Comparison of Analytical and Computational Thermal Models for Gas Metal Arc Braze Welding

Sanjiv E. Shah  
University of Texas at Austin  
Austin, TX, USA

Jason C. Lee  
University of Texas at Austin  
Austin, TX, USA

Carlos Rios-Perez  
University of Texas at Austin  
Austin, TX, USA

Carolyn Conner Seepersad  
University of Texas at Austin  
Austin, TX, USA

ABSTRACT
Analytical and computational models are constructed for predicting the temperature distribution in a workpiece during gas metal arc braze-welding (GMABW). Specifically, the weld zone is modeled with Rosenthal’s analytical model, a finite difference model (FDM) and a finite volume computational model, constructed in the FLUENT® software package. Each model relates controllable braze-welding process parameters, such as traverse speed and applied voltage, to the temperature field that develops during the braze-welding process. Model-based predictions are validated by comparison with experimental data obtained from braze-welded specimens of C22000 commercial bronze alloy (CuZn90/10%wt). Temperature data are collected during the braze-welding process via thermocouples and infrared pyrometers aimed at the top surface of the specimen. Recommendations are made regarding the range of applicability and limitations of the models. In addition, opportunities are discussed for applying these models as part of an automated control framework for GMABW.

1. INTRODUCTION
Braze welding is a special type of welding process that uses a filler metal with a liquidus temperature below the solidus of the base metal [1]. In braze welding, the filler metal is added to the joint via deposition from a filler rod or a consumable electrode, rather than by capillary action, as in brazing. Since the base metal does not melt, bonding takes place between the deposited filler metal and the hot unmelted base metals in the same manner as conventional brazing [2]. One parameter of importance during the braze welding process is the temperature field that develops in the base metal, which affects the resulting microstructure and strength of the final welded assembly. It is advantageous to resolve the temperature field in a workpiece, because the temperature evolution in the base metal directly affects the size of the heat affected zone (HAZ), the weld fatigue life, and the residual stresses remaining in the workpiece after welding [3]. Knowledge of the temperature field also serves as the basis for specific approaches to welding control that use temperature as a control variable. For these reasons it is advantageous to predict the temperature field during braze welding.

Several efforts have been made to accurately model the temperature evolution in a base metal during the braze-welding process [4,6,7,8,9,10,11], but few efforts have been made to compare analytical, computational, and experimental results for the purpose of predictive control. While research has been conducted to model the heat and mass transfer into the workpiece to control weld quality [12], the present study aims to use temperature evolution models as the basis for the braze welding set points.

In this work, specimens of commercial bronze are braze-welded and temperature measurements are taken at four points on the top surface of the workpiece. The temperature data are compared to temperature field estimations that are tabulated from an analytical model and two computational models. Uncertainty with respect to the temperature measurements is quantified via sensitivity studies and error norms. Conclusions are drawn from a comparison of the experimental data and model-based results in the context of those uncertainty bounds. Based on these comparisons, recommendations are made on the feasibility of using this combination of temperature measurements and thermal modeling for braze welding control applications.

Nomenclature

- $T$: temperature, (K)
- $k$: thermal conductivity, (W/m.K)
- $t$: time, (s)
- $C_p$: specific heat, (J/kg.K)
- $a$: width of the braze welding specimen (m)
- $g$: thickness of the braze welding specimen (m)
Consider the braze-welding process shown in an elementary schematic in Figure 1, which shows one-half of the symmetrical workpiece about the x-axis. If convection and radiation are ignored, the temperature evolution in a substrate irradiated by a moving heat source is described by Eqn. 1a. The rate of change of temperature in the substrate is equal to the thermal diffusivity, alpha (\(\alpha\)), times the Laplacian of the temperature, plus contributions from volumetric heat sources, such as chemical reactions that occur inside the material in Figure 1.

