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Design for manufacturing: application of collaborative multidisciplinary decision making methodology

Angran Xiao*, Carolyn C. Seepersad†, Janet K. Allen ‡, David W. Rosen§, and Farrokh Mistree**

Systems Realization Laboratory

George W. Woodruff School of Mechanical Engineering

Georgia Institute of Technology, Atlanta, GA 30332-0405, USA

* Assistant Professor, Department of Industrial & Engineering Technology, Southeast Missouri State University, Cape Girardeau, MO 63701. email: axiao@semo.edu

† Assistant Professor, Mechanical Engineering Department, The University of Texas at Austin, 1 University Station C2200, Austin, Texas 78712-0292. email: ccseepersad@mail.utexas.edu

‡ Corresponding author, Associate Professor, Phone/FAX: 912-963-6901/912-966-7910. e-mail: janet.allen@me.gatech.edu

§ Professor. e-mail: David.Rosen@me.gatech.edu, Phone/fax 404-894-9668/9342

** Professor. e-mail: Farrokh.Mistree@me.gatech.edu, Phone/Fax 912-963-6900/912-966-7910.

Abstract

Design for manufacturing is often difficult for mechanical parts since significant manufacturing knowledge is required to adjust part designs for manufacturability. The traditional trial and error approach usually leads to expensive iterations and compromises the quality of the final design. The authors believe the appropriate way to handle product design for manufacturing problems is not to formulate a large design problem that exhaustively incorporates design and manufacturing issues, but to separate the design and manufacturing activities and provide support for collaboration between engineering teams. In this paper, the Collaborative Multidisciplinary Decision-making Methodology (CMDM) is used to solve a product design and manufacturing problem. First, the compromise Decision Support Problem is used as a mathematical model of each engineering teams' design decisions and as a medium for information exchange. Second, game theoretic principles are employed to resolve couplings or interactions between the teams' decisions. Third, design capability indices are used to maintain design freedom at the early stages of product realization in order to accommodate unexpected downstream design changes. A plastic robot arm design and manufacturing scenario is presented to demonstrate the application of this methodology and its effectiveness for solving a complex design for manufacturing problem in a streamlined manner, with minimal expensive iterations.

Keywords: Collaborative Design, Design for Manufacturing, Game Theory, and Multidisciplinary Decision Making

1. Design for manufacturing

Concurrent engineering involves separating product realization activities so that design activities can be executed independently while simultaneously incorporating relevant information from downstream domains such as manufacturing, assembly, or recycling (Prasad 1996). Concurrent engineering is challenging to implement for complex design problems, however, because designers often lack sufficient knowledge of downstream domains. For example, design for manufacturing (DfM) of mechanical parts is often difficult because significant manufacturing knowledge is required to adjust part designs to aid manufacturability for a specific process. Small design changes may cause large changes in the manufacturing process or render that process infeasible. Alternatively, a manufacturing team that understands the purpose of a design and its functional requirements may be more capable of adjusting it to facilitate manufacturing.

The authors believe the appropriate way to handle complex product design problems such as DfM is not to formulate large design problems but to support cooperation and collaboration between multidisciplinary engineering teams. Towards this end, the Collaborative Multidisciplinary Decision-making Methodology (CMDM) is established (Xiao *et al.* 2005) to enable collaborative decision making between design and manufacturing teams through 'collaboration by separation'. Separation signifies that the responsibility for DfM is transferred from the design team to the downstream manufacturing team; whereas, collaboration signifies that satisfactory systems-level solutions are coordinated with minimal information exchange and iteration. Unlike many mathematical multidisciplinary optimization (MDO) approaches (Balling and Sobieszczanski-Sobieski 1996, Wujek *et al.* 1996, Sobieszczanski-Sobieski and Haftka 1997, Kodiyalam and Sobieszczanski-Sobieski 2000, Sobieski and Kroo 2000, Sobieszczanski-Sobieski *et al.* 2000), this concept is especially well-suited for distributed environments in which geographical dispersion and decentralized management styles make information exchange and cooperation difficult. With the CMDM, several challenges in a DfM process are addressed:

1. *Exchanging Information*. The information required for decision-making in an activity must be transferred completely from one team to another, and the recipient teams should be able to understand the team's intentions without requiring additional information flows or causing iterations. The compromise Decision Support Problem (DSP) (Mistree *et al.* 1990) is used as the information medium in the CMDM to represent information 'in a computer readable and

retrievable format, share (it) among collaborative team members, and facilitate design reuse for new concept generation (Wang *et al.* 2002).

2. *Accommodating interactions between activities.* Some activities in a DfM process may be coupled, such that each design team makes decisions that affect the decisions of other teams. Game theory is used in the CMDM to model different degrees of collaboration and manage interactions between engineering teams, with little or no expensive, systems-level iterations.
3. *Maintaining feasible and satisfactory overall designs.* When design activities are separated, design teams must make decisions without full knowledge of their impact on downstream activities. If single point solutions are exchanged, downstream designers are prevented from adjusting designs for feasibility or satisfactory local performance, and iterations often ensue. With set-based approaches, however, ranges or sets of solutions are shared and gradually narrowed during the design process, thereby reducing or eliminating the need for global, systems-level iterations (Sobek *et al.* 1999). One way to generate ranges of solutions in a collaborative decision-making context is to use robust design methods to identify robust ranges of coupled design variable values for adjustment by subsequent, collaborating designers without adversely affecting previous disciplines (Chang and Ward 1995, Chen and Lewis 1999, Kalsi *et al.* 2001). Here, design capability indices (DCI) are used (Chen *et al.* 1999) as a metric for identifying ranges of solutions that can be adjusted subsequently by collaborating designers.

In a product realization problem, the dependencies between any two activities, such as design-manufacturing, design-design, or manufacturing-manufacturing, may be interactive or sequential. Game theory is used to resolve the interactive couplings and design capability indices are used to handle the sequential relationships. The authors believe the sequential relationships are more significant in DfM problems, mostly due to the upstream/downstream nature of the design-manufacturing relationship. As shown in Figure 1, while using the CMDM, the upstream design team presents a ranged set of solutions, all of which satisfy design requirements. Downstream manufacturing teams then specify, or select from, the ranged set of solutions based on manufacturability requirements without jeopardizing design feasibility. Effectively, the design team preserves design freedom for the manufacturing team, where design freedom is defined as ‘the extent to which a system can be adjusted while still meeting its design requirements’ (Simpson *et al.* 1998). Collaboration by separation enables a product design and manufacturing process to be accomplished in a streamlined manner, by gradually reducing design freedom along the product realization timeline, Figure 1. The purpose of this paper is to present a robot arm design and manufacturing scenario to demonstrate the effectiveness of the CMDM for solving a complex DfM problem.

<Figure 1 goes about here>

2. Collaborative multidisciplinary decision making methodology

The CMDM is implemented in three steps:

- Step 1 Representing decision making information in a compromise DSP which serves as an information medium to eliminate iterations caused by information exchange and communication;
- Step 2 Representing cooperation styles among engineering teams with game theoretic protocols to eliminate iterations caused by interdisciplinary interactions; and
- Step 3 Reformulating the compromise DSPs using design capability indices for finding superior ranged set of solutions that eliminate or reduce costly iterations caused by unexpected downstream requirements and constraints.

2.1 Step 1, modeling product realization activities using compromise DSPs

In order to resolve the first challenge, that of exchanging information, Section 1, a compromise Decision Support Problem, DSP, is used. A compromise DSP is a multi-objective decision model — a hybrid formulation based on mathematical programming and goal programming — that is used to find the values of design variables that satisfy a set of constraints and achieve a set of conflicting goals as closely as possible (Mistree *et al.* 1990). The mathematical formulation of the compromise DSP is given in Figure 2.

For a given product realization activity, a compromise DSP is capable of representing a team's decision-making knowledge, as well as the design rationale underlying its decision. A team's decision is represented with a feasible design space, a set of design objectives, and a tradeoff strategy between these design objectives. As shown in Figure 2, the feasible design space is located by the bounds of system variables, $[lb_j, ub_j]$, and constraints, g_k. The team's design objectives (and requirements) are mathematically modeled as design goals, G_i. The tradeoff strategy between these goals is represented in the form of a deviation function, $Z = \sum W_i(d_i^- + d_i^+)$. The deviation variables, d_i^- and d_i^+, measure the difference between the target value and the actual achievement, A_i, of each goal. The solution of a compromise DSP is a decision that satisfies the constraints while achieving the conflicting goals as closely as possible, as measured by the deviation function. Hence, a team's capabilities, constraints and design rationale are modeled using a compromise DSP, which can also be reused by assigning different parameter values, reformulating mathematical functions, and linking to different computing codes to help the team explore new concepts.

