \( \Theta_20 = 56^\circ \) indicated.

Results of the stability analysis are shown in Fig. 39 where the growth rate vs \( x \)-direction wavenumber is displayed for a normal mode and a base flow intensity \( \Theta_20 = 56^\circ \). The wavenumber with the maximum growth rate \( (k^+ = 0.02) \) is roughly \( L_x^+ = 300 \). Recall that the Görtler experiments give \( \lambda_1 = 175 \) and the Couette calculation yields \( \lambda_1 = 195 \); all about the same. Also in Fig. 39 is the maximum growth rate for \( y^+ = 0.02 \) waves as a function of the intensity factor \( \Theta_20 \). Only base flows with \( \Theta_20 > 48^\circ \) are unstable.

Schoppa and Hussain do not believe that linear stability is a dominant mechanism in the self-sustaining process. They interrogate the KMM [54] data-base and find that unstable streaks are only a small fraction of the streak population. Thus, the likelihood of a normal mode mechanism is small. In addition, they note that the time scale for viscous diffusion to destroy the streak profile is short. For example, a streak profile with intensity \( \Theta_20 = 56^\circ \) decays to \( \Theta_20 = 50 \) (48° being stable) in \( r^+ = 30 \). The third reason is that normal mode growth rates are very small. For example the most unstable mode for \( \Theta_20 = 56^\circ \) grows by a factor of two when diffusion of the base flow is included.

As described in 7.0, researchers have come to appreciate that a normal mode stability analysis is only a partial answer. The nonself-adjoint nature of the Navier–Stokes equations means that unstable normal modes are not the only mechanism of perturbation growth. Perturbations exist, both for stable and unstable base-flow intensities \( \Theta_20 \), that grow algebraically to large levels and then die off exponentially (or become an unstable normal mode). The important point is that when the disturbance is at a large level it is essentially a new flow pattern that can initiate other events.

Shoppa and Hussain chose a perturbation of the form that is not a normal mode

\[
w' = W \sin(\alpha x) y \exp(-\eta y). \quad (20)\]

Fig. 40 gives the energy of a normal mode and a transient perturbation, denoted as streak transient growth (STG), as a function of time. In this calculation, viscous decay of the base flow is accounted for as indicated by the curve of \( \Theta_20 \) decreasing with time. The transient perturbation grows by a factor of 20 compared to the normal mode’s growth of a factor of two. In order for this vortex-streak-transient growth-vortex formation process to occur, a certain level of perturbation is needed. The initial level of perturbation required depends on the base flow intensity. For example, \( w_{rms}^+ = 0.25 \) for \( \Theta_20 = 56^\circ \). This is roughly 25% of the typical turbulence intensity.

According to the author’s minimal channel calculations, the perturbation continues to growth to level
where nonlinear equations must be used. Eventually the DNS calculation yields a regeneration of the streamwise vortex. Upon nonlinear saturation, stretching of the streak waviness by \( \partial \omega / \partial x \) causes formation of a streamwise vortex. Presumably the cycle is then completed by formation of a new streak profile by the vortex. Cross-sections in the \( y-z \) plane are shown in Fig. 41 at various times in the process. Contours of constant \( \omega_x \) vorticity are depicted in frame of reference convecting at \( 0.6 U_0 \).

A further result of import is that Shoppa and Hussain observe the three-dimensional structure of the regenerated vortex and its neighbors. These are overlapping vortices of opposite sign. It is interesting to note that Waleffe and Kim [98] envision that the fundamental sinusoidal mode yields overlapping vortices, and that the sub-harmonic sinusoidal mode produces horseshoe vortices. As noted by Moser [4] the Schoppa–Hussain scenario provides many specific details that can be tested against data.

An interesting footnote to this section is the paper of Kim and Lim [102]. They arrange the DNS equations for channel flow with the vertical velocity and normal vorticity as unknowns. The equations explicitly display the nonlinear terms and the coupling term between \( v \) and \( \omega \). The coupling term leads to “larger” nonnormal behavior. That is, the coupling term give 3-D disturbances a larger transient growth. Three sets of calculations are performed; full N–S equations, equations without coupling terms, and equations without nonlinear terms. In the absence of either term the turbulence dies. They propose that formation of the near-wall vortices is essentially nonlinear, but that maintenance is linear. These ideas are in agreement with the other mechanisms of this section.

