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REPLACEMENT STRATEGIES

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Abstract: We define what is a replacement policy for a system that fails randomly in time and its main characteristics. There are several parameters that drive the structure of the optimal policy. We provide a detailed discussion of the time horizon, objective functions, and failure time distributions as part of any replacement policy.

1 Definitions and main characteristics

Our daily lives involve the observation, use, construction, and destruction of many systems. We replace the light balls when they burn out, take our cars to the repair shop when the breaks need to be replaced or the oil is due to be changed. The situations requiring a *replacement* are usually connected to wear-out, ageing, deterioration, or failure of the item/system involved. In this presentation we will assume that all of these processes are random.

Definition: A replacement policy π is a decision making rule that defines the time and type of replacement of an item or a system such that an objective defined by the decision maker is optimized.

The main characteristics of any replacement policy are:

- *Objective function* - A common criteria in deciding when to perform a replacement is economically justified, i.e. a cost function (called *objective function*), $G(x)$, is optimized with respect to a set of parameters. Examples are the smallest average replacement cost per unit time or the total discounted cost. Alternative objectives are the maximum reliability, minimum net present cost, internal rate of return, or any general utility function defined by the decision maker.

- *Time of replacement* - A replacement can be performed: as soon as the item fails (also known as corrective maintenance), before failure (preventive maintenance), or in a certain amount of time after failure. This will be one of the objective function's parameters, call it T. We will try to find the value of T that optimizes the objective function.

- *Type of replacement* - The item can be replaced with a new one or with an old (used) one. This is another one of the objective function parameters, denote it by a. As before, we would like to find the age of the system with which we will replace the "old" one that optimizes the objective function.

- *Failure time* - The failure occurrences are assumed to be random following a general counting stochastic process $\{N(t), t \geq 0\}$, see Rausand and Høyland (2004, page 231). The "failure time" will be the time between two consecutive failures.

We can formulate the general stochastic optimization problem of finding the optimal replacement policy $\pi = (T^*, a^*)$ as

$$\min_{T, a \in R^+} E[G(T, a, t)] \tag{1}$$

where E stands for the expectation operator taken with respect to a filtration up to time t defined by the counting process $N(t)$. The decision variables are T - time of replacement and a - age of the replacing item. Note that if we want to find the maximum of the objective function, it will be equivalent to finding the minimum of its negative value.

Here are some common replacement policies one can obtain for specific values of the time of replacement, T:

- Age replacement policy: the item/system is replaced upon failure or at age Y, whichever comes first (see eqr111).

- Block replacement: the item/system is replaced at regular time intervals $X, 2X, \ldots$ regardless of age (see eqr361).
- Group replacement: a group of items is replaced at the same time to take advantage of economies of scale (see eqr105).
- Condition-based replacement: a collection of variables measuring the state of system's degradation is monitored and replacement decision is made based on their values (see eqr123).
- Opportunity-based: the replacement is performed at time when *opportunity* arrives. Examples of opportunities are: scheduled downtime, lunch breaks, failure of a system in close proximity to the item of interest. The opportunities' arrival process is assumed to be random.

There are three main replacement types based on the values of a:

- $a = 0$ The failed item/system is replaced with a brand new, i.e. with age of 0. This type of replacement is called "as good as new" or also known as *preventive maintenance*.
- $a = a_t$, where t is the time since last replacement and a_t is the age of the failed item. This replacement is called "as good as old", or *minimal repair* (see eqr116).
- The age of the replacing item is different from 0 or the age of the failed item. This is called *imperfect repair* (see eqr107 and eqr114).

The literature on replacement policies is enormous. The earlier work was done in the early 1960s by Barlow and Proschan (1965). Excellent survey of the literature is presented by Valdez-Florez and Feldman (1989) and Dekker (1996). Rausand and Høyland (2004) is one of the contemporary textbooks on reliability. Marquez and Heguedas (2002) present a review of the recent research on maintenance policies and solve the problem of periodic replacement in the context of a semi-Markov decision processes methodology. We will review some of the most recent work on replacement policies and refer the readers to the above references for earlier research.