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\dot{Q}^*}{\rho C_p} \tag{1b}
\]

Some thermal models, such as the numerical FLUENT and finite difference models described in Section 2.2, are based on the assumption of a fixed coordinate system on the top surface of the substrate. When appropriate terms are removed, Eqn. 1b reduces to:

\[
\frac{\partial T}{\partial t} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\dot{Q}^*}{\rho C_p} \tag{2}
\]

Other thermal models, such as the analytical model described in Section 2.1, are based on the assumption that a coordinate system is aligned with the moving heat source as pictured in Figure 1. When appropriate terms are cancelled and the coordinate frame is rotated, Eqn. 1b reduces to:

\[
\frac{\partial T}{\partial t} - u \frac{\partial T}{\partial x} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\dot{Q}^*}{\rho C_p} \tag{3}
\]

where \(u\) is the velocity of the torch in the \(x\) direction. Eqn. 3 is based on the assumption that the torch moves only in the \(x\) direction.

Eqs. 2 and 3 may be solved analytically for specific types of boundary conditions, as described in Section 2.1. However, those analytical solutions consider only the problem of heat conduction. The temperature distribution may be influenced by other physical phenomena, such as convection, radiation, fluid dynamics in the weld pool, and temperature-dependent material properties, but those phenomena require additional equations that become intractable to solve analytically [13]. More sophisticated computational models, such as the FLUENT and finite difference models described in Section 2.2, can take these phenomena into account, but the associated execution times make them difficult to use for real-time, automated control applications. Hence, more computationally efficient analytical models are used in this research for preliminary analysis and sensitivity studies. As described in Section 5, candidate models for real-time control include the analytical models and surrogate or reduced order models based on data from the numerical models. The analytical and numerical models are described in the remainder of this section.

### 2.1 Analytical Model

One of the earliest solutions of the braze welding equation, Eqn. 3, is Rosenthal’s analytical model [4]. In Rosenthal’s model, the torch is modeled as a point heat source, such that the heat is assumed to be transferred from the torch to the substrate at an infinitesimally small point. The point source (torch) is assumed to move relative to the substrate, and the origin of the coordinate axes is fixed to the top of the domain as shown in Figure 1. Rosenthal alters the fixed coordinate system by defining a moving coordinate system aligned with the heat source.
source. In the moving coordinate system, $\xi$ represents the distance between a point on the substrate and the heat source, moving with velocity $u$, as shown in Figure 1. Accordingly, $x$ is replaced with $\xi$ in Eqn. 2, which is related to $x$ by the velocity of the heat source (i.e., $\xi=x-ut$). The resulting formulation is shown in Eqn. 4:

$$\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial \omega^2} + \frac{\partial^2 T}{\partial \eta^2} = \frac{-u}{\alpha} \frac{\partial T}{\partial \xi} + \frac{1}{\alpha^2} \frac{\partial T}{\partial \omega}$$

(4)

Rosenthal’s treatment of the governing equation is a quasi-steady approximation in which the temperature distribution surrounding a heat source, moving with constant velocity, is assumed to be steady with time [3].

Rosenthal’s solution for Eqn. 4 is documented in Eqn. 5. The solution is based on several simplifying assumptions. The substrate is assumed to be a semi-infinite solid subjected to a quasi-steady moving point source. The material properties of the substrate are assumed to be temperature-independent. The solution accounts for conduction within the solid but neglects convection, radiation, and melting/solidification. Finally, the heat source is modeled as a boundary condition. The solution is provided for three dimensional heat flow in terms of the radial distance, $R$, from the heat source, as follows:

$$T(R) = T_e + \frac{Q}{4\pi k R} \exp\left(-\frac{u}{2\alpha}(\xi + R)\right)$$

(5)

Rosenthal found that his solutions matched experimental data closely except near the actual weld zone. Rosenthal attributed the error in this zone to the fact that the heat source was modeled as a point source rather than a source of finite size and defined geometric distribution.

Rosenthal’s solution has several advantages and disadvantages. The solution is simple and very fast for calculating the temperature distribution in a welded solid. Also, it retains the 3-D characteristics of the problem and seems to be relatively accurate for specific welding regimes [5]. However, temperatures become less accurate and approach infinity as the radius, $R$, approaches zero because the heat source is modeled as a point source [4]. In addition, the solution is a quasi-steady solution; therefore, transient effects (e.g., startup) are not represented. Also, the assumption of a semi-infinite domain causes underestimation of the temperature field in the substrate because of heat conduction away from the weld zone into an infinite heat sink. Lastly, lack of temperature-dependent properties causes inaccuracies for braze welding applications that have a broad range of temperatures.