8. Dynamic systems

Another approach for understanding turbulence is to study the behavior of a system of ordinary differential equations. The ordinary differential equations are in some sense a simplification of the Navier–Stokes equations. One goal of this approach is to produce a system of low dimension that will mimic the essential processes of turbulence. Two classes can be distinguished;
those that use a set of basis functions and those that propose flow elements. The basis function approach is deductive and the physical information specific to wall turbulence is contained in the basis functions. This method relies on experiments or DNS calculations to determine the basis functions. On the other hand, the flow element approach is inductive and based on observations to formulate the elements. The “flow element” method will be presented first.

8.1. Systems using flow pattern elements

In a series of papers Waleffe et al. [97,98] propose an eighth order model and a fourth-order model for plane Couette flow between moving plates (no pressure gradient). The hope is that the major aspects of the dynamics of Couette flow will also apply to wall turbulence in general. Unknowns in the model are the amplitudes of four flow elements as functions of time.

The elements and their velocity components are as follows:

- Mean profile:
  - $u = \sin(\pi y/2)$, $v = 0$, $w = 0$,

- Streak profile:
  - $u = \cos(\gamma z)$, $v = 0$, $w = 0$,

- Roll profile:
  - $u = 0$, $v = \gamma \cos(\beta y) \cos(\gamma z)$, $w = \beta \sin(\beta y) \sin(\gamma z)$,

- Instability mode:
  - 3-component profile with $\exp(izx)$ dependence (21)

Slip at the walls, $y = \pm 1$ is allowed. Note that the streak and roll profiles are de-coupled. This is compatible with the old idea that the mean $u$ profile is by itself unstable (for instance Blackwelder [83]). The assumption is that the rolls produce the streak profiles, but are not otherwise important in the instability. Robinson [16] found numerous situations where streak profiles exist without rolls. The roll mode above covers the entire channel width with a single roll. Thus, the model does not have two interacting wall layers, but a set of rolls constrained by two walls. The $W(t)$ instability mode is the neutral sinuous mode in $x$ with a complicated $y-z$ dependence. Waleffe [97] finds that the neutral mode has the same shape as unstable modes with longer wavelengths. This flow element is inviscid and nonpropagating.

The governing equations are arrived at by a Galerkin projection of these flow elements with the N–S equations, Waleffe [99]. Taking the lowest order truncation yields five coefficients for the $W$ mode. Hence, the
eight-th order system results. Further assumptions reduced this to a single amplitude coefficient and the fourth-order system

Mean: \[ \frac{d}{dt} \left( \frac{\kappa_m^2}{\text{Re}} \right) M = \frac{\kappa_m^2}{\text{Re}} + \sigma_m W^2 - \sigma_a UV, \]

Streaks: \[ \frac{d}{dt} \left( \frac{\kappa_u^2}{\text{Re}} \right) U = \sigma_a MV - \sigma_u W^2, \]

Rolls: \[ \frac{d}{dt} \left( \frac{\kappa_v^2}{\text{Re}} \right) V = \sigma_v W^2, \]

Instability: \[ \frac{d}{dt} \left( \frac{\kappa_w^2}{\text{Re}} \right) W = \sigma_a UW - \sigma_m MW - \sigma. \]

The first term on the right drives each mode and the others show transfers of effects. It is observed that the mean \( M \) is maintained by the external forcing \( \kappa_m^2/\text{Re} \), while the streaks are formed by the redistribution of the mean by the rolls \( MV \). Rolls are sustained by the nonlinear instability \( W^2 \) while the instability of the streaks grows by the \( UW \) term.

