COMPLEX VARIABLES PROBLEMS

1) Using Euler’s identity, express \( \exp\{(-8 + 9j) t\} \) in terms of sinusoids.

2) Given the polynomial function \( F(z) = z^3 + 4z^2 - 7z + 3 \) in the complex variable \( z = x + jy \). Determine \( \text{Re}(F) \), \( \text{Im}(F) \), \( |F| \), and \( \arg F \).

3) Given the differential equation \( \ddot{q} + 4 \dot{q} + 2q = 2e^{2j \omega} \) excited by the complex exponential \( 2e^{2j \omega} \).
   a) Determine the particular solution. \( \text{Hint:} \) try \( Be^{2j \omega} \).
   b) Sort out the real and imaginary parts of the particular solution.
   c) Show that the real part of the solution is the solution of the differential equation excited by \( 2 \cos 2t \).

4) Given the function
   \[ G(s) = \frac{0.01s + 1}{s^2 + 0.2s + 100} \]
   With \( s = j \omega \), express the numerator and denominator as complex numbers in polar form.
   Then find \( |G| \) and \( \arg G \).

5) For \( G(s) \) given in problem 4, with \( s = j \omega \),
   a) Express the numerator and denominator as Cartesian complex numbers.
   b) Multiply the numerator and denominator by the complex conjugate of the denominator.
   c) Show that \( |G| \) and \( \arg G \) of the resulting complex number is identical to those of problem 4.