The reliability literature on replacement models and policies that allow for a change of the future failure behavior of the system is limited. There are several papers that could be classified as either models where replacement actions reduce the rate of failures, or models where the replacement action reduce the (virtual) age of the system, see Rausand and Høyland (2004, page 287), for details.

Such problems where the replacement decisions may influence the future stochastic nature of the system are referred as *decision-dependent-randomness* problems. For a general overview of the existing literature that relates to this class of problems see Morton and Popova (2001). Models with decision dependent uncertainty are discussed by Jonsbråten (1998), and T.W. Jonsbråten and Woodruff (1998).

Lai et al. (2006) analyze a single-unit system subjected to external shocks. The unit can fail due to ageing or shock. This is an example where the failure rate of the system increases after a lethal shock or with ageing (see eqr115). The policy is to replace the system after the n^{th} shock or failure, whichever occurs first. They minimize the long run expected cost per unit time to obtain the optimal value of n. Lai and Tang (2006) consider a two-unit system where the failure of each unit either increases the failure rate of the other or brings it to an instantaneous failure. The system is replaced at age T or at failure whichever occurs first. The value of T is obtained by minimizing the long run expected cost per unit time.

2 Time horizon and objective functions

An important characteristic of the replacement decisions is the time horizon over which we want to solve the optimization problem - it can be either finite or infinite.

2.1 Infinite time horizon

In the infinite case one can use existing limit theorems (for example from renewal theory and Markov decision process) to obtain the structure of the optimal replacement policy, see Ross (1996) (see eqr085 and eqr127). The long-run expected cost per unit time and the total discounted cost are the two common objective functions used in this case.

Juang and Sheu (2003) analyze a $k-$ out-of-n system and find the optimal age replacement policy (see eqr102). They consider two approaches - one that minimizes the long-run cost per unit time, and a graphical approach based on the total time on test concept. Jiang and Ji (2002) consider the age replacement policy by introducing a different objective - multiple attribute utility. In addition to cost they include the availability, reliability, and lifetime as attributes of the objective function.

The optimal replacement of item under warranty is important problem both for the manufacturer and the user. Iskandar and Sandoh (1999) consider a system with warranty period $[0, S]$ where opportunities for replacement occur according to a Poisson process. The policy is as follows: when the system fails at age $x \leq S$, a minimal repair is performed, if an opportunity occurs at age $S < x < T$ the system is replaced with probability p, and it is also replaced at age T. The optimal values of the parameters are obtained by minimizing the long-run expected cost. Iskandar and Murthy (2003) study a combination of repair and replace strategy for a system under warranty. They obtain the optimal policy by minimizing the expected cost of servicing the warranty (see eqr133).

2.2 Finite time horizon

The finite time horizon is a bit more challenging since there are no existing general results from the stochastic/reliability literature to indicate which replacement policy is optimal. Su and Chang (2000) find the periodic maintenance policies that minimize the life cycle cost over a predefined finite horizon. Galenko et al. (2005) solve the combination of preventive replacement and minimal repair policy over a finite horizon by minimizing the total replacement cost where the time of replacement is an integer variable.

There are other objective functions used in the finite time horizon case. Net Present Value (NPV) is the difference between the sum of present values of the project's (replacement policy) future cash flows (computed as the difference between inflows and costs) and the initial cost of the project. The NPV approach is the most common method used in capital budgeting (since the decision when and how to replace fits within the capital budgeting framework).

When making the replacement decision one chooses the option with the highest NPV. This simple rule does not work very well when the replacement decision involves choosing between two machines with unequal lives. Suppose that both machines can do the same job, but they have different operating costs and will last for different time periods. A simple application of the NPV rule suggests taking the machine whose costs have the lower present value. This choice might be suboptimal because the lower-cost machine may need to be replaced before the other one. The easiest way to compare the two machines involves calculating something called the *equivalent annual cost* of each machine. In other words, the NPV of the costs computed over the finite time horizon is converted into an annuity cost assuming that the machines will have to be part of the production process forever, i.e. into infinite horizon case. Examples of such necessities can be found in the health care system where a new admitting system may be needed every 5 years; making a decision about copy machines; replacing computers, etc.