A phase plane analysis of the fourth-order system shows a fixed stable point corresponding to laminar flow at low Reynolds numbers. At high Reynolds numbers the system has, in addition to the laminar point, a saddle point, and a stable limit cycle. This is seen in Fig. 42. From many initial conditions the solution will settle into a periodic orbit rather than approach the laminar point. This produces a self-sustaining process. Paradoxically the eighth-order system contains some anomalous behavior. In any event, the mechanism depicted in the fourth-order model shows that the \( x \)-direction sinuous instability sustains the rolls. The rolls in turn produce the streak profiles from convecting the mean velocity. Streak profiles are the base flow upon which the sinuous instability grows. Some aspects of the Couette flow scenario of Waleffe and Kim (roll \( \Rightarrow \) streak profile \( \Rightarrow \) x-instability \( \Rightarrow \) roll) are the same as the wall-layer mechanisms proposed by Kline, Blackwelder, Schoppa and Hussain. Similarities and detailed differences between mechanisms for Couette flow and channel flows have been discussed by Schoppa and Hussain [31] and Waleffe and Kim [97].

8.2. Systems using orthogonal basis functions

The basis function approach is associated with the terms “proper orthogonal decomposition, (POD)” and “Karhunen–Loève” (KL) decomposition. In seeking an unbiased method of defining a turbulent eddy, Lumley [103,104] proposed the KL decomposition of experimental turbulence measurements. Both space and time dimensions are decomposed. An improvement is to recognize that the flow in the spanwise (\( z \)) and streamwise (\( x \)) directions is homogenous and the decompositions for these directions are Fourier series. A crucial change is to expand the space dimensions, but to allow the coefficients to be functions of time.
The coefficients \( A(t) \) are solved for by a system of ordinary differential equations. The basis functions \( \Psi(y) \) are the heart of the Karhunen–Loève decomposition. They are determined from either experimental data or from full DNS data. The flow is solved in a box \( L_x \), by 1, by \( L_z \) and the integers \( m \) and \( n \) indicate periods for the Fourier components in the \( x \) and \( z \) directions. Sometimes \( q \) is called the vertical quantum number. The basis functions are the solution to an eigenvalue problem and they have several nice properties. They are orthonormal and have optimal convergence in the sense that the energy of the flow is approximated by a fewer number of terms than by any other expansion. The basis functions \( \Psi \) are determined from the integral equation

\[
\lambda^{(mnp)} \Psi^{(mnp)}(y) = \int_0^1 R_{qj} (y, y') \Psi^{(mnp)}(y') \, dy'.
\]

(24)

Here \( \psi_i \) is the \( i \) direction of the vector \( \Psi \). There are \( q \) solutions ordered in decreasing size of eigenvalue \( \lambda \). A sum on \( j \) is implied, and \( R_{qj} \) is the two point velocity correlation tensor \( \langle u_q(y)u_j(y') \rangle \) with the \( ' \) indicating the Fourier transform. The physical elements of turbulent flow enter through the experimental or DNS data used for \( R_{qj} \). Hence, the basis functions are unique to the problem under consideration. The interaction of the basis functions is determined from equations governing the amplitude coefficients.

When using the basis functions \( \Psi(y) \), the question of transient growth of a linear system is moot. The functions are orthogonal so we are looking at an essentially different way of representing the physical behavior. Any transient growth physics is submerged in the basis functions.

Holmes et al. [106] give the details of deriving the dynamic equations used by Aubry et al. [107]. The flow considered is a channel flow with a constant pressure gradient. To begin, the Navier-Stokes equations are written and a spatial “Reynolds” decomposition inserted. The decomposition yields an \( x-z \) spatially averaged quantity and a fluctuation. Averaging the N-S equations over \( x-z \) planes yields equations that include a “\( x-z \) mean” flow, \( U(y, t) \). Note that the \( x-z \) mean flow \( U(y, t) \) is a function of time. The reason for this is that the extent of the computation box is finite in the \( x-z \) directions. As fluctuations grow and decay the amount of energy in the fluctuations and the mean flow interchanges. If the flow were infinite in the \( x-z \) directions, the \( x-z \) mean velocity would be independent of time. The same reasoning applies the \( x-z \) spatially averaged Reynolds stress \( \langle w' \rangle(t) \).