<Figure 2 goes about here>

A compromise DSP represented in its mathematical formulation or XML (Extensible Markup Langrage) format is an object-oriented data model which is a super class above the CAD model, data file, etc. (Whitfield *et al.* 2002). It is different from product data models such as STEP (Standard for the Exchange of Product Model Data) or XML which are capable of providing a complete and semantically rich definition of the physical and functional characteristics of a product (Keith and Hunten 1997, ISO10303-209 2004) but fail to capture the design rationale behind the product model. By using the compromise DSP to model the multidisciplinary teams' decisions, the concept of information model/media is extended to include all types of design knowledge such as design requirements, design rationale and an understanding of system capabilities and constraints. The compromise DSP is used in an over-the-wall manner to simplify information exchange into the transfer of a package of decision-making information.

The compromise DSP resolves the first challenge of exchanging information, but it does not address the second challenge of enabling the separation of activities. There are three possible relationships between any two compromise DSPs; they may be solved concurrently, sequentially, or as coupled problems. Given the disk brakes in a passenger vehicle as an example, there is no direct information exchange between the brake design and exhaust system design. From a decision-making perspective, the two compromise DSPs (brake and exhaust system) do not share any unknown variables. They can be solved concurrently, and the solution remains the same regardless of the teams' cooperation styles. Meanwhile, the brake pad cannot be designed without knowledge of the rotor design team's results, whereas the rotor has to be designed with knowledge of the geometric shape, surface finish, and other details of the brake pad. This situation is reflected as shared variables between the rotor and brake pad design compromise DSPs. Neither compromise

DSP can be solved independently, and the result is always affected by the two design teams' cooperation styles, namely, which team solves its compromise DSP first. Game theoretic protocols are used to address the second challenge of separating the coupled activities. In addition, a manufacturing team must design the fixtures, determining the processing parameters based on the final rotor and brake pad designs. The manufacturing compromise DSP includes variables that are determined only by solving the design compromise DSPs. This is a sequential process and the teams' cooperation styles do not affect the solution. However, the downstream manufacturing team may need to modify the design, causing potential iterations. Design capability indices are used to address this third challenge. Since design and manufacturing activities are separated and the responsibility for DfM is transferred from the upstream design team to the downstream manufacturing team, the third challenge becomes more significant in this study.

2.2 Step 2, representing cooperation styles using game theory

The second challenge is resolving couplings between activities. Traditionally, a trial and error approach is used to solve coupled compromise DSPs. Since a team has to make assumptions about another team's decisions to initiate the trial and error process, this traditional approach may not guarantee consensus (convergence) and usually fails to achieve superior results. Game theory facilitates interaction among multiple engineers without integrating a product realization process into a single large optimization problem or causing iterations. There are three game protocols representing different types of interactions between teams (or players in game theory terminology): cooperative, noncooperative, and leader/follower. Rao and colleagues (Rao 1987, Rao *et al.* 1988, Dhingra and Rao 1995) and Badhrinath and colleagues (Badhrinath and Rao 1996) demonstrated cooperative and leader/follower protocols for product realization. Chen and Li (Chen and Li 2002) investigate the interaction between product design and manufacturing using all three protocols. Lewis and Mistree (Lewis and Mistree 1997, Lewis and Mistree 1998) present mathematical constructs for collaborative decision-making using these protocols. Detailed explanation of the game protocols can be found in (Lewis and Mistree 1997, Lewis and Mistree 1998).

If design and manufacturing activities are coupled, cooperative protocols yield Pareto efficient solutions but require solution of a single combined optimization problem. The noncooperative protocol enables separation of design and manufacturing but the solution process is tedious and difficult; in some cases, the solution may not converge. Furthermore, non-cooperative or leader/follower solutions are not necessarily Pareto efficient although Hernandez and coauthors have suggested a technique for obtaining Pareto efficient solutions for these protocols (Hernandez *et al.* 2002a, Hernandez *et al.* 2002b). The authors recommend the use of leader/follower protocol to facilitate the separation of design and manufacturing activities because it accommodates scenarios in which one engineering team dominates the decision-making process. The manufacturing team is designated as the leader in a leader/follower game because significant manufacturing knowledge is required to solve DfM problems and ensure manufacturability, and it is usually much cheaper to adjust a product design instead of manufacturing equipments and processes.

In a leader/follower game, the leader makes a set of rational decisions by predicting the followers' reactions and representing them with a Best Reply Correspondence (BRC), also called a Rational Reaction Set in (Lewis and Mistree 1997, Lewis and Mistree 1998). For instance, X_A and X_B are the design variable sets in team A and B's compromise DSPs, respectively. x_A is a subset of X_A which must be determined using information from team B, and x_B must be determined using information from A. Hence, x_A and x_B are the coupled variables. Assuming team A is the leader, if B's BRC cannot be derived analytically, design of experiment (DOE) techniques can be used to approximate team B's BRC. That is, team B's BRC is obtained by assuming a set of x_A values and calculating B's reaction which is reflected as a set of corresponding x_B values. Then BRC_B is represented as $x_B = f(x_A)$, which is a rational prediction of player B's behavior. The mathematics of the game is given in (Lewis and Mistree 1997, Lewis and Mistree 1998):

$$\begin{aligned} &\text{minimize} \quad Z_A(x_A, x_B) \\ &\text{satisfying} \quad x_B \in x_B^N(x_A) \end{aligned} \tag{1}$$

The feasibility of the solution is ensured by using a BRC to predict the follower's behavior. Generally, a leader-follower game protocol facilitates collaborative decision making without requiring iteration, hence the coupled activities can be accomplished separately. This solves the second challenge.

2.3 Step 3, maintaining design freedom using design capability indices

The third challenge is to eliminate, or at least reduce, costly iterations between upstream and downstream activities by having the upstream team identify ranges of design variables, rather than single point values, that are as broad as possible without deviating from a desired range of performance, as shown in Figure 1. Chen and coauthors present design capability indices to evaluate performance variations caused by ranged design variables, and to determine whether the performance ranges satisfy design requirements (Chen *et al.* 1999). In Figure 3, *URL* and *LRL* are respectively the upper requirement limit and lower requirement limit that bound the target range for a specific performance criterion. A performance variable is approximated by a distribution with mean μ_y and deviation Δy, to distinguish the mean and deviation from those obtained using statistical techniques (instead of μ and σ). If X and $f(X)$ represent a set of design variables and a performance model, respectively, the deviations of performance, Δy, can be approximated with a first order Taylor series expansion, as shown in Equation (2). Other ways of obtaining the performance range include Latin hypercube sampling, design of experiments (DOE), and Monte Carlo analysis.

$$\mu_y = f(\mu_X), \quad \Delta y = \sum_{j=1}^{m} \left| \frac{\partial f}{\partial x_j} \Delta x_j \right| \quad X = [x_1, x_2 \ldots x_m] \tag{2}$$

<Figure 3 goes about here>

As presented in Figure 3 (Chen *et al.* 1999), for the case in which the design goal is as large as possible, all possible design solutions meet design requirements when $\mu_y - LRL \geq \Delta y$, and $C_{dk} = C_{dl}$ in Equation (3). For the case in which the design goal is required to be as small as possible, all possible design results fall into the target range of design requirements when $URL - \mu_y \geq \Delta y$ and $C_{dk} = C_{du}$ in Equation (3). In the case in which nominal is preferred, the design requirements are acceptable *URL* and *LRL*, and the target value is the midpoint between these two limits. C_{dk} is equal to the value of the smaller of C_{dl} and C_{du}. In all these cases, $C_{dk} \geq 1$ indicates that the ranged performance satisfies design requirements.

$$C_{dl} = \frac{\mu_y - LRL}{\Delta y}; C_{du} = \frac{URL - \mu_y}{\Delta y}; C_{dk} = \min\{C_{dl}, C_{du}\} \tag{3}$$

In the design variable set, X, if any design variable is discrete, say x_j, the location and deviation of the performance measures have to be conservatively estimated using:

$$\left.\begin{aligned}\mu_y &= \frac{\max f(X) + \min f(X)}{2} \\ \Delta y &= \frac{\max f(X) - \min f(X)}{2}\end{aligned}\right. \quad \text{when } x_j \text{ is discrete} \tag{4}$$

In many cases, calculating the min/max values of a performance measure requires exhaustive search, but the performance range estimated in this manner will cover all the possible values even though they are not continuous. If a performance variable is discrete, the design capability indices are not applicable.