Another frequently used approach is the Internal Rate of Return (IRR). The basic rationale behind the IRR method is that it provides a single number summarizing the merits of a project. That number does not depend on the interest rate prevailing in the capital markets. The IRR is the discount rate that equates the NPV of the project to zero. The general decision rule is very simple: accept the project if the IRR is greater than the discount rate; reject the project if IRR is less than the discount rate. One very common cited difficulty is when this approach is used to choose between mutually exclusive projects, which might be the case in the replacement policy framework. The problem arises since the IRR method ignores issues of scale. In other words, one project may have a higher IRR but its NPV may be low compared to the other project that has lower IRR but higher NPV. This is a very important issue in capital budgeting since the main objective of every company is to increase the shareholders value (i.e. the value of the firm). Making a decision to choose between mutually exclusive projects by accepting the one with the highest IRR may not be the optimal decision from maximizing the firm value point of view. There are several ways that one can solve this problem. One is to use the NPV method and make a decision based on highest NPV. Another way is to compute the incremental NPV and accept if it is greater than zero. A third one is to compare the incremental IRR to the discount rate and accept if the IRR is greater than the discount rate.

Some companies use the Profitability Index (PI) method to evaluate projects. It is the ratio of the present value of the future expected cash flows after initial investment divided by the amount of the initial investment. This approach shares the same advantages and disadvantages with the IRR rule. For mutually exclusive projects it ignores the issue of scale. Incremental cash flows have to be constructed and than the PI rule can be used. The project will be accepted if $PI > 1$. This rule is also useful in the capital rationing context. Suppose that the firm does not have enough capital to fund all positive NPV projects. In the case of limited funds, we cannot rank projects according to their NPVs. Instead, we should rank them according to PI. The Profitability Index measures the dollar return for the dollar invested, i.e. ”the bang for the buck”, and is useful for capital rationing. We should note that the PI does not work if funds are also limited beyond the initial time period, i.e. not useful over multiple time periods.

Both objectives, IRR and PI, can be applied to the replacement problem in the following situation. Suppose we have to compare an existing replacement policy to a ”proposed” one (claimed to be economically better). Then we can compute the cost savings when the new policy is used, and construct both the IRR and PI objectives.

The above methods briefly describe some of the most common used approaches by companies. For additional examples and more detailed explanation see Ross et al. (2005). In reality, additional to them companies use methods like payback, discounted payback and accounting rate of return. The use of the methods varies with the industry. For example, firms that are better able to estimate cash flows are more likely to use NPV. Companies in the oil business or in the energy related business can do such estimation. But companies in the entertainment business, like motion-picture production may not be able to produce reliable cash flow estimates.

3 Failure time

An important input to the optimization problem (1) is the failure time distribution, which measures the time between two consecutive failures. The exponential distribution is a common assumption which implies that the system's failure rate function is constant over time. In this situation preventive replacement will not be optimal and one needs to only consider replacement at failure.

It is also important to recognize the implications of the type of replacement to the future failure time distribution. The replacement with a new item corresponds to having independent and identically distributed (i.i.d.) times between failures. This scenario is the best one if we have to estimate the failure time distribution's parameters from a data set since standard statistical techniques will be applicable.

The replacement with an old item has a variety of options: the age of the "new" item is the same as the age of the item that it is replacing, or it can be different. The first case is referred to as "replacement with as good as old" (or minimal repair) and the resulting stochastic failure process is the nonhomogeneous Poisson process, see Brown and Proschan (1983).

Recently, several papers analyzed a system which failure and repair times follow a geometric process. Zhang et al. (2001) consider a deteriorating system and find the optimal replacement time. They investigate two replacement policies: one based on the accumulated working time T of the system, and the other based on the accumulated number N of failures. The optimal values of T and N are obtained by minimizing the long-run expected cost per unit time. Zhang (2004) study a deteriorating system and obtain the optimal replacement policy based on the number of failures by minimizing the long-run expected cost per unit time.

