Chapter 4

System Representation: Problems

Figure 4.1: Bond graph for problem 1.

1. Complete the bond graph of figure 4.1, and from it extract the complete system of state equations.

2. Complete the bond graph of figure 4.2, and from it extract the complete system of state equations.

3. Figure 4.3 depicts a crude seismograph, a geologic instrument for measuring ground motions $x(t)$. Motions $x(t)$ (or $\dot{x}(t)$) transmit through the very stiff structure and excite vertical motions $z$ of mass $M$, suspended from the structure by spring $k$ and damper $b$. The needle attached to the mass/spring/damper rotates about point $O$; deflections of the needle’s end record $z(t)$. From the record of $z(t)$, ground
2  \textit{CHAPTER 4. SYSTEM REPRESENTATION: PROBLEMS}

Figure 4.2: Bond graph for problem 2.

Motions $x(t)$ can be inferred. Obtain the system equation(s): a bond graph may or may not be necessary.

4. Extract the state equations from the bond graphs constructed for problems ??-??, ??, ??, ??, in section ?? of chapter ??.

5. Modify the block and tackle hoisting mechanism of Figure ??a of example ?? of chapter ?? so the cable loops through twice, instead of once. Construct a bond graph for this mechanism, and extract the state equations.

6. Derive the transfer function $G_2(s) = X_2(s)/F(s)$ from the state equations

$$\dot{x}_1 = -4x_1 + x_2$$
$$\dot{x}_2 = -2x_1 - x_2 + f(t), \quad t > 0.$$

7. Given the system of state equations

$$\dot{x}_1 + 4x_1 = f(t)$$
$$\dot{x}_2 + 2x_1 + 7x_2 - 2x_3 = 0$$
$$\dot{x}_3 + x_1 + 2x_2 + x_3 = g(t)$$
with initial conditions \( x_1(0) = x_2(0) = x_3(0) = 0 \). Calculate the transfer function \( H(s) = X_1(s)/F(s) \). Determine all system eigenvalues. For \( f(t) = \cos 3t \) and \( g(t) = 0 \), determine the steady state (particular) response for \( x_1(t) \).

Figure 4.4: Block diagram for problem 8.

8. Determine the ordinary differential equation that corresponds to the block diagram of figure 4.4.

9. For the seismograph of figure 4.3 and problem 3, obtain the transfer function \( G(s) = X(s)/Z(s) \), to infer the ground motions \( x(t) \) from the measured motions \( z(t) \).

10. Given the transfer function \( G(s) = \frac{2(s+2)}{s^2+3s+2} \), calculate the poles and zeroes and determine the steady state response for sinusoidal excitation \( f(t) = 2 \cos 2t \).