Other researchers have subsequently relaxed some of Rosenthal’s assumptions, as outlined in Table 1. For example, some of the subsequent models preserve the transient term in the model to reflect the time dependence of the problem [6,7,8,9,10,11]. Additional improvements on Rosenthal’s solution include: (1) coupling the heat equation with other physical phenomena and modeling the multiphysics system [13] and (2) formulation and solution of the heat equation in a finite domain as opposed to an infinite domain [9,10]. Furthermore, many subsequent models use different expressions to represent the heat source, thereby removing the singularity of the point source [6,8,9,10,11]. Representation of the heat source is very important for the practical relevance of the thermal model, and it is one of the most difficult conditions to model accurately.

<table>
<thead>
<tr>
<th>Year</th>
<th>Heat Source</th>
<th>Solution Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1946</td>
<td>Point source (Quasi-Steady)</td>
<td>Analytical</td>
</tr>
<tr>
<td>~1947</td>
<td>Point source (Transient)</td>
<td>Analytical</td>
</tr>
<tr>
<td>1975</td>
<td>Gaussian Distribution</td>
<td>Computational</td>
</tr>
<tr>
<td>1983</td>
<td>2-D Gaussian Distribution</td>
<td>Analytical</td>
</tr>
<tr>
<td>1985</td>
<td>Semi-Ellipsoidal Heat Source</td>
<td>Computational</td>
</tr>
<tr>
<td>1999</td>
<td>Double Ellipsoidal Heat Source</td>
<td>Analytical</td>
</tr>
</tbody>
</table>

Table 1: Table of Heat Source Models and Solutions

In this work, a variant of Eqn. 5 is used as the analytical model. In this variation (Eqns. 6a-6c), the solution retains the quasi-steady assumption and the point heat source of the original solution, but the substrate is assumed to be bounded by adiabatic planes oriented parallel to the direction of motion [4]. The effective result is a domain of finite width and finite thickness and a temperature profile that is lumped in the thickness direction (i.e., a thin plate assumption). The corresponding solution is:

For $\xi > 0$,

$$T - T_e = \frac{Q}{C_p \rho g u \alpha} \exp\left(-\frac{u \xi}{\alpha} + \sum_{n=1}^{\infty} \frac{2}{\mu_n} \exp\left(-\mu_n + 1\right) \frac{u \xi}{2\alpha} \cos\left(\frac{\pi n \mu_n}{a}\right)\right)$$

(6a)

For $\xi < 0$,

$$T - T_e = \frac{Q}{C_p \rho g u \alpha} \left[1 + \sum_{n=1}^{\infty} \frac{2}{\mu_n} \exp\left(\mu_n - 1\right) \frac{u \xi}{2\alpha} \cos\left(\frac{\pi n \mu_n}{a}\right)\right]$$

(6b)

where

$$\mu_n = \frac{\sqrt{2 \alpha \pi n}}{u a}$$

(6c)

In Section 4.1, this analytical model is used to investigate the sensitivity of temperature to several important variables.

2.2 Numerical Models
Two computational models of the weld zone were constructed: a customized finite difference model and a FLUENT-based model. Results from these models were compared with experimental data and an analytical solution. Both models take advantage the symmetry of the problem by setting an adiabatic wall condition on the surface that falls beneath the path of the welding torch.

Radiative and convective boundary conditions are applied to exterior surfaces of the substrate, with a convective coefficient of $h = 10\ \text{W/m}^2\text{K}$ and an emissivity of $\varepsilon = 0.9$. The surrounding environment is treated as a non-participating medium at a uniform, steady room temperature of 300 K.

The energy source employed in both numerical models corresponds to a 3.18 mm square heat source of $\dot{Q} = 1976\ \text{W}$ with a uniform heat flux distribution (top hat). Although the shape and distribution of the heat source has an important influence on the temperature distribution and history as shown by Nguyen [11], its effect is confined to the area near the weld zone where no thermocouple data can be obtained. Applying the FD model on the geometry and material properties mentioned below, square heat sources of 3.18, 7 and 10 mm, produce temperature profiles that varied less than 9% and 5% at 5 mm and 10 mm away from the center of the torch, respectively. The validation of the heat source size, shape and distribution will require the measurement of temperature very close to the center of the weld zone.

