Physics review & bond graphs: Mechanical Rotational

Example: Automotive drive train
   vector mechanics approach

Bond Graph Approach

   Physical Model
   Map power flows in system

Compare
Example: Automotive drive train

Vector mechanics approach would consist of

- Physical Model
- Free Body Diagrams
- Physical Analysis
  - action/reaction
  - Euler's law: \[ \Sigma T = I \alpha = \frac{d(I \omega)}{dt} = \frac{dh}{dt} \]

OR Dynamic equilibrium with inertial torque:
\[ T_I = -\frac{dh}{dt} \Rightarrow \Sigma T = 0 \text{ (rotational KVL)} \]

- Constitutive equations / system elements
- Kinematics
  \[ \Rightarrow \text{Get Equations of motion} \]
Bond Graph Approach

- Physical Model
- Map power flows in system
- Physical Analysis
  - Causality
  - Dynamics
  - Kinematics
  - Constitutive equations
- Label efforts & flows on bonds in terms of energy variables

⇒ Equations of motion
Physical Model

Map power flows in system
Power distribution junctions

"I" junction at each inertia:

- balances efforts (torques), i.e., incorporates Euler's (D'Alembert's) law
  \[ \sum \text{ [torque drops]} = 0 \]
  over inertia

- enforces kinematics: common flows (same angular velocities)
Power distribution junctions

"0" junction at compliance:

- common efforts (torques at ends) from free body diagram
- balances flows (angular velocities) \( \dot{\theta}_1 = \omega_{\text{right}} - \omega_{\text{left}} \)

which incorporates kinematics of spring twist

\[ \theta_1 = \theta_{\text{right}} - \theta_{\text{left}} \]
Bond Graph Construction

- Physical Analysis
- Power Flow Map
- Include
  - Causality
  - Dynamics
  - Kinematics
  - Constitutive equations
- Label efforts & flows on bonds in terms of energy variables
### Compare

<table>
<thead>
<tr>
<th>system type</th>
<th>displacement</th>
<th>momentum</th>
<th>flow</th>
<th>effort</th>
<th>effort balance</th>
<th>flow balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>general</td>
<td>q</td>
<td>p</td>
<td>f</td>
<td>e</td>
<td>1 junction $\Sigma e = 0$</td>
<td>0 junction $\Sigma f = 0$</td>
</tr>
<tr>
<td>electrical</td>
<td>charge q</td>
<td>flux linkage $\lambda$</td>
<td>current i</td>
<td>voltage V</td>
<td>KVL</td>
<td>KCL</td>
</tr>
<tr>
<td>mech. translation</td>
<td>displacement x</td>
<td>linear mom. p</td>
<td>velocity v</td>
<td>force F</td>
<td>d'Alembert dyn. equil.</td>
<td>translation kinematics</td>
</tr>
<tr>
<td>mech. rotation</td>
<td>angular disp. $\theta$</td>
<td>ang. mom. h</td>
<td>ang. vel. $\omega$</td>
<td>torque T</td>
<td>d'Alembert dyn. equil.</td>
<td>rotation kinematics</td>
</tr>
</tbody>
</table>

Common relationships between variables:

- dynamics on inertance (KE storage element) $e = \dot{p}$
- kinematics on capacitance (PE storage element) $f = \dot{q}$