Given that all the performance variables are continuous, design capability indices are embedded into the compromise DSP by formulating the design goals using C_{dk}, adding constraints $C_{dk} \geq 1$, and formulating the deviation function to *maximize* the overachievements of C_{dk}. Moreover, the constraints are re-formulated using Equation (2) or (4). That is, for continuous design variable set, $g_k(X) \leq 0$ in Figure 2 is changed to:

$$g_k(\mu_X) + \sum_{j=1}^{m} \left| \frac{\partial g_k}{\partial x_j} \Delta x_j \right| \leq 0 \quad k = 1, 2 \ldots p \tag{5}$$

Clearly, constraint $g_k(X)$ must be differentiable. If any design variable is discrete, the constraints can be calculated using Equation (4). The bounds of design variables are still formulated using constant values. The resulting compromise DSP is shown in Figure 4.

<Figure 4 goes about here>

In Figure 4, $C_{dk} \geq 1$ ensures that each performance range falls into the corresponding target range. Maximizing overachievement ensures that the performance range achieves the specific goal as well as possible. Therefore, as shown in Figure 3, when the design goal is as large as possible, the performance range falls at the right (large) side of LRL and is as far away from it as possible. In the case the design goal is as small as possible, the performance range falls at the left side of URL, and is as far away from URL as possible. When a nominal target is preferred, the performance range is as close to the target value (nominal value) as possible. In this re-formulated compromise DSP, the unknown variables include the mean value and deviation of each design variable, μ_{x_j} and Δx_j. For simplicity, the Δx_j values are assigned by the decision making team. Hence, μ_{x_j} are the only system variables to be solved in this compromise DSP. Please note μ_{x_j} is the nominal value of the resulting range of the design variable, which is not necessarily the mean value of that range because each design variable is subject to its bounds, [lb_j, ub_j]. The resulting design variable ranges are calculated as follows:

$$x_j \in [\mu_{x_j} - \Delta x_j, \mu_{x_j} + \Delta x_j] \cap [lb_j, ub_j] \tag{6}$$

Design capability indices help engineering teams determine the locations or nominal values of ranged design variables to ensure that performance ranges fall into target ranges. Its most significant benefit is helping to avoid costly iterations between upstream and downstream teams by providing downstream teams with a range of options for the values of coupled variables. However, in some cases, a ranged set of design variables cannot yet satisfy the requirements in downstream activities. Therefore, the authors claim that the CMDM will reduce this category of iterations but cannot guarantee their elimination.

In summary, the CMDM consists of three constituent elements and the associated steps to instantiate them, each of which eliminates or reduces one type of iteration in a product DfM process. In Step 1, the challenge is to provide a method for exchanging information, and it is solved by representing the decision making information in compromise DSPs. In Step 2, the challenge is accommodating interactions between activities, and it is addressed with game theory. In Step 3, the challenge is to maintain feasible and satisfactory overall designs, and it is addressed by reformulating the compromise DSPs using design capability indices. The CMDM provides a normative framework that facilitates collaborative product realization by separating the decision making activities.

3. A robot arm design and manufacturing scenario

The authors have developed a distributed product realization environment called the Rapid Tooling Testbed (RTTB) (Gerhard *et al.* 2001). The basic product realization process consists of several activities: product geometric modeling, generating the CAD model of an injection mold of the product, fabricating the mold halves on a rapid prototyping machine, and building the product with injection molding equipment. In this case, a customer needs a batch of robot arms for demonstration and testing purposes, and requires delivery within 40 hours, with cost less than \$2000. The basic size of the robot arm is given by the customer, Figure 5(a), *i.e.*, L = 63.5 mm, D_0 = 10.32 mm, and d_0 = 5.16 mm. The robot arm must function as required with a maximum deformation of approximately 0.5mm under working loads, a maximum von Mises stress of approximately 6MPa, and a weight of approximately 3.5 g. The maximum deformation and stress are simulated using FEA software, as explained in (Xiao 2003).

<Figure 5 goes about here>

A design team, a rapid tooling team, and an injection molding team are assigned to this task. Correspondingly, the product realization process is partitioned into three activities, as shown in Figure 5(b). In product design, the design team selects general-purpose polystyrene as the material and designs the geometric shape. Since no manufacturing information is available at this stage, product design is an independent activity, with inputs of customer requirements and output of robot arm CAD model. In rapid tooling, the injection mold (mold halves) for the robot arm are designed; SLA3500 rapid prototyping machine and SL7510 resin are selected to build the mold; and a set of rapid prototyping parameters are determined. In addition to the robot arm CAD model, the rapid tooling team needs to know the life (number of molding cycles before failure) of the plastic mold from the injection molding team to decide how many pairs of molds to build, which affects the rapid prototyping parameters. In injection molding, the team determines appropriate molding parameters and fabricates the batch of robot arms on a Morgan-Press (G-100T) injection molding machine. The mechanical properties and detailed geometric shape of the mold must be known to determine the mold life.

In this scenario, DfM includes not only adjusting the geometric shape, but also the entire rapid tooling activity. Designing the mold pairs requires knowledge about rapid tooling; hence, it is very difficult for the design team. Since the robot arm is designed without knowledge of the downstream manufacturing process, the design team will have to modify its design based on feedback from manufacturing experts. For a simple product realization process like this one, it is possible to collect all the manufacturing related information and formulate a large design problem to solve all the design variables, i.e., robot arm geometry shape, mold geometry shape, and some manufacturing parameters, like that in Concurrent Engineering. For complex real-world problems, this implies a design problem containing large numbers of design variables and complex analyses; therefore it is not practical to solve as a single problem. The traditional approach to solving a DfM problem is a trial and error approach, which will cause extensive information exchange and iteration.

In this example, all three of the challenges addressed by the CMDM exist. Information exchange is required between these activities (challenge 1). Rapid tooling and injection molding are coupled activities; thus iterations exist between them (challenge 2). The geometric shape of the robot arm may have to be modified in order to fabricate the batch with given time and cost; this forces the upstream design team to redesign the geometry (challenge 3).

3.1 Engineering teams' compromise DSPs

The first step of the CMDM is modeling each activity as a standard compromise DSP as shown in Figure 2. The design team controls three design variables, D, d and t as shown in Figure 5(a). Three design goals are determined based on the customer requirements: (i) the maximum deformation under working load should be as close to 0.5mm as possible, (ii) the maximum von Mises stress under working load should be as close to 6MPa as possible, and (iii) the weight of the robot arm should be as close to 3.5g as possible. The design compromise DSP is shown in Figure 6. Please note at this step, no design capability indices or game protocol is yet involved. Mathematical equations for deformation, stress, and weight can be found in (Xiao 2003).

<Figure 6 goes about here>

The rapid tooling team designs the injection mold halves, Figure 7, and determines the process parameters to build them [††]. The basic dimensions of the mold halves, *e.g.*, length, width and height, are determined based on the fixture of the injection-molding machine with the shape of the mold cavity determined by that of the robot arm. In order to eject the molded part after the cooling period, a draft angle Θ is added onto the two boss features in the mold cavity, corresponding to the two holes on the robot arm. The draft angle affects the mold life in the injection molding activity. On the other hand, the mold life determines the number of mold halves, N_m, that must be built in the rapid tooling activity, which thus affects all of the process parameters in this activity. Therefore, injection molding and rapid tooling are coupled activities. As shown in Figure 8, the rapid tooling team controls four system variables: layer thickness, *LT*; hatch overcure, *HOC*; fill overcure, *FOC*, and draft angle, Θ. The five goals are: (i) the shortest time, PT_m, (ii) the least cost, CS_m, (iii) surface finish on the boss surfaces, *SF*, as smooth as possible, (iv) the mold's Young's modulus, *YM*, as large as possible, and (v) the mold's tensile strength, *TS*, as large as possible. Detailed derivations of these equations are presented in (Rosen 2000, Sambu 2001). Obviously the rapid tooling team cannot decide the four system variables without knowing the final geometric shape of the robot arm, D, d and t, or the injection molding information, mold life, *ML*.

[††] English units have to be used for the parameters related to SLA3500 and Morgan-Press® (G-100T).

<Figure 7 goes about here>

<Figure 8 goes about here>

The injection molding team fabricates the batch of robot arms, as illustrated in Figure 9, after the design team determines *D*, *d*, *t*, and the rapid tooling team determines *LT*, *HOC*, *FOC*, Θ. In case a team uses software tools, instead of mathematical equations, to calculate the performance variables, compromise DSP can still be formulated. The injection molding team uses a simulation code to decide the cooling time (*CT*), mold life (*ML*), molding time (PT_i) and cost (CS_i), and its compromise DSP is shown in Figure 10. Please refer to (Pham 2001, Sambu 2001) for detailed explanations of injection molding, part ejection and mold life. Here, the injection molding team is responsible for specifying input, running the simulation code to get output parameters.