The \( x-z \) mean flow can be related to the spatially averaged Reynolds stresses by integrating the momentum equation to get

\[
\frac{U(y^+, t)}{u_*} = \int_0^{y^+} \frac{\langle w' \rangle}{u_*^2} \, dy^+ + \left[ y^+ - y^+ \right] \frac{2\text{Re}_*}{2\text{Re}_*}.
\]

(25)

(26)
In deriving this equation the Cornell group assumed \( \partial U / \partial t = 0 \). This implies that the time scale of \( U \) is much slower than that of the fluctuations. This equation ultimately leads to terms in the \( A(t) \) differential equations that are cubic. Aubry et al. (1988) notes that assuming a constant \( U(y) \), leads to diverging solutions.

If the expansion Eq. (23) is truncated at a few terms, smaller scale fluctuations that dissipate energy are missing. To account for the missing modes another decomposition into resolved and unresolved components is inserted. Subsequently, terms with products of unresolved components are arranged into deformation and pseudo-pressure terms. Then, the unresolved components are expressed in terms of the resolved components by an eddy viscosity model and an assumption about excess kinetic energy production. At the same time two adjustable coefficients, \( z_1, z_2 \) are introduced and set equal. Inner scales are used to normalize the equations in preparation for substituting a K–L expansion for the fluctuations and Fourier transforming on \( x \) and \( z \).

Finally, the orthogonal nature of the basis functions is used in a Galerkin projection where the integration domain in \( y^+ \) is 0–40. This produces a system of ordinary differential equation for the \( A^{(\text{non})}(t) \) coefficients. The reason that the integration limits are \( y^+ = 0 \) and 40 is that the authors have experimental data for the \( R_{ij} \) in that range. The data came from a pipe flow experiment at \( Re_* = 265 \). Mathematically, ending at \( y^+ = 40 \) introduces some boundary terms; including a term that represents the fluctuating pressure at that level. This builds into the model a specific interaction between the “outer”, and inner layers. Here the outer is actually pressure sources \( y^+ > 40 \). The basis functions, Eq. (24), are also determined by integration \( y^+ = 0 \)-40.

Final equations for the coefficients have the form

\[
\frac{dA}{dt} = [L_1 + z_1 v_T L_2]A + \left[ \frac{Q_3(A, A) + \frac{2}{3} y z_2 Q_2(A, A)}{1} \right] + C(A, A, A) + d(t) \tag{26}
\]

where \( L_1 \) is the bracket term in mean flow Eq. (25), \( L_2 \) the neglected modes, \( Q_3 \) the convection terms, \( Q_2 \) the pseudopressure at \( y^+ = 40 \), \( C \) the \( \langle w^+ \rangle \) term in mean flow Eq. (25) and \( d(t) \) the pressure fluctuations at \( y^+ = 40 \).

Aubry et al. [107] investigate these equations for the very severely truncated system \( n = 1, k_1 = 0, k_2 = 1, 2, \ldots, 5 \); a five mode model. This includes basis functions that described the streamwise vortices. However, there were no modes containing \( \gamma \)-direction dependence. In the first calculations they set the pressure term at \( y^+ = 40 \) equal to zero. For high values of the dissipation coefficient \( z \), Aubry observed that stable streamwise rolls (vortices) exist. In the language of dynamic systems the equations have a stable fixed point in phase space. By decreasing \( z \), a regime was encountered with intermittency. The fixed point bifurcates into stable saddle points. The solution stays near one point for a relatively long time, oscillates with growing amplitude for a period, followed by a jump to the other saddle point. Flow at the new point is the previous streamwise vortices translated in the \( z \)-direction by half a cycle. Fig. 43 from Aubry et al. [107] shows the velocity patterns in the \( x-z \) computation plane during the jump period. The authors note that the rush of activity and jump is similar to a burst. The Reynolds stress during the jump does increase, however, the increase is moderate; 20–25%. Holmes et al. [106] explain that a reason for the modest increase may be that the vertical velocity is inadequately represented by this severe truncation. For slightly lower values of \( z \) they observe “chaotically modulated” cycles with asymmetric, less-regular flow fields. As one continues the calculation, the quiescent periods between jumps become infinitely long.