The thermal material properties of the base material were obtained by TPRL from the room temperature to 1000°C [16]. Varying temperature material properties were modeled initially using the FLUENT model. The variations in the results were very small because the changes to the thermal diffusivity of the material were not large. Both models use constant thermal material properties obtained at 550 K because they are the average set of values in this range of temperatures. These values are shown in Table 2. For both models, temperature history data is taken from a time that begins when the arc strikes the specimen at the origin of the coordinate system as outlined in Figure 1.

<table>
<thead>
<tr>
<th>$\rho$ (Kg/m$^3$)</th>
<th>$k$ (W/m.K)</th>
<th>$C_p$ (J/kg.K)</th>
<th>$\alpha$ (m$^2$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7735</td>
<td>179.68</td>
<td>413.3</td>
<td>5.62E-05</td>
</tr>
</tbody>
</table>

Table 2: Material Properties for CuZn10 (UNS C22000) [16]

The Finite Difference Model (FDM) is based on Eqn. 1 and will use a 2-D, implicit method. Eqn. 1 is simplified by retaining the terms that correspond to a 2-D diffusion process and includes terms that account for radiative and convective losses from the top and bottom surfaces of the work piece yielding to the following energy balance equation:

$$\frac{\partial T}{\partial t} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \frac{h}{\Delta t \rho C_p} (T - T_\text{wall}) + \frac{\varepsilon \sigma}{\Delta t \rho C_p} \left( (T^+ - T^-) \right) + \frac{\dot{Q}^i}{\rho C_p} \text{ (7) }$$

Defining the non-dimensional parameter, $\theta$ as follows:

$$\theta = \left( T - T_\text{wall} \right)$$

yields the following energy balance equation:

$$\frac{\partial \theta}{\partial t} = \alpha \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - 2\left[ \frac{h}{\Delta t \rho C_p} + \frac{\varepsilon \sigma}{\Delta t \rho C_p} \left( (T^+ - T^-) \right) \right] + \frac{\dot{Q}^i}{\rho C_p} \text{ (8) }$$

After applying a second-order central difference discretization scheme to the space derivatives and a backwards difference scheme to the time derivatives (implicit method), Eqn. 8 and the associated boundary conditions yield a set of discretized equations upon which the FDM is based. Equations 9, 10 and 11 are examples of the equations that apply to a corner node (opposite to where the coordinate system is located), border node (adiabatic border) and body node on the interior of the plate, where the subscripts $i$ and $j$ are indices that correspond to the location of the node along the $x$ and $y$ axes while $k$ represents the time index.

$$\frac{\theta_{i,j}^{k+1} - \theta_{i,j}^k}{\Delta t} = \alpha \left( \frac{\theta_{i+1,j}^{k+1} + \theta_{i-1,j}^{k+1} + \theta_{i,j+1}^{k+1} + \theta_{i,j-1}^{k+1} - 4\theta_{i,j}^{k+1}}{4 \Delta x^2} \right) \left( \frac{h}{\rho C_p} + \frac{\varepsilon \sigma}{\rho C_p} \right) + \frac{\dot{Q}^i}{\rho C_p} T_{\text{wall}} \text{ (9) }$$

$$\frac{\theta_{i,j}^{k+1} - \theta_{i,j}^k}{\Delta t} = \alpha \left( \frac{\theta_{i+1,j}^{k+1} - 2\theta_{i,j}^{k+1} + \theta_{i-1,j}^{k+1} + 2\theta_{i,j+1}^{k+1} - 2\theta_{i,j-1}^{k+1}}{4 \Delta x^2} \right) \left( \frac{h}{\rho C_p} + \frac{\varepsilon \sigma}{\rho C_p} \right) + \frac{\dot{Q}^i}{\rho C_p} T_{\text{wall}} \text{ (10) }$$

$$\frac{\theta_{i,j}^{k+1} - \theta_{i,j}^k}{\Delta t} = \alpha \left( \frac{\theta_{i+1,j}^{k+1} - 2\theta_{i,j}^{k+1} + \theta_{i-1,j}^{k+1} + 2\theta_{i,j+1}^{k+1} - 2\theta_{i,j-1}^{k+1}}{4 \Delta x^2} \right) \left( \frac{h}{\rho C_p} + \frac{\varepsilon \sigma}{\rho C_p} \right) + \frac{\dot{Q}^i}{\rho C_p} T_{\text{wall}} \text{ (11) }$$

Since the FD model was developed to evaluate the relative importance of the different heat dissipation mechanisms, the radiation term is not linearized in temperature and makes this a non-linear and time dependant problem. The solution of the temperatures for the system of equations for the FD model is conducted by an iteration process using the False Position Method [17].

