OPTIMAL PREVENTIVE MAINTENANCE UNDER DECISION DEPENDENT UNCERTAINTY

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ABSTRACT

We analyze a system of N components with dependent failure times. The goal is to obtain the optimal block replacement interval (different for each component) over a finite horizon that minimizes the expected total maintenance cost. In addition, we allow each preventive maintenance action to change the future joint failure time distribution. We illustrate our methodology with an example from South Texas Project Nuclear Operating Company.

INTRODUCTION

In this paper we address two important problems for the maintenance and operation of nuclear power plants: proper modeling and analysis of dependent failures and the impact of maintenance interventions on the future failure behavior of the system being analyzed.

Event/fault trees is the common methodology used in practice to model internal dependencies and estimate the probabilities of the ”top” events. One disadvantage of this approach is that it is a static model of the system and time-dynamics (i.e. how the failure behavior of the system evolves over time) cannot be incorporated. The use of Markov processes to describe the time evolution of the system, makes the assumption that the history of the failure process is not important, and only the current state of the system is sufficient to forecast the future.

There is an enormous literature on optimal maintenance policies for a single item that dates back to the early 1950s. The majority of the work covers maintenance optimization over an infinite horizon, see Valdez-Flores and Feldman [1] for an extensive review. The problem that we address in this report is over a finite planning horizon, which comes from the fact that every nuclear power plant has a license to operate that expires in a finite predefined time. In addition the form of the policy is effectively predefined by the industry as a combination of preventive and corrective maintenance, as we describe below. Marquez and Heguedas [2] present an excellent review of the more recent research on maintenance policies and solve the problem of periodic replacement in the context of a semi-Markov decision processes methodology. Su and Chang [3] find the periodic maintenance policies that minimize the life cycle cost over a predefined finite horizon.

A review of the Bayesian approaches to maintenance intervention is presented in Wilson and Popova [4]. Chen
and Popova [5] propose two types of Bayesian policies that learn from the failure history and adapt the next maintenance point accordingly. They find that the optimal time to observe the system depends on the underlying failure distribution. A combination of Monte Carlo simulation and optimization methodologies is used to obtain the problem’s solution. In Popova [6], the optimal structure of Bayesian group-replacement policies for a parallel system of $n$ items with exponential failure times and random failure parameter is presented. The paper shows that it is optimal to observe the system only at failure times. For the case of two items operating in parallel the exact form of the optimal policy is derived.

The reliability literature on models and policies that allow for a change of the future failure behavior of the system is limited. There are several papers that could be classified as either models where repair actions reduce the rate of failures, or models where the repair action reduce the (virtual) age of the system, see Rausand and Hoyland, page 287, [7], for details. Such problems where the decisions made influence the future stochastic nature of the system are referred as decision-dependent-randomness, hence the title of the paper. For a general overview of the existing literature that relates to this class of problems, see Morton and Popova, [8]. Models with decision dependent uncertainty are discussed by Jonsbråten [9], and Jonsbråten et. al. [10].

We analyze a system of $N$ components that could be in any dependent structure. The time horizon $T$ is finite. At the beginning of each time period, $t \in \{1,2,\ldots,T\}$ we observe the state of the system and must decide whether or not to perform preventive maintenance (PM) to each of the $N$ items. Each PM restores the state of the item to “as good as new”. At the end of each time period we investigate system and if any of the items have failed during the past time interval, we perform corrective maintenance (CM) that keeps the age of the repaired item the same (i.e. it is repaired to “as good as old” state). We will assume that the time increases in increment of 1 unit.

Each item $i, i = 1,\ldots,N$ can fail independently and in addition can trigger the failure of any of the other $N - 1$ items. The collection of all $N$ items constitute the system and it’s failure will be a function of the items’ failures. We introduce the notion of failure pattern below and show that the system failure is a subset of the set of all failure patterns.