In another set of calculations Aubry et al. [107] include the pressure term at \( y^+ = 40 \). They take pressure data from the DNS calculations of KMM (\( Re_* = 180 \)). Their conclusion is that the pressure term does not significantly change the dynamics during a jump (burst), but that during quiescent periods the pressure can move the solution state to instigate a jump. Although the addition of the pressure term gives a finite time between jumps, the time is still much longer than the observed time between bursts. Stone and Holmes [116] propose that the time also depends on the level of the pressure fluctuations. Adjusting the required pressure fluctuation level can reproduce the correct experimental frequencies.

In Sanghi and Aubry [108] two modes with streamwise variation are included. This allows for the essential process of vortex line stretching. They observe the same bursting behavior as before, somewhat less sensitivity to the pressure term, and that the length of the vortices becomes much smaller during the bursting event. There are some additional instabilities of streamwise modes when \( z \) is reduced below the window for which the jump phenomena is observed. The Cornell workers believe, although there are many simplifications and approximations that dilute the quantitative results, that the jump phenomena is the essential dynamical “skeleton” of bursting (Holmes et al. [106, p. 374]). The mechanism is that outer pressure perturbations push the system from one roll-state to another, and that bursting is associated with this transition.

With respect to wall layers, the other group that has worked in this area is Sirovich and coworkers. Their approach has similarities and also distinct differences with the Cornell group. While Aubry form the basis functions from experimental data at \( y^+ < 40 \), Sirovich always uses DNS results from channel flows. By using the full channel, Sirovich eliminates the pressure term at
The outer flow effects are in principle included in the basis functions. The conclusions of this group are quite contrary to those of the Cornell group. Sirovich et al. [111] and Sirovich et al. [110] take a DNS channel flow ($Re_* = 125$) and decompose the instantaneous records in KL expansions. They find that modes with streamwise dependence, $k_1 \neq 0$, are convected with approximately the mean flow speed. These papers give arguments that the propagating modes trigger turbulence and that the frequency of bursting is comparable to the experimental values. Zhou and Sirovich [113] formulate a five mode model similar to that of Aubry et al. [107], except the basis functions are found from DNS calculations. As noted above, the pressure term is avoided by integrating across the channel. They find a jump behavior similar to Aubry. However, when they include $x$-dependent, propagating (convecting) modes, in 16 and 27 mode calculations, they find that the process becomes essentially different. The jump behavior gives way to chaotic solutions. In light of this they argue that the jump behavior of Aubry et al. [107] is a remnant of the low-order truncation. From these calculations they surmise that the convecting (propagating), $x$-dependent modes are essential triggers for the bursting process.

In all work cited above the time-dependent, $x-z$ spatially averaged, mean flow $U(y, t)$ is calculated by Eq. (25) which includes the assumption $\partial U/\partial t = 0$. The results, on the other hand, show that the time scale for $U$ is quite short. As one might expect from Eq. (25) the time scale for $U$ and $\langle uv \rangle$ are similar. Sirovich [112] avoids the use of Eq. (25) by also expanding the mean velocity by Eq. (23). The mean velocity is represented by combinations of basis functions for modes with $m$ and $n = 0$. He calls these “flux” modes. The DNS data base for the basis functions is calculated with constant $dp/dx$ so the flow rate and $x-z$ mean velocity are actually time dependent because the calculation domain is finite. In addition to the flux modes ($m = 0, n = 0, q$) for $U(y, t)$, they define roll modes ($m = 0, n \neq 0, q$) with velocities $u = 0, v \neq 0, w \neq 0$. The swirling motion of the roll modes has no $x$-dependence. Remaining modes ($m \neq 0, n, q$) are termed propagating (convecting) modes and they have $x$-dependence. Sirovich et al. [110] find that these modes have a nearly constant phase speed. The roll and propagating modes together are known as the fluctuating modes.

