Measurement of the resonance frequency of single bubbles using a laser Doppler vibrometer

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Abstract: The behavior of bubbles confined in tubes and channels is important in medical and industrial applications. In these small spaces, traditional means of observing bubble dynamics are often impossible or significantly perturb the system. A laser Doppler vibrometer (LDV) requires a narrow (<1 mm diameter) line-of-sight access for the beam and illumination of the bubble does not perturb its dynamics. LDV measurements of the resonance frequency of a bubble suspended in a small tank are presented to illustrate the utility of this measurement technique. The precision of the technique is similar to the precision of traditional acoustic techniques.

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1. Introduction

The behavior of bubbles and bubbly liquids is important in many applications ranging from underwater sound and sonar,1,2 to industrial processes, sonochemistry, and cavitation,3-6 to medical acoustics.7-9 Understanding the behavior of single bubbles is fundamental to the study of collections of bubbles in any of these applications. Recently, the behavior of bubbles confined in tubes or channels and near surfaces has become important for medical and industrial applications.10-12 In confined spaces, traditional means of observing bubble dynamics, either acoustically with hydrophones13 or optically with Mie scattering14 or stroboscopy,15 are often impossible or they significantly perturb the system. A laser Doppler vibrometer (LDV)16 only requires a narrow (less than 1 mm diameter) line-of-sight access for the beam and illumination of the bubble does not perturb its dynamics. To illustrate the utility of this measurement technique, bubble resonance frequencies, obtained from LDV measurements of the acoustically excited response of a bubble suspended in a small tank are presented and compared to theory. No absolute standard exists to assess the accuracy of bubble resonance frequency measurements, therefore the precision of the technique was considered and found to be similar to the precision of a traditional acoustic technique17 that inferred the bubble resonance from a pair of acoustic pressure measurements.

2. Description of the apparatus and measurement procedure

A small acrylic-walled tank (35 cm × 35 cm × 13 cm, 0.625 mm wall thickness) with a tight fitting lid was filled with degassed distilled water. A single air bubble generated by a syringe and a needle was captured under a pair of parallel nylon monofilament lines (0.15 mm diam.) and positioned in the tank, as shown in Fig. 1. It has been shown that positioning a bubble of the size range used here (radii between 0.8 and 1.5 mm) with fine fibers has a negligible effect on the bubble’s resonance frequency.17 The tank was completely filled and closed so that no air remained in the tank. Acoustic excitation was provided by an electromagnetic shaker and a circular piston (2.5 cm diam.) through a hole in the tank wall and a rubber membrane. The source
signal (band limited pseudorandom noise, 1–5 kHz) was generated by the data acquisition computer and directed to a power amp and the shaker. Standing waves were thus set up inside the tank and the bubble was forced into oscillation. A pressure spectrum of the tank recorded with a miniature hydrophone (Bruel & Kjaer 8103) located near the position of the bubble, but in absence of the bubble, is shown in Fig. 2(a). Discussion of this response is deferred until Sec. 4. The normal velocity of the bubble wall was observed using a laser Doppler vibrometer (Polytec OFV-534). This procedure was repeated for four bubble sizes.

3. Description of the vibration measurement and data analysis

A LDV is based on the principle of detection of the Doppler shift of a monochromatic, directional, coherent light beam that is scattered from the surface of interest. The frequency of the scattered light (compared to the frequency of a reference beam) is used to determine the component of velocity along the axis of the incident beam. A helium-neon laser with a wavelength of 633 nm is used in the vibrometer utilized in this study. A number of challenges are associated with using a LDV to measure bubble motion. The measurements must be conducted through a volume of water and the tank wall, which causes additional attenuation of the laser beam as compared to a beam path of the same length in air. If an absolute measurement of velocity is required, the optical index of refraction of the water and the tank wall must be accounted for.