**NOMENCLATURE**

Sets:

- $t \in \{1,2,\ldots,T\}$ optimization horizon
- $i \in \{1,2,\ldots,N\}$ number of items in the system
- $p \in \{1,2,\ldots,P\}$ enumerated failure patterns
- $a \in \{1,2,\ldots,A\}$ enumerated ages of the system

Constants:

- $Pm_i^t$ preventive maintenance cost of item $i$ in the time period $t$
- $Cm_i^t$ corrective maintenance cost of item $i$ in the time period $t$
- $U_p$ failure pattern, it shows which items have failed and which are still working
- $B_p$ additional aftermath costs associated with failure pattern $U_p$
- $f_{a,p}$ the probability of getting failure pattern $U_p$ with age structure $a$
- $D$ large constant used in the optimization model
- $Resources^t$ the available resources to perform preventive maintenance in time period $t$
- $MaxAge^t_i$ an upper bound on the age of item $i$

Variables:

- $d_i^t$ decision variable, it equals 1 if we perform preventive maintenance on item $i$ at time $t$, 0 otherwise
- $age_i^t$ age of item $i$ at time period $t$
- $z_i^t$ index of the current age structure of the system
- $y_i^t$ artificial variable used to calculate the age of item $i$ at time period $t$ with a given maintenance policy

**ILLUSTRATIVE EXAMPLE WHEN $N = 4$**

In this section we explain in detail the notation introduced above using a system of 4 items, i.e. $N = 4$. First, a particular $i, i = 1,\ldots,4$ corresponds to exactly one item in the system. If we say 3-rd item has failed then we know which item had failed and where this item is positioned in the system.

Table 1 shows the list of all failure patterns for a system consisting of 4 items. For instance, $U_9 = (0,1,0,1)$ means that we have observed failure pattern $p = 9$ where items numbered 2 and 4 had failed, and items numbered 1 and 3 are still functioning. For the case $N = 4$ there are 16
Aftermath Cost,

assume we start with a new system at time 0, then the size item with index 1 is two-period old in time period 2, item have an age equivalent to observing the failure pattern. Table 2 is an example of this additional cost.

\[ U_0 = (0,0,0,0) \]
\[ U_6 = (1,0,1,0) \]
\[ U_7 = (1,0,0,1) \]
\[ U_8 = (0,1,0,1) \]
\[ U_9 = (0,1,1,0) \]
\[ U_{10} = (0,0,1,1) \]
\[ U_{11} = (1,1,1,0) \]

**Table 2. AFTERMATH COST MATRIX**

<table>
<thead>
<tr>
<th>Pattern, ( p )</th>
<th>Aftermath Cost, ( B_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$0</td>
</tr>
<tr>
<td>2</td>
<td>$100</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>10</td>
<td>$1500</td>
</tr>
</tbody>
</table>

For instance, for \( p = 10 \) the aftermath cost is 1500. It means that if we see items numbered 3 and 4 failed (which will be equivalent to observing the failure pattern \( U_3 = (0,0,1,1) \)) we expect to lose additional 1500. In our setting, the aftermath cost will be different for each failure pattern \( p \).

We obtain the optimal preventive maintenance intervals for all \( N \) items by solving the following optimization problem:

\[
\min_{(d,y,z)} \sum_{i=1}^{N} \sum_{j=1}^{T} P_{ij} d_{ij} + \sum_{t=1}^{T} \sum_{t=p=1}^{P} (\sum_{m=0}^{N} C_{m} U_{p} + B_{p}) \sum_{a=1}^{A} f(a,p) z_{a}^{p}
\]

s.t.  \[ \sum_{a=1}^{A} a z_{a}^{p} = \sum_{i=1}^{N} \prod_{i=1}^{t} (\text{MaxAge}^{t-1} + 1) age_{i}^{t} \]
\[ t = 1, \ldots, T \]
\[ \sum_{a=1}^{A} a z_{a}^{p} = 1 \quad t = 1, \ldots, T \]
\[ age_{i}^{t} = t - y_{i} \quad i = 1, \ldots, N \quad t = 1, \ldots, T \]
\[ y_{i}^{j} \geq m d_{i}^{j} \quad m = 1, \ldots, t \quad i = 1, \ldots, N \]
\[ t = 1, \ldots, T \]
\[ y_{i}^{j} \leq \text{MaxAge}^{j} \quad t = 1, \ldots, T \quad i = 1, \ldots, N \]
\[ \sum_{i=1}^{N} r_{i} d_{i}^{j} \leq \text{Resources}^{j} \quad t = 1, \ldots, T \]
\[ d_{i}^{j} \in S_{i}^{j} \quad i = 1, \ldots, N \]
\[ y_{i}^{j} \geq 0 \quad i = 1, \ldots, N \quad t = 1, \ldots, T \]
\[ age_{i}^{t} \geq 0 \quad i = 1, \ldots, N \quad t = 1, \ldots, T \]
\[ d_{i}^{j} \text{ binary}, \quad i = 1, \ldots, N \quad t = 1, \ldots, T \]
\[ z_{a}^{p} \text{ binary}, \quad a = 1, \ldots, A \quad t = 1, \ldots, T \]

The objective function is to minimize the total expected cost in \( T \) time periods. The total cost consists of two terms - the first one, \( \sum_{i=1}^{N} \sum_{j=1}^{T} P_{ij} d_{ij} \), is the planned (PM) maintenance cost, and the second one, \( \sum_{i=1}^{T} \sum_{t=p=1}^{P} (\sum_{m=0}^{N} C_{m} U_{p} + B_{p}) \sum_{a=1}^{A} f(a,p) z_{a}^{p} \), is the unplanned (due to failure) cost.