<Figure 9 goes about here>

<Figure 10 goes about here>

In a traditional product realization process, the data flows of the design variables are shown in Figure 11(a). Clearly, the product design activity is not coupled with the manufacturing activities, while rapid tooling and injection molding activities are coupled with each other.

If we combine all compromise DSPs into one problem with a variable set that includes all of the variables of design, rapid tooling, and injection molding and an objective set that includes all of the objectives with the same weights, the results are shown in Table 1. Since formulating a large design problem is not practical; the results are used only for comparison. The trial and error approach is also used to solve these compromise DSPs, and the results are reported in Table 2. The design activity is solved independently, the result from the design DSP, Figure 6, is *D* = 20.52mm, *d* = 9.25mm, *t* = 3.10mm. In this case, all design goals achieve their target values and the overall deviation is 0. Then, the rapid tooling and injection molding teams make decisions based on this result. Since these two activities are coupled, the rapid tooling team assumes a value of *ML*, and expects to acquire converged results after several iterations. Unfortunately in this case, rapid tooling and injection molding teams' solutions do not converge. The reason is the design team makes decisions only considering its own design goals; hence the thickness of robot arm *t* is too large and the mold life becomes so short that the rapid tooling team must build 10 pairs of molds. This violates the constraints of time and cost. Therefore, the product design must be modified. For simplicity, the intermediate results are not listed here. After several rounds of iterations, the converged results from the traditional approach are as shown in Table 2.

<Figure 11 goes about here>

In the trial and error process, iterations happen not only between the coupled rapid tooling and injection molding activities, but also with the upstream design activity. Furthermore, the number and styles of iterations are affected by some unpredictable or uncontrollable factors, such as the teams' experience, and the initial values the teams choose to start the iterative process. So far, this case has demonstrated the difficulties of DfM, and why the traditional approach cannot guarantee the superiority of the final result. By using the CMDM to separate the activities, we expect to change the process shown in Figure 11(a) to a much simpler one shown as (b), in which the activities are relatively independent of each other, without unnecessary information exchange or costly iteration. Therefore, the manufacturing teams can adjust the product design to accommodating its manufacturability, and accomplish product DfM in a streamlined manner.

<Table 1 goes about here>

<Table 2 goes about here>

3.2 Resolving couplings using Leader/Follower protocol

Since the design team's decision is not coupled with the decisions of either of the manufacturing teams, as shown in Figure 11(a), only the couplings between rapid tooling and injection molding are resolved using game protocols, Step 2 of the CMDM. The rapid tooling team is selected as leader and the injection molding team as follower, considering that mold life *ML* is influenced primarily by the design and mechanical strength of the mold pair which are determined by the rapid tooling team. In other words, the rapid tooling team is the dominant player. The detailed process of deriving an engineering team's BRC can be found in (Xiao *et al.* 2005). Briefly, the injection molding team assumes a set of values of the coupled variables and conducts a set of simulation runs. BRC_M is then obtained by fitting the results as a function of the coupled variables. In this paper, a full factorial experiment is used to select the values of the coupled variables, and quadratic response surface equations are fit to the results. Since layer thickness, *LT*, is discrete, the BRC_M is:

when LT = 2 mils (7)

$$ML=1696.10-135.46d-311.63t+423.03\Theta+61.17(d-8.11)^2+192.98(t-2.61)^2+233.25(\Theta-1)^2+113.78(t-2.61)(d-8.11)-163.42(\Theta-1)(d-8.11)-373.64(\Theta-1)(t-2.61)$$

when LT = 4 mils

$$ML=754.23-58.17d-133.96t+170.8\Theta+26.17(d-8.11)^2+82.46\,(t-2.61)^2+81.01(\Theta-1)^2+48.47(t-2.61)(d-8.11)-66.63(\Theta-1)(d-8.11)-152.25(\Theta-1)(t-2.61)$$

when LT = 8 mils

$$ML = 330.97-24.55d-56.63t+66.34\Theta+11.01(d-8.11)^2+34.71(t-2.61)^2+25.82(\Theta-1)^2+20.28(t-2.61)(d-8.11)-26.30(\Theta-1)(d-8.11)-60.06(\Theta-1)(t-2.61)$$

This equation can be used in the rapid tooling team's compromise DSP to solve it without causing iteration. Some multidisciplinary optimization methods also use response surface models to approximate an engineer's model (Burgee *et al.* 1996), which is fundamentally different from using them to approximate the BRCs. Beyond predicting a player's behavior using its BRC, game protocols also govern issues such as the sequence of the players' decision making activities and control over specific variables. All of these factors are determined by the players' cooperation styles.

If the injection molding team is selected as the leader, the BRC_T is:

$$\begin{aligned}&\text{when } 200 \geq ML \geq 150\\&\quad LT = 4.0 \text{ mils},\ HOC = 3.24 \text{ mils},\ \Theta = 0,\ FOC = 4.0 \text{ mils}\\&\text{when } 150 > ML \geq 75\\&\quad LT = 8.0 \text{ mils},\ HOC = 1.00 \text{ mils},\ \Theta = 0,\ FOC = 2.0 \text{ mils}\end{aligned} \tag{8}$$

From the equation, when the plastic mold is capable of withstanding more than 150 injection cycles, the rapid tooling team has to build only one pair of molds. This allows the mold to be built with smaller layer thickness, LT = 4 mils, which results in higher Young's modulus and tensile strength. When $150 > ML \geq 75$, two pairs of molds have to be build, thus LT = 8 mils has to be selected in order to meet the time constraint. Here, we do not consider the situation that several pairs of molds can be built simultaneously in an SLA3500 machine. It can also be observed from BRC_T that Θ remains 0 because the rapid tooling team does not know how the draft angle will affect mold life, and strives to reduce surface roughness, *SF*, of the robot arm which is achieved with $\Theta = 0$. Obviously, the injection molding team will be unable to eject the parts with a zero draft angle. This is the main reason why the traditional trial and error approach does not converge between the rapid tooling and injection molding teams.

3.3 Compromise DSP for a ranged set of decisions

As described in Section 1, the third step is reformulating the compromise DSPs using design capability indices. In the design activity, all the design variables are continuous; therefore the locations and deviations of the performance variables are calculated using Equation (2). The design team's compromise DSP for a ranged set of decisions is shown in Figure 12, only the aspects that differ from Figure 6 are shown here. New constraints $C_{dk} \geq 1$ ensure that every specific point within the ranged set of decisions satisfies the requirements of the design team. The target ranges and deviations of design variables are assigned by the design team. In future work, a set of system variables can be used to calculate the maximum ranges of the design variables. An acceptable target range for each specific design goal is provided by the customers. Some design variables in the rapid tooling and injection molding activities are discrete, thus their locations and deviations can only be conservatively estimated using Equation (4). The rapid tooling teams' compromise DSP for a ranged set of decisions is shown in Figure 13.

<Figure 12 goes about here>

<Figure 13 goes about here>

<Table 3 goes about here>

<Table 4 goes about here>

Given the same weight to each goal, the result of the design team's compromise DSP for a ranged set of design specifications is shown in Table 3. In this table, a nominal value is the *location* of a ranged design variable calculated from the compromise DSP, which is not necessarily the mean value of its range. The actual ranges are calculated using Equation (6). Deviation of a performance variable is calculated using Equation (2). In the table, every C_{dk} is larger than 1, meaning that any specific point in the performance range falls into the ranges of design requirements shown in Figure 12. Then product realization responsibility is transferred to the following manufacturing teams, which select within these ranges to satisfy their own design goals as well as possible. The leader, rapid tooling team solves its compromise DSP, Figure 13, using BRC_M, (7). The resulting mold CAD model is sent to the injection molding team, who finalize the results, as shown in Table 4.

<Table 5 goes about here>

<Figure 14 goes about here>

In Figure 14, the results of the CMDM are compared with (a) the results of the trial and error approach (cf. Table 2) and (b) the results from combining all of the teams' decisions into a single, integrated, compromise DSP (cf. Table 1). The results in Figure 14 are presented as deviations from target values, calculated with the deviation functions (Z) in Figures 6 and 8 for the design and rapid tooling teams, respectively. The results of the CMDM are presented in three forms: nominal results, best case results, and worst case results. The nominal results, labeled CMDM in Figure 14, are calculated with the nominal values for design variables in Table 4. The best and worst case results, as presented in Table 5, are identified by searching for the combinations of design variable values, within the ranges identified in Table 4, that yield the smallest (best) and largest (worst) possible deviations from target values for each design team. A detailed comparison of the results in Figure 14 reveals that the trial and error approach provides the worst overall deviations, the combined approach provides the best overall deviations, and the CMDM's results fall between the two extremes. The combined approach performs well because the design process is centralized with an integrated decision and a single decision-maker, although solving such a large design problem is not practical in most DfM settings. The trial and error approach performs poorly because the manufacturing team must modify the design team's results without knowledge of the effects of those changes on the product's overall performance. This situation is very common in product DfM. When implementing the CMDM, however, the design team identifies a ranged set of design parameters, and keep the ranges as large as possible if they are shared with other teams or if they impact the objectives of other teams. Any specific point in these ranges offer satisfactory design performance. Accordingly, the following manufacturing team enjoys the freedom to explore the ranged design space to identify parameter values that are also satisfactory from a manufacturing perspective. As shown in Figure 14, the nominal CMDM results are much better than the trial and error method. The best case CMDM results exhibit deviations that are almost as small as those of the combined approach. Even the worst case CMDM results are much better than the trial and error approach from the manufacturing perspective. These results demonstrate the effectiveness of the CMDM at resolving DfM problems.