![Diagram](image)

Fig. 1. (Color online) The measurement instrumentation is shown in schematic on the left. The empty arrowheads represent signal paths. A photo (obtained with the stereo microscope described in the text) of the bubble positioned underneath a pair of monofilament nylon fibers is shown above right. This pair of fibers was held in place by a wire frame that was attached to the top of the tank. The bubble was centered in the tank for the two dimensions not explicitly indicated.

![Graphs](image)

Fig. 2. The acoustic pressure spectrum measured inside the tank near the location of the bubble is shown in (a). The acoustic pressure was normalized by the maximum pressure. The frequency range of the bubble resonance measurements that appear in Fig. 3(a) is also indicated. A typical spectrum of the bubble wall velocity measured with the LDV is shown in (b). The velocity was normalized by the maximum velocity. The open circle identifies the resonance frequency.


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The spherical shape of the bubble’s surface further reduces the intensity of light scattered back to the interferometer, compared to that scattered from a flat surface. Finally, scattering from the tank walls must not interfere with the light scattered from the bubble.

To overcome these difficulties, an LDV sensor head (Polytec OFV-534) that projects and receives the laser beam through a microscope objective was used. A coaxial video image of the bubble was also acquired through the objective lens and displayed in real time on the data acquisition PC, which afforded precise alignment of the laser beam with the normal point on the bubble surface. The sensor head was mounted on a photographic tripod and positioned through manual manipulation of the tripod controls. The beam was focused to a diameter of approximately 2 μm, and therefore it illuminated a relatively small portion of the bubble surface. For the smallest bubble in this study, the patch illuminated by the laser deviated from a plane by at most 6 picometers, which is a negligible fraction of the optical wavelength. Hence, the LDV was effectively observing motion that was normal to the bubble surface.

An objective lens with a magnification factor of 10 was used to ensure that sufficient light was reflected back to the photodetector, as determined by near-unity coherence between the input excitation signal and the measured velocity. The working distance provided by the objective allowed for unfocused light to pass through the tank wall, which in turn reduced spurious reflections to a negligible level. A time domain voltage signal that is a direct analog of the normal surface velocity of the illuminated patch on the bubble was output from the controller/demodulator (Polytec OFV-5000). A PC-based data acquisition system was used to acquire and process the velocity signals. For a given bubble size, the time domain signal was windowed and a fast Fourier transform was performed. Fifty frequency-domain averages were computed and the average spectra (resolution bandwidth=3.125 Hz) was saved. A typical spectrum is shown in Fig. 2(b). The resonance frequency of the bubble was taken to be the frequency that corresponded with the maximum amplitude of the spectra. Finally, a stereo microscope with a charge coupled device camera and diffuse white backlighting (oriented on an axis normal to the LDV axis) were used to measure the bubble size.

4. Results

The resonance frequencies extracted from the LDV velocity spectra are shown in Fig. 3(a). The error bars represent uncertainty in the measured bubble radii due to the resolution (pixelization) of the digital micrographs, which were obtained with various degrees of magnification. The solid line is the prediction of the bubble resonance frequency for free field conditions

$$\omega_0 = \left[ \frac{P_0}{\rho a^2} \left( \text{Re} \Phi - \frac{2\sigma}{aP_0} \right) \right]^{1/2},$$

where the hydrostatic pressure at the bubble was $P_0 = 1.03 \times 10^5$ Pa, the density of the water was $\rho = 998$ kg/m$^3$, $a$ was the measured bubble radius, and the surface tension of the water was $\sigma$. The two curves nearly overlay one another, except for bubble radii near 0.8 mm. Results obtained with an acoustic technique (see Ref. 17) appear in (b). In both cases, the error bars represent uncertainty in the bubble size measurement.
The frequency-dependent thermal behavior of the gas is described by \( \Phi \) and is given by Eq. (27) in Ref. 19. The gas properties diffusivity \( D_g = 2.08 \times 10^{-5} \text{ m}^2 / \text{s} \) and the ratio of specific heats \( \gamma = 1.4 \) were used to calculate \( \Phi \).