A study of a minimal channel is then done. It shows that convecting (propagating) modes had greatest Reynolds stress in the core of the channel while roll modes had greatest contributions near the wall. Furthermore, Sirovich [112] finds a typical time sequence where the roll modes lose their peak energy and then convection (propagating) modes attain their peak energy. Another conclusion is that if a realistic low-dimensional model is possible it must consist of...
carefully selected roll and convecting (propagating) modes.

Omurtag and Sirovich [115] find that in order to capture a high level of realism in a dynamic system, one needs a large number of flux modes in the system. In the investigation of flux modes they find that four flux modes represent the proper bulk (time independent) velocity, but still has some small wiggles. Eight flux modes do much better, and 16 produce nearly the exact result. The number of equations in these systems is 168 for four flux modes, 304 for 8 flux modes, and 576 for 16 flux modes. Thus, the meaning of “low dimensional” is expanding. In all these calculations they adjust the eddy viscosity to produce the same total dissipation as that in the DNS calculations.

Fig. 44 shows the Reynolds stress from the roll and flux components and the total for the DNS calculation. In these calculations \( q = 1, 2, 3, 4 \), \( m = 0, \pm 3, \pm 6 \), and \( n = 0, \pm 3, \pm 6 \), and \( \pm 9 \). A conclusion is that the roll and propagating modes contribute in different regions of the wall-layer. The component intensities, shown in Fig. 45, have the proper shape, but are not quite strong enough. Another observation is that convecting modes in the outer layer instigate a disruption of the inner roll modes that, in turn, transfers energy to the outer region.

There are several general observations one can make. The very low-dimensional roll-jump process of the Cornell group is mathematically elegant in its simplicity. However, it is not yet firmly related to the bursting process. On the other hand, in systems with many modes, it is very difficult to discern simple dynamics. Sirovich indicates that one must carefully select the proper modes if one is to produce a low order dynamic system that has the essential elements of turbulence. This cannot be done on the basis of the energy content of the modes alone. There may be modes of small energy content that are critical to the dynamics. A correlation method is needed to select modes that are dynamically important.

Another characteristic of note is that the basis function approach does not separate the mean flow and the streak velocity profile. Both are in the flux or mean modes. Thus, the Roll \( \Rightarrow \) Streak profile \( \Rightarrow \) X-instability \( \Rightarrow \) Roll mechanism is hidden. Waleffe [99] proposes a separation method for these modes.

Berkooz et al. [117] and Sirovich and Zhou [118] comment on the effect of Reynolds number on the models and they criticize each other’s conclusions. Sirovich’s group use eigenfunctions from a low Reynolds number channel flow that has a certain mixture of inner and outer effects. Any pressure effect from the outer region is incorporated into information in the eigenfunctions and not readily apparent. The Cornell group, on the other hand, assumes a pressure signal at \( y^+ = 40 \) as the interaction method and truncates the eigenfunction integrations at that level. Since the Reynolds stress is a typical part of \( R_{ij} \) in Eq. (22), it is interesting to note the value used in the data set used by Aubry at \( y^+ = 40 \), \( Re_* = 265 \) is \( \frac{-\langle w \rangle / u_*^2}{80} \). At infinite \( Re_* \), this value would be 0.93. Thus, the \( Re_* \) based on \( y^+ < 40 \) are not quantitatively accurate for all \( Re_* \). Actually in both approaches the hope is that, as it is with other DNS studies and structure experiments, the dynamics uncovered is typical and qualitatively similar to that at higher \( Re_* \).

If one seeks eigenfunctions that are truly independent of Reynolds number then one must assume that the two-layer structure of the wall layer applies to all \( Re_* \). Since the purpose is to understand the Reynolds stress producing mechanisms the actual assumption would be that the active parts of \( R_{ij} \) can be represented by the wall-defect decomposition, Eq. (13). Inserting the

![Fig. 44. Reynolds stress contributions from roll modes and flux modes according to the calculations of Omurtag and Sirovich [115].](image1)

![Fig. 45. Computed rms fluctuation intensities for all components. From Omurtag and Sirovich [115].](image2)
decomposition of $R_{ij}$ into Eq. (24) would allow basis functions for the inner region (wall layer plus log layer) to be computed.