The CMDM is especially useful in the early stages of product design, when little is known about the product and approximating a set of correct designs is much more efficient than conducting rounds and rounds of guessing and correcting. The advantage of the CMDM in DfM is that design and manufacturing activities are separated systematically; hence the product realization activities are accomplished in a more streamlined process as shown in Figure 11(b). Due to the separation, the design team can focus on the product design activity and have the knowledgeable manufacturing teams resolving the manufacturing problems. This not only ensures the overall superiority of the final result, but also eliminates information exchange and iterations which make the product realization process costly and time-consuming.

3.4 Design freedom in the process

The reason the CMDM results in more superior results than the traditional trial and error approach is the design freedom in the product realization process. A design freedom metric is presented in (Simpson *et al.* 1998) by measuring the overlap between target ranges of system performance measures and their achievable performance ranges:

$$DF = \frac{1}{n}\sum_{i=1}^{n}\frac{Overlap_i}{PR_{i,initial}} = \frac{1}{n}\sum_{i=1}^{n}\frac{TR_i \cap PR_i}{PR_{i,initial}} \tag{9}$$

where n is the number of performance measures of the system. For the i^{th} performance measure, TR_i is the target range, PR_i is the feasible performance range, and $PR_{i,initial}$ is the initial feasible performance range. In Table 6, the target ranges given by the customer are listed in the first row. At the second row, the performance ranges and design freedom at the initial state of this product realization process are listed. The initial design freedom is 0.734, which is smaller than 1 due to the natural limitation of this process and the couplings between activities. In the trial and error process, when the design team makes a specific decision, design freedom is quantified as 0.368, shown at the third row of Table 6. Then after the rapid tooling team makes a decision, design freedom is reduced to 0 even before the final decision is made. This is why the quality of the final result cannot be guaranteed. Figure 15 depicts the design freedom change using the decision making activities to mark the timeline. In the lower section of Table 6, the product realization process is accomplished using the CMDM: the design team makes a ranged set of decisions at Step 1; the injection molding team (player) constructs BRC_M at step 2; and the rapid tooling team (player) makes a ranged set of decisions at Step 3. The injection molding team finally makes its decision at Step 4. It is observed that when the design team makes a ranged set of decisions, design freedom is 0.504 at Step 2. At this moment of the product realization process, the rapid tooling and injection molding teams can pick any point within $D \in [21.9, 25.4]$ mm, $d \in [7.6, 8.62]$ mm and $t \in [2.5, 2.72]$ mm. Even after the rapid tooling team further shrinks the ranges to $D \in [21.9, 23.53]$ mm, $d \in [7.61, 8.46]$ mm and $t \in [2.5, 2.64]$ mm, design freedom still remains at 0.170. Meanwhile, the injection molding team can explores the entire range of the design variables to increase the mold life.

<Table 6 goes about here>

<Figure 15 goes about here>

4. Closure

In this paper, the idea of collaboration by separation is tested in product DfM problems. The CMDM is used to enable the separation without causing costly information exchange and iterations. Generally, the compromise DSP is used as an information medium to separate the activities at the information communication level, game theoretical principles separate the coupled activities, and design capability indices separate upstream and downstream activities. The robot arm design and manufacturing process demonstrates that by using the CMDM, a complex product realization process is implemented in a streamlined manner, with each engineering team focusing on its areas of expertise. With the CMDM, final results are obtained with fewer iterations between design teams and significantly less deviation from target performance, relative to using the traditional trial and error approach.

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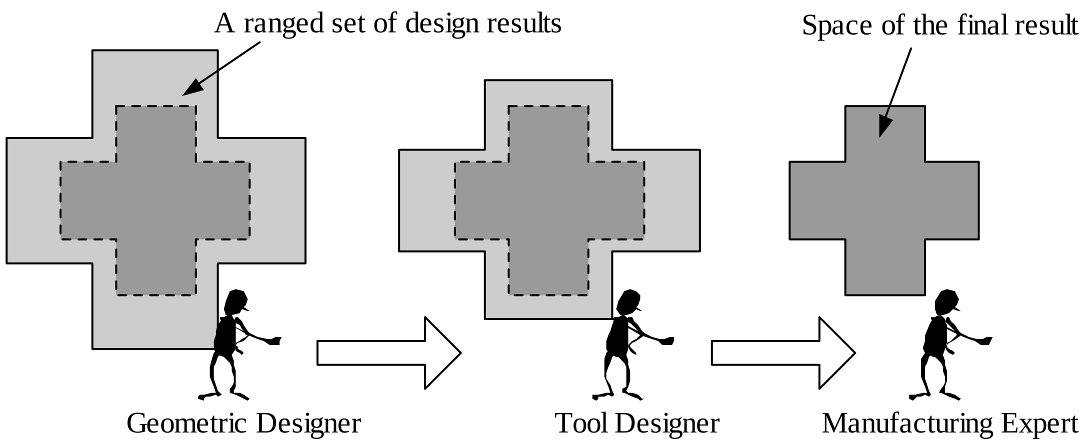


Figure 1. Coordination by Separation in Design for Manufacturing

Given

An alternative to be improved

Target value for goals, G_i, $i = 1,\ldots,n$

Relative importances of goals, w_i

Find

System Variables: $x_j \in X$, $j = 1,\ldots,m$

Deviation Variables: d_i^-, d_i^+

Satisfy

Goals: $A_i(X) + d_i^- - d_i^+ = G_i,$

Constraints: $g_k(X) \leq 0$, $k = 1,\ldots,p$

$d_i^- \cdot d_i^+ = 0$, $d_i^-, d_i^+ \geq 0$

Bounds: $lb_j \leq x_j \leq ub_j, j = 1,\ldots,m$

Minimize

Deviation Function:

$$Z = \sum_{i=1}^{n} W_i (d_i^- + d_i^+) \text{, where } \sum_{i=1}^{n} W_i = 1,\ W_i \geq 0$$

NOTE: Alternative formulations for the deviation function are available in (Mistree *et al.* 1993).

Figure 2. Mathematical Formulation of the Compromise DSP (Mistree *et al.* 1993)

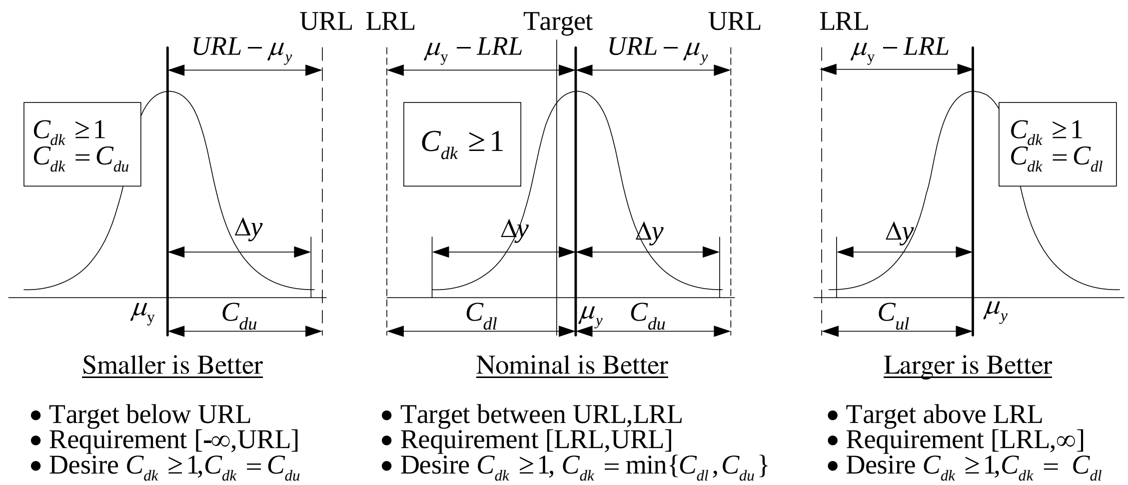


Figure 3. Design Capability Indices (Chen *et al.* 1999)