Equation (1) is for a bubble in a free field, but the resonance frequency of a bubble can be modified by the presence of tank walls, as quantified by Eq. (35) of Ref. 13 in terms of an infinite series

\[
\omega_{res} = \omega_0 + \frac{4 \pi c^2 \alpha}{\omega_0 V} \sum_{n=1}^{N} T_n,
\]

where \( T_n \) is given by Eq. (14) of Ref. 13 and is proportional to the mode shape functions. The fully enclosed, acrylic-walled tank in this work exhibited a lowest-order resonance frequency near 2.2 kHz, as shown in Fig. 2(a), and no other resonances are present below 6 kHz. The tank used in this work can support a static pressure, hence it appears acoustically rigid at low frequencies. This assertion is supported by calculation of the lowest-order eigenfrequency of a rigid-walled tank of the same dimensions, using Eq. (9.2.7) of Ref. 20, which yields 2.1 kHz, closely matching the frequency of the peak labeled \( \alpha \) in Fig. 2(a). At higher frequencies, the tank appears pressure release. The evidence for this is threefold. First, the higher-order resonances that are predicted for a rigid-walled tank between 2 and 6 kHz are absent in Fig. 2(a). Second, the reflection coefficient for the three medium problem given by Eq. (6.3.7) of Ref. 20, for a water-acrylic-air system between 2 and 4 kHz, is approximately \(-0.999\), which is nearly pressure release. Third, the lowest-order resonance frequency predicted for a pressure-release tank of the same dimensions is 6.4 kHz, which is similar to the frequency of the peak labeled \( \beta \) in Fig. 2(a).

To bound both behavior regimes, Eq. (2) was evaluated for rigid and pressure release walls. The modal summations were taken to \( N = 1000 \), where convergence was achieved to less than 1 part in \( 10^4 \). The quality factors of the tank’s resonances are inputs to Eq. (2). For these calculations, the quality factors were all set to 100, which exceeds the actual quality factor measured inside the tank for the frequency range of the bubble measurements. The sound speed inside the tank was set to 1481 m/s. The results are shown in Figs. 3(a) and 4. Within the experimental bubble size range, the deviation between the pressure release tank model and the free field model Eq. (1) has a mean value of 0.36% and at most the deviation is 0.54%. The deviation between the rigid tank model and the free field model Eq. (1) has a mean value of 0.27% and at most the deviation is 0.64%. Since these deviations are small relative to the uncertainty in bubble size measurement, and since determining the exact effect of the tank reverberation is beyond the present scope, the free field bubble model Eq. (1) is used in Sec. 5.

For comparison to the LDV technique, bubble resonance measurements obtained with an acoustical technique \(^{17}\) are shown in Fig. 3(b). The acoustic pressures inside a tank both with and without a bubble present were measured. Manipulation of these measured pressures yielded the bubble resonance frequency. The acoustic measurements are compared to Eq. (1) with \( P_0 \)
and the remaining physical parameters unchanged from the previous case.

5. Discussion

There is no accepted standard for the assessment of the absolute accuracy of a bubble resonance measurement, therefore relative measures of precision were used to characterize the two measurement techniques. The RMS error between the measurements in Fig. 3 and Eq. (1) are 1.5% for the LDV technique and 2.1% for the acoustic technique. We therefore conclude that a LDV is a viable alternative for observing bubble resonance and is capable of relative precision equal to that of a traditional acoustic technique. The noninvasive nature of the LDV technique permits use in confined spaces, such as in a narrow tube with a diameter on the order of the bubble diameter, where acoustic techniques would be difficult due to the size requirement imposed on the measurement hydrophone, and where the Mie scattering technique would be impossible, since it requires the light source and receiver axes to be separated by an angle of about 80°.

Acknowledgments

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References and links