The first two constraints make sure that the probability weights from the probability matrix \( f(a,p) \) in the objective
function correspond to the right age structure of the system. The second constraint allows us to set to 1 only one $z$ variable in each time period $t$. The righthand side of the first constraint transforms the age structure vector into age structure index. The third, fourth and fifth constraints modify the age structure in the time period $t$ that depends on our decisions $d$ up to time $t$.

THE PROBABILITY MODEL

In this section we describe how to obtain the probabilities, $f(a, p)$ from a set of real data for the CW example described above.

The main steps of the procedure are:

1. Compute the probability of independent failure of each of the items in the next time period given it’s age, $P_{age}^i$.
2. Compute $P(\bar{U}_{\text{ind}} = U_j|age_1^t, age_2^t, \ldots, age_N^t)$, the probability of observing failure pattern $j$ in the next time period as a result of independent item failures.
3. Compute the “dependent-adjusted” probabilities, $f_{a,p} = P(\bar{U}_{\text{dep}} = U_p|age_1^t, age_2^t, \ldots, age_N^t)$, where $S_{pj}$ corresponds to the probability of jumping from pattern $j$ to pattern $p$ due to interaction between items in the system.

In summary,

$$f_{a,p} = P(\bar{U}_{\text{dep}} = U_p|age_1^t, age_2^t, \ldots, age_N^t)$$
$$= \sum_{j=1}^{p} S_{pj} P(\bar{U}_{\text{ind}} = U_j|age_1^t, age_2^t, \ldots, age_N^t)$$
$$= \sum_{j=1}^{p} S_{pj} \prod_{i=1}^{N} \left[U_j^i P_{age}^i + (1 - U_j^i)(1 - P_{age}^i)\right] \quad (1)$$

SOUTH TEXAS PROJECT EXAMPLE

There are four circulating water (CW) pumps in each of two STPNOC plants. The water pumped by the CW pumps is necessary for the plants to generate electrical power. We describe one of the pumps and the equipment that supports its operation.

Referring to Figure 1, the pump is driven by an electrical motor supplied with 13,800 volt ac current through a circuit breaker used to control power to the motor. The motor is quite large, requiring about 1 megawatt when fully loaded to the pump. Contained within the motor is a mechanism that prevents the pump from spinning in reverse when it is shut off (current interrupted by the circuit breaker).

If the mechanism fails, the motor and pump will be destroyed by spinning rapidly in the reverse direction under certain circumstances occurring during plant operation. The circuit breaker must also operate correctly to prevent damaging the motor. In particular, all phases of the 3-phase circuit must be interrupted simultaneously to avoid motor electrical failure.

The electric motor drives the pump with a shaft and moves water up from a large man-made reservoir into a piping network through a shut-off valve. Because the pump capacity and flow rate are so large, the shut-off valve operates under a program to prevent sudden operation from causing a shock to the pump casing. Sudden shutting of the valve can cause a shock large enough to destroy the pump casing.

When a failure of one of the components in the system leads directly to the failure of another component, we will refer to that type of transition as ”dependent” other transitions are referred to as ”natural”.

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Table 3. FAILURE STATES

<table>
<thead>
<tr>
<th>State</th>
<th>Type</th>
<th>Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>dependent</td>
<td>Motor reverse mechanism failure leads to pump and motor failure</td>
</tr>
<tr>
<td>1</td>
<td>dependent</td>
<td>Breaker failure leads to motor failure</td>
</tr>
<tr>
<td>2</td>
<td>dependent</td>
<td>Failure of valve leads to failure of pump</td>
</tr>
<tr>
<td>3</td>
<td>natural</td>
<td>Valve failure prevents operation</td>
</tr>
<tr>
<td>4</td>
<td>natural</td>
<td>Breaker failure prevents operation</td>
</tr>
<tr>
<td>5</td>
<td>natural</td>
<td>Motor failure prevents operation</td>
</tr>
<tr>
<td>6</td>
<td>natural</td>
<td>Pump failure prevents operation</td>
</tr>
</tbody>
</table>