9. Summary

There are several viable self-sustaining mechanisms in wall turbulence. Two general classes of mechanisms are the parent vortex-offspring vortex class and the streak instability class. In addition, some schemes are better categorized by the mathematical approach. In each category there are some major themes that are agreed upon, but on the other hand, essentially different mechanisms exist. All of the well regarded mechanisms are verified by simplified experiments and/or DNS calculations.

One type of parent–offspring mechanism involves generation of a streamwise vortex by an existing streamwise vortex. This occurs near either end of the original vortex. Other mechanisms focus on the regeneration of a horseshoe vortex by a parent horseshoe vortex. Inviscid processes of the interaction of the legs and lift up of the head are dominant in this mechanism. What may be a third distinct mechanism is the bridging between two streamwise fingers of vorticity as calculated form the Q2 initial condition. A slightly different viewpoint emphasizes curvature of the parent vortex line and the associated adverse pressure gradient. This occurs at the head of the hairpin and also where the vortex lines of the legs turn to the spanwise direction. These mechanisms all envision activity within the inner region. A variation is the concept that vortices regenerate into packets. The hairpin packets grow to inhabit the outer region. A final mechanism is the concept that vortices from the outer region interact with the inner region to engender an offspring near the wall.

Workers in the instability group have abandoned the idea that a simple inflection in the $U(y)$ profile is important. It is generally agreed that a streak profile, $U(y, z)$, with a periodic variation in $z$ matching the streak spacing has unstable normal modes. The unstable modes are sinuous in the flow direction. Evidently shows that a certain strength of the base profile is needed for linear instability and that the population of these streak profiles in DNS data bases is very sparse. An alternate theme is that transient growth of finite sinuous disturbance is the dominant instability. In any event, subsequent growth and nonlinear development generates an intense streamwise vortex. This process has been calculated for both turbulent Couette and Poiseuille flow. In principle the streamwise vortex sweeps slow moving fluid together and reproduces the streak velocity profile.

Regarding the instability mechanisms, it is an open question whether the outer layer is the source of the sinuous perturbation. The majority of researchers favor a mechanism confined to the inner layer. This is compatible with the required Reynolds number scaling.

Minimal channel studies show that the streak-instability mechanism can exist at low Re in a situation without an outer layer, whereas the vortex parent–offspring mechanism cannot. This should be regarded as support for the instability mechanisms, but one should not conclude that the parent–offspring mechanisms are not viable. They are probably absent because of the low Reynolds number and small computation region that prohibits large scale activity.

The dynamics systems research area has offered three different self-sustaining mechanisms. Waleffe has constructed a model for Couette flow that has a roll $\Rightarrow$ streak profile $\Rightarrow x$-instability $\Rightarrow$ roll mechanism. For some initial conditions the phase plane shows periodic orbits around an attractor. The work of the Cornell group interprets bursting as a jump in the solution from one roll-state to another. Both roll states are stable nodal points in phase space. The transition is initiated by a strong pressure perturbation from the outer region. Although the jump between stable nodal points is a mathematically elegant theory, critics contend that it is a mathematical remnant of the truncation. In the third mechanism, Sirovich and co-workers decompose the velocity into flux, roll, and convecting modes and conclude that many carefully selected modes are needed to model turbulence. With such a high order system one cannot easily determine the phase plane behavior. However, they do identify the Reynolds stress activity as triggered by the interaction of convecting modes with the roll modes. Identification of dominant self-sustaining mechanisms awaits further progress.

Much of our knowledge of the details of turbulence has been learned at low Reynolds numbers and self-sustaining mechanisms is no exception. Even at low Reynolds numbers, we are unsure of the importance of different self-sustaining mechanism. Behavior at higher Reynolds numbers is much more speculative. Two other extremely important areas where our current knowledge is inadequate are; flows over rough walls, Antonia and Djenid [119], and three-dimensional boundary layers. How much of our knowledge of self-sustaining mechanisms from smooth-wall, two-dimensional flows applies to these flows is unknown. There is much work to be done.

References