Given

An alternative to be improved

Deviation of the design variables, $\Delta x_j \in \Delta X$, $j = 1,\ldots,m$

Target ranges of performance measures, URL_i and LRL_i

Relative importances of goals, w_i, $i = 1,\ldots,n$

Find

Location of system variables: μ_{x_j} ,

Deviation Variables: d_i^-, d_i^+

Satisfy

Goals: $C_{dk\text{-}i}(\Delta X, \mu_X) + d_i^- - d_i^+ = 1$

Constraints: $C_{dk\text{-}i}(\Delta X, \mu_X) \geq 1$

$g_k(\Delta X, \mu_X) \leq 0, \quad k = 1,\ldots,p$

$d_i^- \cdot d_i^+ = 0, \; d_i^-, d_i^+ \geq 0$

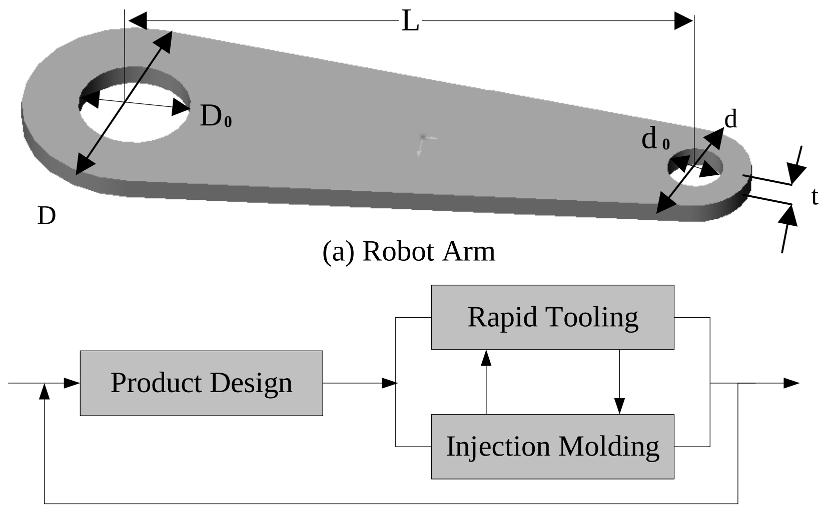
Bounds: $lb_j \leq x_j \leq ub_j$

Minimize

Deviation Function:

$Z = \Sigma w_i \cdot (- d_i^+)$, where $\sum_{i=1}^{n} w_i = 1, \; w_i \geq 0$

Figure 4. The Compromise DSP for Ranged Set of Decisions (Chen *et al.* 1999)



(a) Robot Arm

(b) The Product Realization Process

Figure 5. Robot Arm Design and Manufacturing Process

Given

- Customer requirements:
  - Target values of design goals, 0.5mm, 6MPa, and 3.5g
  - Deliver a batch of 50 parts (N_P) in 40 hours (*time*), with less than $2,000 (*cost*)
- Robot arm is injection molded in polystyrene
  - Young's modulus:3.2 GPa, Tensile strength: 37.4 MPa
- Basic CAD model of the robot arm, as shown in Figure 5 (a)
- Equation of relative system variables: *deform(D, d, t), stress(D, d, t), weight(D, d, t)* (Xiao 2003)

Find

- Geometric variables (design variables): *D*, *d* and *t*
- Deviation variables d_i^-, d_i^+; i = 1,2,3

Satisfy

- Goals:
  - Maximum deformation under the working loads is around 0.5 mm (We prefer to match this deformation as closely as possible – either underachievement or overachievement of this goal is undesirable):

    $$\frac{deform}{0.5} + d_1^- - d_1^+ = 1$$
  - Maximum von Mises stress under working loads is around 6 Mpa:

    $$\frac{stress}{6.0} + d_2^- - d_2^+ = 1$$
  - Weight is around 3.5g: $\dfrac{weight}{3.5} + d_3^- - d_3^+ = 1$
- Constraint:
  - Maximum stress smaller than tensile strength: $stress < 37.4MPa$
  - Surface finish of the inner surfaces smaller than 0.5 mils: $SF \le 0.5$ mils
  - Deliver product within 40 hours: $time \le 40$
  - Product development cost less than $2000: $cost \le 2000$
- Bounds on design variables
  - $15.2mm \le D \le 25.4mm$
  - $7.6mm \le d \le 12.7mm$
  - $2.5mm \le t \le 3.6mm$
- Deviation variables: d_i^-, $d_i^+ \ge 0$; $d_i^- \cdot d_i^+ = 0$ i = 1, 2, 3

Minimize

The deviation function (Archimedean Formulation)

$$Z_D = \sum_{i=1}^{3} W_i (d_i^- + d_i^+) \quad \text{where} \quad \sum_{i=1}^{3} W_i = 1, \quad W_i \ge 0$$

Figure 6. The Design Team's Compromise DSP without Design Capability Indices

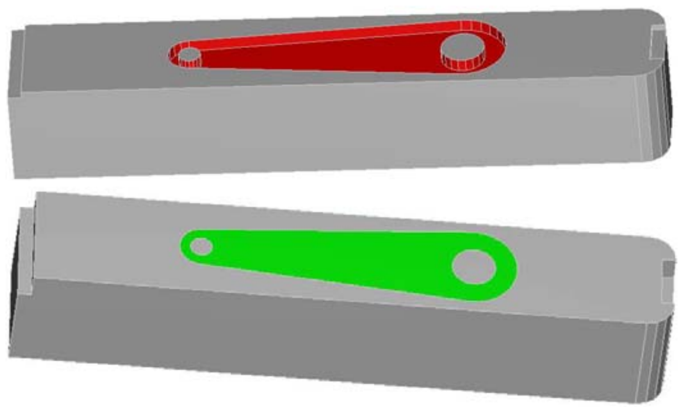


Figure 7. Mold Halves for the Robot Arm (Chen 2001)

Given

- From product design team:
  - CAD model of the robot arm including *D*, *d*, and *t* and part batch: $N_p = 50$
- From rapid tooling team (self):
  - CAD model of the injection mold and cavity dimensions from the robot arm (*D*, *d*, and *t*)
- From injection molding Team
  - Mold life, *ML*
- Equations for the system variables are:
  Time: $PT_m\ (BT)$, Cost: $CS_m\ (BT)$, Surface Finish: $SF\ (LT, \Theta)$
  Young's Modules: $YM\ (LT, HOC)$, Tensile Strength: $TS\ (LT, HOC)$ (Rosen 2000, Sambu 2001).
- Number of mold halves to be built: $N_m = \min\{x \mid x \geq \frac{3N_P}{ML}, x \text{ is integer}\}$ (safety factor 3)

Find

- Design variables:
  - The process planning parameters: *LT*, *HOC*, *FOC* and draft angle of two holes, Θ
- Resulting system variables:
  - Mold properties: Young's modulus *YM*, tensile strength *TS*
- Deviation variables d_i^-, d_i^+;

Satisfy

Goals:

- Rapid tooling time 20 hours (smaller value preferred): $\frac{20}{PT_m} - d_1^- + d_1^+ = 1$
- Rapid tooling cost $1000 (smaller values are preferred): $\frac{1000}{CS_m} - d_2^- + d_2^+ = 1$
- Surface finish 0.3 mils (smaller values are preferred): $\frac{0.3}{SF} - d_3^- + d_3^+ = 1$
- Young's modules 3.5 GPa (larger values are preferred): $\frac{YM}{3.5} + d_4^- - d_4^+ = 1$
- Tensile strength 60 MPa (larger values are preferred): $\frac{TS}{60} + d_5^- - d_5^+ = 1$

- Constraints
  - $SF_m \leq 0.5$ mils and $CS_m + CS_i \leq \$2000$ and $PT_m + PT_i \leq 40$ hours
- Bounds on design variables:
  - $0 \leq \Theta \leq 2$
  - *LT* is 2, 4, 8 (mils) – Discrete variable
  - $2 \leq HOC_2 \leq 6$ (mils); $3 \leq HOC_4 \leq 7$ (mils); $1 \leq HOC_8 \leq 5$ (mils) (depend on *LT* value)
  - $12 \leq FOC_2 \leq 16$ (mils); $4 \leq FOC_4 \leq 8$ (mils); $2 \leq FOC_8 \leq 6$ (mils)
- Deviation variables: d_i^-, $d_i^+ \geq 0$; $d_i^- \cdot d_i^+ = 0$ $i = 1, 2, \ldots 5$