As mentioned before, the FD model assumes the plate is thermally lumped in the z-direction i.e. there is no gradient of temperatures through the thickness of the plate; therefore, $\Delta z = 6.35$ mm. As a first attempt, a square grid is used in this
numeral model ($\Delta x=\Delta y$). This grid symmetry can be altered if the temperature gradient in one direction requires higher resolution.

The grid independence study was conducted analyzing the temporal variation of the highest temperature of the plate since the influence of varying the grid sizes is the greatest at the peak temperature. Three different grid sizes were analyzed: 1.5 mm, 1 mm and 0.7 mm using the C22000 measured material properties, $\dot{Q} = 2980$ W, and 6.35 mm/s of traverse speed. Peak temperature histories are shown in Figure 2. This figure also shows the presence of oscillations in the peak temperature history as a result of the time step, grid size and velocity employed. The heat flux is modeled such that a proportional heat flux is applied to an element partially within the heat zone. Depending on the time step and grid size, the proportion of the element that is partially in the heat zone may change with time, causing a varying heat distribution. To avoid this phenomenon, the time step could be chosen based on the grid size and weld velocity such that at each time step the weld boundary travels one element. However, due to the comparably small time step used on the FDM model, the oscillations in the peak are not completely eliminated. However, the amplitude of the oscillations for an energy source stepping 38 times on each element in cases with $\Delta x=\Delta y=1.5$ mm, $\Delta x=\Delta y=1$ mm and $\Delta x=\Delta y=0.7$ mm are 2.5 %, 1 %, and 0.6 % respectively with respect to the peak temperature. A variation lower than 3 % is considered acceptable for the current model; therefore, the model proved to be grid independent at $\Delta x=\Delta y=1.5$ mm with a heat source giving 38 steps each $\Delta x$.

![Figure 2: FDM Peak Temperature](image)

FLUENT is commercially available computational fluid dynamics software, suitable for solving heat transfer problems with a finite volume approach. The finite volume method is similar to the finite difference method. However instead of using the differential form of the heat equation, an integral form is used. This method ensures that energy is conserved because it models the fluxes across surfaces.

FLUENT has a number of different thermal wall boundary conditions available. For this model three different boundary conditions are used. (1) A symmetric boundary condition is chosen for the symmetry plane. As described above, this is equivalent to an adiabatic wall. (2) The remaining walls, except for the top wall which the torch traverses over, are modeled as a wall with mixed thermal conditions. These walls are specified with the environment temperature, convective heat coefficient and emissivity described above. (3) The top wall is modeled with a heat flux boundary condition. A user defined function (UDF) is written for the applied heat flux. In areas that are not under the torch zone, the UDF specifies the heat flux as the radiative and convective flux, and in areas under the torch zone, the UDF specifies the heat flux as the torch flux in addition to the radiative and convective fluxes.

The specimen geometry is meshed in GAMBIT, a commercial software designed to build meshes for CFD commercial packages, and imported into FLUENT. A cubic mesh is chosen. The quality of the mesh created is very important in determining how well FLUENT can accurately model the problem. Because the geometry is a simple rectangular prism, creating a uniform cubic mesh is not difficult. This is ideal because the aspect size and angle ratio are both 1. Three different sizes of these cubic meshes were created to study grid independence: 0.635 mm, 0.318 mm, and 0.254 mm. The study performed is identical to that performed using the FDM (the C22000 measured material properties, $\dot{Q} = 2980$ W and 6.35 mm/s of traverse speed). The peak temperature histories shown in Figure 3 has an average percent difference of 5.6 % for $\Delta x=\Delta y=0.635$ mm and 0.8 % for $\Delta x=\Delta y=0.318$ mm, with respect to $\Delta x=\Delta y=0.254$ mm showing relative grid independence for the grid size of $\Delta x=\Delta y=0.318$ mm.