Table 4. ESTIMATED PARAMETERS OF THE MARGINAL TIME-TO-FAILURE DISTRIBUTIONS

<table>
<thead>
<tr>
<th>Item</th>
<th>Shape</th>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pump</td>
<td>3.13</td>
<td>817.37</td>
</tr>
<tr>
<td>Motor</td>
<td>1.79</td>
<td>2333.21</td>
</tr>
<tr>
<td>Breaker</td>
<td>0.94</td>
<td>13954.20</td>
</tr>
<tr>
<td>Valve</td>
<td>0.99</td>
<td>8034.33</td>
</tr>
</tbody>
</table>

Table 5. ESTIMATED TRANSITION PROBABILITIES

<table>
<thead>
<tr>
<th>From pattern</th>
<th>To pattern</th>
<th>Transition probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td>0.00051</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>0.000254</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>0.000508</td>
</tr>
</tbody>
</table>

Table 3 describes all possible states of the system with the corresponding failures.

We now have to compute the probabilities $P_{age_i}^t$ for $i = 1, \ldots, 4$ and $t = 1, \ldots, 12$. The time steps will be measured in years, i.e. we will solve the maintenance scheduling problem assuming a finite time horizon of 12 years.

Assume that the marginal time-to-failure time distributions for each of the 4 items, $X_i$ are $Weibull(\lambda_i, \alpha_i)$, where $\lambda_i$ are the shape and $\alpha_i$ are the scale parameters. We estimate them using data (right censored) that start in 1998 and end in 2005. Table 4 shows the estimated parameters for each of the items where the time is measured in days. Then the probabilities $P_{age_i}^t$, for $age_i < t$, will be equal to

$$P_{age_i}^t = P[X_i < t | X_i > age_i] = F_i(t) - F_i(age_i) / (1 - F_i(age_i)),$$

where $F_i(t), i = 1, 2, 3, 4$ are the corresponding Weibull cumulative distribution functions evaluated at $t, t = 1, \ldots, 12$ for all $age_i < t$.

The transition matrix, $S_{pj}, p = 1, \ldots, 16, j = 1, \ldots, 16$, contains the probabilities for transition from a failure pattern $j$ to a failure pattern $p$. Since our system is made of 4 units, then we will have a total of 16 failure patterns and as a result the dimension of the matrix $S$ is $16 \times 16$. Table 3 shows that we have 3 dependent states. Translating this into our failure pattern terminology: the non-zero transition probabilities will be for transition from pattern 2 to 5, 3 to 8, and 4 to 7. The rest will equal to 0. We assumed that the time-to-dependent-failures, $D_j, j = 1, 2, 3$, are exponentially distributed with parameters $\gamma_j$. We estimated them from the set of failure data. Table 5 shows the estimated transition probabilities.

The other parameters that are part of the objective function are the preventive ($C_{pm}$), and corrective ($C_{cm}$) maintenance costs, the aftermath cost for each failure pattern, and the max age for each of the four items. We used historical data to estimate $C_{pm}$ and $C_{cm}$ for time 0. Assuming 4% rate, we projected these costs for the rest of the time horizon (8 years) using present value discounting. The aftermath cost was assessed from past data and experts’ opinions. The maximum age is assumed to be 10 years for all of the four systems. Table 6 shows the estimated preventive...
and corrective maintenance costs at time 0, and Table 7 the aftermath cost matrix, $B$.

The optimization problem was run on a computer with Xeon 3.00Ghz CPU and 2Gb of RAM. CPLEX 9.1 [11] branch and bound algorithm was used to solve the problem described above. The optimal solution was obtained in 4436 seconds (1 hour and 14 minutes).

Table 8 shows the optimal replacement schedule. The results are intuitively appealing: the marginal failure time distributions of the breaker and valve have a decreasing failure rate functions and therefore it is not optimal to do preventive replacement; the cost for buying a new motor is less than the cost of repair, hence, it is optimal to replace it than to repair it; the pump fails too often and that is why we have to preventively replace it every period. The total expected cost when following the optimal policy is $445,215.00.

Conclusions

We present a new model for optimal scheduling of preventive maintenance when the future behavior of the system is influenced by the maintenance performed. The size and complexity of the problem grows exponentially with the number of components. We plan to address the large scale problem in the near future.

ACKNOWLEDGMENT

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deteriorating single-unit systems”. Naval Research Logistics, pp. 419–446.