Minimize

The deviation function (Archimedean Formulation)

$$Z_T = \sum_{i=1}^{5} W_i (d_i^- + d_i^+) \quad \text{where} \quad \sum_{i=1}^{5} W_i = 1, \quad W_i \geq 0$$

Figure 8. Rapid Tooling Team's Compromise DSP without Game Theory or Design Capability Indices

Figure 9. Injection Molding of Robot Arm (Chen 2001)

Input

- From the rapid tooling team:
  - CAD model of the injection mold (D, d, and t)
  - Draft angle, Θ
  - Rapid tooling parameters: *LT*, *HOC*, *FOC*
- From the injection molding team (self):
  - Circle time of each injection molding shot: $t_i = 1.5 \cdot CT$
  - Approximately 3 shots to get 1 quality part, time of injection molding: $T_i = 3 \cdot N_P \cdot t_i$
  - Cost of injection molding: $30/Hour
- Constraints
  - $ML \leq 200$

Output

- CT, ML, PT_i and CS_i

Figure 10. Injection Molding Team's Compromise DSP

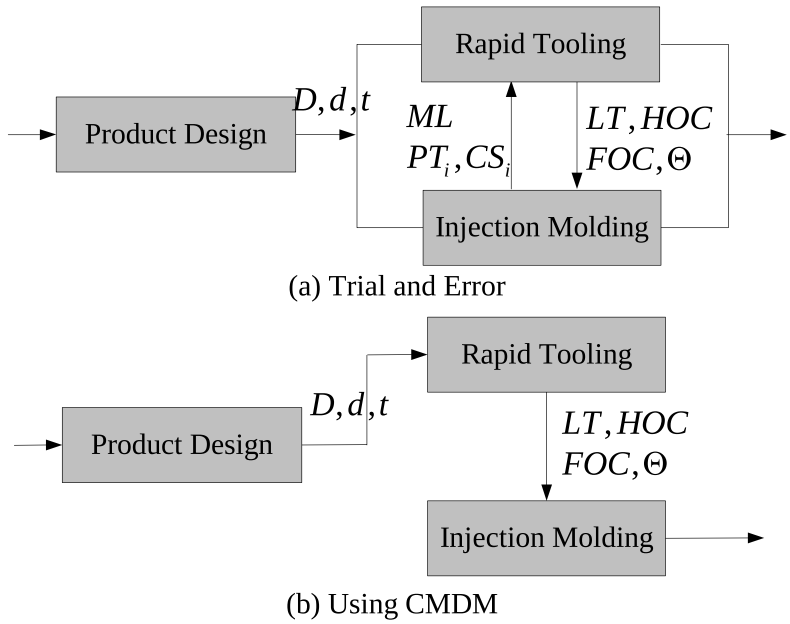


Figure 11. Data Flows in the Product Realization Process

Given

…

- Deviation of design goals obtained from Equation (2): $\Delta deform$, $\Delta stress$, $\Delta weight$
- Deviation of design variables:
  - $\Delta D = (25.4 - 15.2)/5$
  - $\Delta d = (12.7 - 7.6)/5$
  - $\Delta t = (3.6 - 2.5)/5$

Satisfy

- Goals:
  - Achieve deformation within specification range (nominal). The design team prefers ranged deformation values that fall into the range [0.35, 0.65] mm, and match the target deformation, 0.5 mm as closely as possible.

    $C_{dk-deform} + d_1^- - d_1^+ = 1$

    $C_{dk-deform} = \min\{C_{dl-deform}, C_{du-deform}\}$ where

    $C_{dl-deform} = (deform - 0.35)/\Delta deform$

    $C_{du-deform} = (0.65 - deform)/\Delta deform$
  - Achieve von Mises stress within specification range (nominal). The design team prefers ranged stress values that fall into the range [4.2, 7.8] N, and match the target force, 6.0 MPa

    $C_{dk-stress} + d_2^- - d_2^+ = 1$

    $C_{dk-stress} = \min\{C_{dl-stress}, C_{du-stress}\}$ where

    $C_{dl-stress} = (stress - 4.2)/\Delta stress$

    $C_{du-stress} = (7.8 - stress)/\Delta stress$
  - Achieve the weight within specification range (nominal). The design team prefers ranged weight values that fall into the range [2.45, 4.55]g, and match the target weight, 3.5g

    $C_{dk-weight} + d_3^- - d_3^+ = 1$

    $C_{dk-weight} = \min\{C_{dl-weight}, C_{du-weight}\}$ where

    $C_{dl-weight} = (weight - 2.45)/\Delta weight$

    $C_{du-weight} = (4.55 - weight)/\Delta weight$
- Constraint:
  - Every ranged design variable falls into the corresponding scope

    $C_{dk-DF} \ge 1$; $C_{dk-S} \ge 1$; $C_{dk-W} \ge 1$
  - $stress + \Delta stress < 37.4 MPa$
  - $SF + \Delta SF \le 0.5$ mils
  - $time + \Delta time \le 40$ hours
  - $cost + \Delta cost \le 2000$ dollars

…

Minimize

The deviation function (Archimedean Formulation)

$$Z_D = \sum_{i=1}^{3} W_i(d_i^+) \quad \text{where} \quad \sum_{i=1}^{3} W_i = 1, \qquad W_i \ge 0$$

Figure 12. The Design Team's Compromise DSP for Ranged Set of Decisions

Given

…

- Deviation of design goals*:* ΔPT_m , ΔCS_m, ΔSF_m, ΔE_m *and* ΔY_m
- BRC_M in Equation (7)
- *Deviation of design variables:*
  - $\Delta HOC = (HOC_{up-LT} - HOC_{low-LT})/4$
  - $\Delta FOC = (FOC_{up-LT} - FOC_{low-LT})/4$ (up and low HOC, FOC values depend on LT)
  - $\Delta\Theta = (2-0)/4$
  - $\Delta D = (D_{up} - D_{low})/3$; $\Delta d = (d_{up} - d_{low})/3$ *;* $\Delta t = (t_{up} - t_{low})/3$ (up and down *D*, *d*, *t* values are obtained from the design team's ranged design variables)

Satisfy

- Goals:
  - Achieve the time range. The rapid tooling team prefers that ranged time value is smaller than 40, and as small as possible. 40 hours is the URL, different from the target value in Figure 8.

    $C_{dk-PT_m} + d_1^- - d_1^+ = 1$

    $C_{dk-PT_m} = C_{du-PT_m} = (40 - PT_m)/\Delta PT_m$
  - Achieve the cost range (small). The rapid tooling team prefers that ranged cost value is smaller than 2000, and as small as possible.

    $C_{dk-CS_m} + d_2^- - d_2^+ = 1$

    $C_{dk-CS_m} = C_{du-CS_m} = (2000 - CS_m)/\Delta CS_m$
  - Achieve the surface finish range (small). The rapid tooling team prefers that ranged surface finish value of the two holes is smaller than 0.5 mils, and as small as possible.

    $C_{dk-SF} + d_3^- - d_3^+ = 1$

    $C_{dk-SF} = C_{du-SF} = (0.5 - SF)/\Delta SF$
  - Achieve the Young's modules range (large). The rapid tooling team prefers that ranged YM value is larger than 2.0 GPa, and as large as possible.

    $C_{dk-E} + d_4^- - d_4^+ = 1$

    $C_{dk-E} = C_{dL-E} = (YM - 2.0)/\Delta YM$
  - Achieve the Tensile strength range (large). The rapid tooling team prefers that ranged TS value is larger than 40 MPa, and as large as possible.