![Figure 3: FLUENT Peak Temperature](image)

Two observations can be made on the peak temperature history plots, used for the grid independence studies. A local maximum peak temperature is observed at 0.25 seconds. At time $t=0$ the center of the heat zone is at the specimen origin. It takes 0.25 seconds for the entire heat zone to be active onto the specimen. Before 0.25 seconds the specimen is supplied a continuously increasing heat source.
causing the sharp gradient. After 0.25 seconds the power supplied is constant, and the specimen levels to steady state peak temperature. The peak temperature sharply increases again as the weld reaches the end of the specimen, around 24 seconds. This global maximum peak temperature occurs because heat dissipates faster through conduction than through convection and radiation. Since the weld is at the end of the plate, the reduced volume available for heat conduction causes a spike in temperature.

The accuracy of the thermally thin assumption used in the FDM model is analyzed by evaluating the temperature thickness distribution from the FLUENT model. The temperature thickness distribution, at the center of the plate when the torch is directly above, has a large gradient, as shown in Figure 4. It is clear from this temperature distribution, that the thermally thin assumption is not valid directly beneath the torch. Since the FDM model uses the thermally lumped assumption, it is expected to vary from the FLUENT model near the torch. As shown in Figure 5, the temperature differences between the FDM and FLUENT models at the top surface are large near the torch. However, these temperature differences are less than 10% at y>5 mm when compared using the grid size of Δx=Δy=1 mm for the FDM model. When the temperature distribution in the z-direction of the FLUENT model is averaged for comparison with the lumped-temperature results assumed in the FDM, the temperature distributions are similar, even near the torch, as shown in Figure 5.

3. BRAZE-WELDING EXPERIMENTS

One rectangular piece of commercial bronze measuring 152 mm long by 50.8 mm wide by 6.35 mm thick was braze welded with gas metal arc braze weld using the parameters outlined in Table 3. In Gas Metal Arc Braze Welding, a Metal Inert Gas (MIG) welder is used to perform the braze weld. Hence, the parameters in Table 3 refer to the settings on the MIG welder used during the experiment. Wire diameter was 0.762 mm, the voltage was set to 24 V, the wire feed rate corresponds to how fast the braze wire was fed into the braze joint, and the traverse speed corresponds to how fast the torch was moving during the braze weld experiment. The parameters of wire feed rate, voltage, wire diameter and traverse speed are crucial to the temperature field modeling as they directly correlate to the heat input of the model. Two 24 gauge K-type thermocouples were mounted to the top surface on the piece of commercial bronze. Two Omega OS554A infrared pyrometers were mounted to the welding torch and followed a point ±6.35 mm in the y-direction from the weld centerline and 28.9 mm in the negative x-direction from the welding heat source. The schematic of the thermocouple and pyrometer locations are outlined in Figure 6 and Table 4, respectively. The weld begins at the origin and moves along the positive x-direction. Experimental results are reported in Section 4.2.
4. RESULTS AND DISCUSSION

Temperature predictions from the analytical and numerical models were compared with temperature data obtained from the experiments. The results are reported in Section 4.2. Deviations between model-based predictions and experimental data may be attributed to both model simplifications and experimental error and uncertainty. Therefore, it is important to quantify the expected magnitude of uncertainty in the data. Of particular interest is the uncertainty in temperature data related to the precision of placement of the thermocouples and pyrometers. With the steep temperature gradients experienced near the weld zone, even a millimeter-scale error in placement could have a significant impact on temperature measurements. In Section 4.1, the analytical model is used to quantify the expected placement-induced uncertainty with respect to temperature measurements.

4.1 Sensitivity Study and Results of Analytical Model

In the case of braze-welding, several parameters can influence the temperature evolution in the welded solid. A sensitivity study serves to quantify the parameters that have the strongest effect on temperature measurements and may also be used to ascertain the impact of uncertainty associated with various measurements taken during experiments. In this work the sensitivity study is used to quantify the amount of error introduced into the temperature measurements as a result of uncertainty on thermocouple and IR pyrometer placement.

