Given the feedback controller with plant transfer function

\[ g(s) = \frac{y(s)}{u(s)} = \frac{4}{s^2 + 2s + 1} \]

and controller transfer function \( k(s) = \frac{u(s)}{e(s)} = K_p \) where \( K_p \) is constant.

(a) Find transfer functions \( \frac{y(s)}{r(s)} \) and \( \frac{e(s)}{r(s)} \).

\[ \frac{y(s)}{r(s)} = G + GH = \frac{kq}{1 + kq} = \frac{4k}{s^2 + 2s + 1 + 4k} \]

\[ e(s) = r - y = \left[ 1 - \frac{4k}{s^2 + 2s + 1 + 4k} \right] r \]

\[ \frac{e(s)}{r(s)} = \frac{s^2 + 2s + (1 - 4k)}{s^2 + 2s + 1 + 4k} \]

(b) Determine the range of values for gain \( K_p \) for which the closed loop system is stable.

Char Eqn: \( s^2 + 2s + (1 + 4k) = 0 \)

Poles: \( p_1, p_2 = -2 \pm \sqrt{2^2 - 4(1+4k)} = -1 \pm 2\sqrt{-k} \)

Stable if: \( -1 \pm 2\sqrt{-k} \leq 0 \) \( \Rightarrow \) \( \pm \sqrt{-k} \leq \frac{1}{2} \)

\( \Rightarrow -k \leq \frac{1}{4} \)

\( \Rightarrow k \geq -\frac{1}{4} \)