    $C_{dk-Y} + d_5^- - d_5^+ = 1$

    $C_{dk-Y} = C_{dL-Y} = (TS - 40)/\Delta TS$
- Constraint:
  - Every ranged design variable falls into the corresponding scope: $C_{dk} \geq 1$
  - $SF + \Delta SF \leq 0.5$ mils
  - $time + \Delta time \leq 40$ hours
  - $cost + \Delta cost \leq 2000$ dollars

…

Minimize

The deviation function (Archimedean Formulation)

$$Z_T = \sum_{i=1}^{5} W_i (d_i^+) \quad \text{where} \quad \sum_{i=1}^{5} W_i = 1, \quad W_i \geq 0$$

Figure 13. The Rapid Tooling Team's Compromise DSP for Ranged Set of Decisions

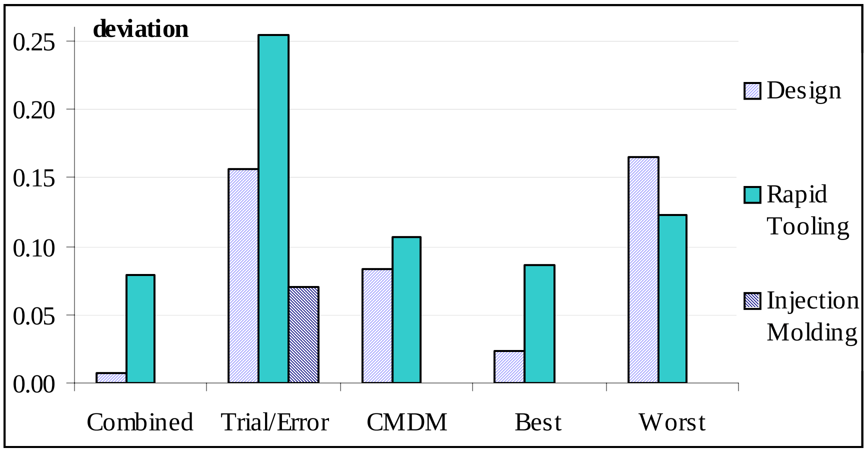


Figure 14. Deviation from Target Values

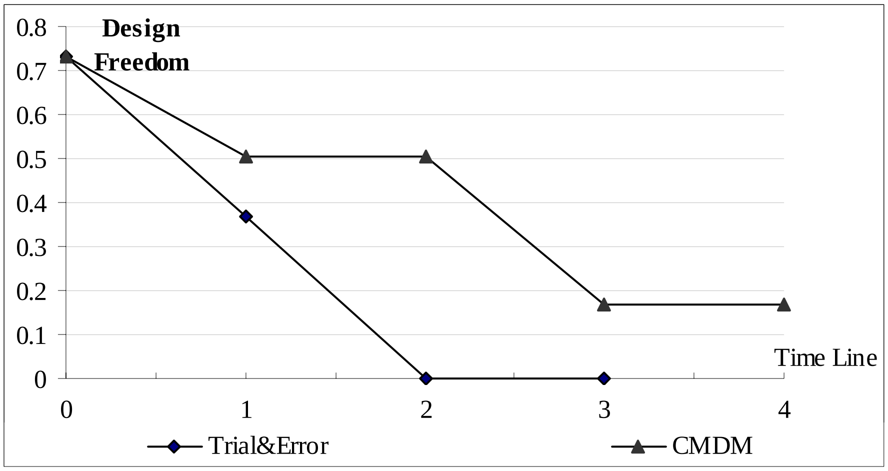


Figure 15. Design Freedom Change in the Product Realization Process

List of Tables

Table 1. Results from Combined DSPs

Design Variable	Value	State Variable	Value
D (mm)	20.92	*deform* (mm)	0.50
d (mm)	8.56	*stress* (MPa)	5.88
t (mil)	3.12	*weight* (mm^3)	3.50
LT (mil)	4.00	*time* (Hour)	28.04
HOC (mil)	4.70	cost ($)	1226.53
FOC (mil)	4.00	*SF* (mil)	0.13
draft,Θ	2.00	*YM* (GPa)	2.54
CT (s)	300	*TS*(MPa)	52.68
		ML	200

Table 2. Results from the Traditional Trial and Error Approach

Design Variable	Value	State Variable	Value
D (mm)	20.52	*deform* (mm)	0.58
d (mm)	9.24	*stress* (MPa)	7.01
t (mil)	2.69	*weight* (mm^3)	3.03
LT (mil)	8.00	*time* (Hour)	26.96
HOC (mil)	1.95	cost ($)	1215.93
FOC (mil)	2.00	*SF* (mil)	0.50
draft,Θ	1.97	*YM* (GPa)	2.24
CT (s)	300	*TS* (MPa)	45.62
		ML	93.07

Table 3. Results of the Design Team's Ranged Compromise DSP

Design Variable	Nominal Value	Deviation	Actual Range
D (mm)	23.94	2.04	[21.90, 25.40]
d (mm)	7.60	1.02	[7.60, 8.62]
t (mm)	2.50	0.22	[2.50, 2.72]
Performance Variable	**Nominal Value**	**Deviation**	**C_{dk} value**
deform (mm)	0.50	0.11	1.29
stress (MPa)	6.00	1.07	1.69
weight (g)	3.11	0.65	1.02

Table 4. Final Result of the Case

Design Variable	Nominal Value	Deviation	Actual Range
D (mm)	22.19	1.34	[21.90, 23.53]
d (mm)	8.03	0.43	[7.61, 8.46]
t (mm)	2.66	0.09	[2.57, 2.75]
LT (mil)	4.00	0	4.00
HOC (mil)	3.20	1.00	[2.20, 4.20]
FOC (mil)	4.00	1.00	[3.0, 5.0]
Θ	1.25	0.50	[0.75, 1.75]
CT (s)	300	0	300
Performance Variable	Nominal Value	Deviation	C_{dk} value
deform (mm)	0.53	0.08	1.53
stress (MPa)	6.35	0.73	1.99
weight (g)	3.07	0.24	2.63
PT (H)	32.13	4.47	1.76
CS ($)	1522.08	320.78	1.49
SF (mil)	0.13	0.00	1.00
YM (GPa)	2.28	0.20	1.39
TS (MPa)	49.81	2.09	4.69

Table 5. Best and Worst Points in the Final Results

Design Variable	Best Point	Worst Point
D (mm)	22.29	21.90
d (mm)	8.39	7.65
t (mil)	2.75	2.57
LT (mil)	4	4
HOC (mil)	4.20	2.20
FOC (mil)	3.00	5.10
draft,Θ	1.63	1.09
CT (s)	300	300
State Variable		
deform (mm)	0.50	0.59
stress (MPa)	6.00	6.88
weight (mm^3)	3.26	2.92
time (Hour)	27.66	27.68
cost ($)	1201.40	1202.61
SF (mil)	0.13	0.13
YM (GPa)	2.47	2.07
TS (MPa)	51.84	47.62
ML	200	150.14

Table 6. Performance Ranges and Design Freedom

Goals	def. (mm)	stress MPa	wt. (g)	PT (H)	CS ($)	SF (mil)	YM (GPa)	TS MPa	ML	Free-dom
Target Values	0.50 ± 0.15	6.00 ± 1.80	3.50 ± 1.05	≤ 40	≤ 2000	≤ 0.50	2.00 ≥	40.00 ≥	100 ≥	
Initial Performance Range	0.74 ± 0.50	8.19 ± 5.11	3.80 ± 1.78	132.44 ± 109.65	8390.1 ± 7504.9	0.29 ± 0.22	2.37 ± 0.46	48.81 ± 6.77	632.32 ± 627.01	0.734
After Design	0.58	7.01	3.03	87.99 ± 61.14	5329.3 ± 4127.5	0.28 ± 0.22	2.36 ± 0.45	48.74 ± 6.70	291.52 ± 281.83	0.368
Using the CMDM										
Step 1	0.49 ± 0.12	5.93 ± 1.15	3.27 ± 0.43	80.46 ± 57.66	4903.4 ± 4018.1	0.28 ± 0.22	2.36 ± 0.45	48.74 ± 6.70	586.24 ± 575.89	0.504
Step 2	0.49 ± 0.12	5.93 ± 1.15	3.27 ± 0.43	80.46 ± 57.66	4903.4 ± 4018.1	0.28 ± 0.22	2.36 ± 0.45	48.74 ± 6.70	586.24 ± 575.89	0.504
Step 3	0.53 ± 0.08	6.35 ± 0.73	3.07 ± 0.24	32.13 ± 4.47	1522.1 ± 320.8	0.13 ± 0.00	2.28 ± 0.20	49.81 ± 2.09	294.37 ± 183.14	0.170
Step 4	0.53 ± 0.08	6.35 ± 0.73	3.07 ± 0.24	32.13 ± 4.47	1522.1 ± 320.8	0.13 ± 0.00	2.28 ± 0.20	49.81 ± 2.09	294.37 ± 183.14	0.170
Goals	def. (mm)	stress MPa	wt. (g)	PT (H)	CS ($)	SF (mil)	YM (GPa)	TS MPa	ML	Free-dom
Target Values	0.50 ± 0.15	6.00 ± 1.80	3.50 ± 1.05	≤ 40	≤ 2000	≤ 0.50	2.00 ≥	40.00 ≥	100 ≥	
Initial Performance Range	0.74 ± 0.50	8.19 ± 5.11	3.80 ± 1.78	132.44 ± 109.65	8390.1 ± 7504.9	0.29 ± 0.22	2.37 ± 0.46	48.81 ± 6.77	632.32 ± 627.01	0.734
After Design	0.58	7.01	3.03	87.99 ± 61.14	5329.3 ± 4127.5	0.28 ± 0.22	2.36 ± 0.45	48.74 ± 6.70	291.52 ± 281.83	0.368