The sensitivity of the temperature evolution with respect to deviations in x and y placement are the quantities of interest, hence the steepest gradients of temperature with respect to x and y are sought [14]. The quasi-steady temperature distribution in a welded material by Rosenthal (Eqn. 5) is amenable to direct analytical differentiation and is used for the sensitivity study. The partial derivative that shows the highest degree of change has the most effect on the change in temperature. Eqns. 12 and 13 are the derivatives of Eqn. 5 with respect to y and x. These partial derivatives represent the parameters of importance regarding the placement of the thermocouples and IR pyrometers.

\[
\frac{\partial T}{\partial y} = \frac{\dot{Q}Y}{4\pi kR^2} \left( \frac{-1 - \frac{v}{R}}{R^2} \right) \exp \left[ -\frac{v(\xi + R)}{2\alpha} \right] \tag{12}
\]

\[
\frac{\partial T}{\partial x} = \frac{\dot{Q}}{4\pi kR^2} \left( \frac{-\xi}{R^2} - \frac{\nu}{\alpha} \right) \exp \left[ -\frac{v(\xi + R)}{2\alpha} \right] \tag{13}
\]

In the evaluation of Eqns. 12 & 13 the material properties listed in Table 2 and the braze welding parameters in Table 3 are employed. Figure 7 and Figure 8 display plots of the absolute value of the temperature derivative with respect to y and x.

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Table 3: Braze Welding Test Parameters

<table>
<thead>
<tr>
<th>X (mm)</th>
<th>Y (mm)</th>
<th>X (mm)</th>
<th>Y (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC 1a</td>
<td>76.2</td>
<td>TC 1b</td>
<td>76.2</td>
</tr>
<tr>
<td>9.53</td>
<td>-9.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pyro 1a</td>
<td>6.35</td>
<td>Pyro 1b</td>
<td>-6.35</td>
</tr>
</tbody>
</table>

Table 4: Thermocouple and IR Pyrometer placement

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Figure 6: Schematic of Thermocouple and IR Pyrometer Layout

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\( \dot{Q} = 2980 \text{ W} \)
In Figure 7 the partial derivative of the temperature with respect to y is plotted for an energy input of $\dot{Q} = 2980$ W with the weldhead at four different locations on the x-axis. The weldhead is at a distance ahead of the axis on which the thermocouples are mounted as outlined in Figure 6 and Table 4. This was done to avoid the singularity in Eqn. 12 that appears when $\xi$ = 0. By examining the plot in Figure 7 the line that corresponds to the weldhead located at 76.4 mm has the highest peak values of sensitivity compared to the plots of sensitivity with the weldhead at the other positions. The peak value that corresponds to the weldhead at 76.4 mm and the locations of the thermocouples are used to calculate the uncertainty and error introduced by misaligning a thermocouple in the y-direction.

In Figure 8 the partial derivative of temperature in the x-direction is plotted for an energy input of $\dot{Q} = 2980$ W with the weldhead located at 76.2 mm from the origin of the coordinate system outlined in Figure 6 for the thermocouple locations on the y-axis. The lines plotted in Figure 8 correspond to the thermocouple locations on the y-axis, specifically located at (76.2 mm, ±9.53 mm) and (76.2 mm, ±15.9 mm) for points in the x-direction behind the thermocouple axis. x corresponds to location of the weldhead on the weld specimen, the sensitivity is plotted with respect to $\xi$, the distance from the weldhead in the negative x-direction. Points plotted in Figure 8 correspond to points where $\xi$ is non-zero. The peak value of each curve in Figure 8 is used to calculate the uncertainty and error introduced by misaligning a thermocouple in the x-direction for each thermocouple. The peak value of each curve is used as it represents the highest deviation from the expected value of the temperature and will serve as an estimate of worst possible error introduced by misplacing the thermocouples.

### 4.2 Comparison of Model Predictions with Experimental Data

All models (Rosenthal, FDM and FLUENT) are used to predict the temperature history at the thermocouple locations documented in Figure 6 and Table 4. The model-based predictions and corresponding experimental data are illustrated in Figure 9. The experimental data were obtained from a braze weld conducted with the parameters outlined in Table 3. Experimental data were collected on both sides of the braze welded specimen. Six sets of data were collected during testing; experimental data shown in Figure 9 and Figure 10 are averaged values of the six tests at each time step. The experimental data shown in Figure 9 has a maximum overall standard deviation of 0.72 K. The experimental data shown in Figure 10 has a maximum overall standard deviation of 2.11 K.