Hu and Yue (2003) model a deteriorating system (according to semi-Markov process) that operates in semi-Markov environment. They show the existence of an optimal control limit policy that minimizes the discounted total cost over finite and infinite time horizon. Moustafa et al. (2004) analyze a system that deteriorates according to a multi-state semi-Markov process. They obtain a control limit policy using the policy-iteration algorithm for the expected long-run cost rate of the system. Chen and Feldman (1997) formulate a Markov decision process model for a deteriorating system and show that the control limit policy for a modified repair/replacement is optimal over the space of all possible policies under the discounted cost criterion.

A different failure time model is presented by Castro and Alfa (2004) who model a single unit system with operational time according to a phase-type distribution. In addition the system is subjected to external failures that arrive as a Bernoulli process. They present two versions of the age replacement policy. Archibald et al. (2004) use Cox regression model to estimate the underlying maintenance behavior. They develop a stochastic dynamic programming approach to obtain the optimal replacement policy for a system subject to a decay. In addition they investigate the stability of the results to changes in the cost parameters.

Castanier et al. (2001) propose a combined replacement policy (based on periodic and aperiodic inspections) for a stochastically deteriorating system. They model the deterioration process as a Gamma process (see eqr098) and derive the long-run expected cost per unit time for this policy. Barros et al. (2006) focus on a parallel system of two units with dependent failures and subject to external shocks that arrive according to a Poisson process (see eqr055). The failures, however, are detected with a given probability. The replacement policy is a group-replacement, i.e. both items are replaced regardless of which one has failed. The objective function is the long-run expected cost per unit time.

A different approach to failure times modeling is via the Bayesian perspective. The classical failure time modeling assumes a parametric distribution (like exponential or Weibull) with parameter values that are, for instance, the maximum likelihood estimates. Standard Bayesian modeling assumes a parametric lifetime distribution and makes inferences about its parameters via the posterior distribution derived using Bayes' theorem, see Bernardo and Smith (1994). A review of the Bayesian approaches to maintenance intervention is presented in Wilson and Popova (1996). Chen and Popova (2000) propose two types of Bayesian policies that learn from the failure history and adapt the next maintenance point accordingly. They find that the optimal time to observe the system depends on the underlying failure distribution. A combination of Monte Carlo simulation and optimization methodologies is used to obtain the problem's solution. Another related reference is Mazzuchi and Soyer (1996). In Popova (2004), the optimal structure of Bayesian group-replacement policies for a parallel system of n items with exponential failure times and random failure parameter is presented. The paper shows that it is optimal to observe the system only at failure times. For the case of two items operating in parallel the exact form of the optimal policy is derived.

A more general approach is to use nonparametric Bayesian approach which, for instance, takes the hazard rate of the failure time to be a random "parameter", and puts the continuum of all distributions as it's prior. This notion of placing a prior on function spaces is termed "nonparametrics". Ferguson (1973) was the first to consider this idea from a Bayesian perspective. Since then, there have been hundreds of papers in a variety of contexts that use Bayesian nonparametric models, see Dey and Rao (2006). Once a prior has been put on the space of hazard rates, the posterior distribution of the hazard rate process is derived. Typically, this posterior distribution will not have a closed form solution. Markov Chain Monte Carlo (MCMC) methods are used to obtain inferences from the posterior distributions of interest; see, for example, Krishnan et al. (1999). Merrick et al. (2003) discuss optimal replacement strategies for machine tools when their failure behavior is modeled via Bayesian semiparametric proportional hazards model. The problem of a finite horizon single item maintenance optimization structured as a combination of preventive and corrective maintenance in a nuclear power plant environment is analyzed by Damien et al. (2007). In addition they present Bayesian semiparametric models to estimate the failure time distribution and costs involved.

A real world application of replacement strategies is presented by Brezavscek and Hudoklin (2003) who develop a block replacement and spare-provision policy for the electric locomotives in Slovenian Railways. Aka et al. (1997) investigate the relationship between a replacement and inventory policy for a manufacturing system.

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5 Related Articles

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