Before comparing data and predictions, it is important to highlight some of the underlying assumptions. The traverse speed and power input for the computational models are set to match the experimental parameters and heat input measured during the experiment. The heat input that the specimen accepts can be determined by the nearly steady temperatures reached by the plate immediately after the weld torch has completed its pass. Assuming that the entire piece is lumped, the heat input required to achieve a specific lumped temperature can be determined with a first law energy balance to calculate the thermal storage of the workpiece as outlined in Eqn. 14.

$$\rho c_p \psi \frac{\partial T}{\partial t} = \dot{Q}$$

(14)

In Eqn. 14, heat input for the welded workpiece, $\dot{Q}$ may be estimated by multiplying the thermal capacitance of the metal by the volume of the material, $\psi$. In addition, knowing the duration of the weld schedule, $\Delta t = \Delta t_s$, and the change in temperature due to the braze welding process, $\Delta T = \Delta T_s$, the heat input into the material, $\dot{Q}$ and may be calculated. This calculated heat input is compared to the measured heat input from the voltage and current data to estimate the arc efficiency, $\eta$. In this case, the heat input is calculated to be 1976 W, corresponding to an efficiency of approximately 60%. The low efficiency may imply that the weld was performed in the improper braze welding regime, which would contribute to lower power and heat input and lower temperatures throughout the specimen.
Comparing the steady state temperatures (temperatures after the peak) of the models with the experimental data shows a disparity of 24 K between the FLUENT model and the experiment, a disparity of 2.5 K between Rosenthal and the experimental values and a disparity of 35 K between FDM and experimental values. The disparity for the FDM lies outside of the error range predicted by the sensitivity studies (i.e., +/- 21 K) in Section 4.1, but the disparities for FLUENT and Rosenthal temperature predictions fall within the error range.

In Figure 10, experimental data for the IR pyrometers is compared with the FLUENT, FDM and Rosenthal models. For the power input of $Q = 1976$ W, both the FLUENT and FDM models slightly under-predict the measured temperature profile. The Rosenthal model does not capture the trend of the transient experimental data because it is a quasi-steady formulation that does not take into account temperature history. Similar to the model predictions shown in Figure 9 most of the data points in Figure 10 fall within the aforementioned error band of ±21 K for the FLUENT, FDM and Rosenthal models. The mismatch in the model data and experimental data in Figure 9 and Figure 10 can be attributed to the method used to estimate the actual power input to the weld specimen. In utilizing Eqn. 14 to estimate the power input, the assumption is made that the temperature throughout the workpiece is lumped and that no heat is lost to convection and radiation.

5. CONCLUSION

Computational predictions from FLUENT, a finite difference model, and Rosenthal’s analytical approximation have been compared to experimental data of the temperature field that develops while braze welding a specimen of commercial bronze. Comparisons between the FLUENT and FDM models and Rosenthal’s analytical approximation show close agreement for developing temperatures but departures for peak temperature and post-peak temperatures. FLUENT and FDM models under-predict the peak and post-peak temperatures. This difference is attributed to the boundary conditions and method of modeling the heat source in each respective model. Both FLUENT, and Rosenthal predictions fell within the expected error bounds on the experimental data, based on a sensitivity study of errors due to thermocouple placement location.

Predictions from FLUENT and FDM and Rosenthal’s models were compared with experimental data from IR pyrometers. The IR pyrometers traveled with the welding torch, therefore offering a view of transient conditions near the ends of the specimen along with steady state conditions near the ends of the specimen along with steady state conditions in the middle. Rosenthal’s approximation agrees closely with experimental data under steady state conditions, but yields poor agreement for transient conditions such as start-up when the temperature is changing rapidly in the workpiece.

For real time control and predictive process control, the Rosenthal model may provide fair results in a steady state regime, whereas other methods that take temperature history into account such as FDM or FLUENT are necessary for non-
steady state and transient regimes. Since FDM and FLUENT models are not computationally efficient enough for real-time control applications, surrogate modeling techniques are being investigated. Examples include neural networks and advanced regression techniques.

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REFERENCES


