Elastohydrodynamic Lubrication

- In highly loaded contacts
  - Roller bearing line contacts
  - Ball bearing point contacts
  - Gear teeth line contacts
- Very high lubricant pressures $p$, order of Hertz pressures: $\sim 100$ ksi $\approx 700$ Mpa
- Elevated temperatures $T$ in lubricant
- Consequences
  - Lubricant viscosity $\eta = \eta(p,T)$ increases
  - Surfaces deform, altering film thickness $h$
  - Altered film thickness affects pressures $p$

Stribec Curve
Unkowns
- Pressures $p$
- Film thickness $h$
- Temperatures $T$

Requires simultaneous solution of
- Reynolds equation
- Deformation of bodies, often elastic
- Energy equation

Geometry of Rollers:

- Roller on flat with same separation function:
  $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

- Equivalent roller radius:
  $$R = \frac{R_1 R_2}{R_1 + R_2}$$

Slip-roll ratio:
$$S = \frac{U_1 - U_2}{U_1 + U_2} = \frac{\Delta U}{2U}, \quad 0 \leq S \leq \infty$$
- $S = 0$: pure rolling ($U_1 = U_2$), no slip
- $S = \infty$: pure slip ($U_1 = -U_2$), no rolling
Elastohydrodynamic Lubrication between Rollers

- Geometric film thickness:
  \[ h(\theta) = h_o + R(1 - \cos \theta) \]

- Convergent & divergent profiles
- Possible cavitation from divergent profile
- Possible elastic deformation of bodies, due to high pressures
- Possible density & viscosity changes of lubricant, due to high pressures

- Local geometry:
ElastoHydrodynamic Lubrication: Formulations Outline

- Modify Reynolds Equation
- Film Thickness Equation
- Oil (Lubricant) Rheology
- Reynolds Equation & Solutions
- Effects
Reynolds Equation

- Assumptions

2D: \( \partial / \partial z = 0 \)

Incompressible: \( \frac{\partial \rho}{\partial \zeta} = 0, \quad \zeta = x, z, t \)

Film thickness: \( h = h(x) \)

Pressure: \( p = p(x) \)

Velocities: \( W_1 = W_2 = 0 \)

\[
\frac{\partial}{\partial x} \left( \frac{h^3 \partial p}{\eta \partial x} \right) = 12(V_2 - V_1) \\
+ 6(U_1 - U_2) \frac{\partial h}{\partial x} + 6h \frac{\partial(U_1 + U_2)}{\partial x}
\]
\[ V_2 - V_1 = \frac{dh}{dt} = \frac{dh}{dx} \frac{dx}{dt} = \frac{dh}{dx} U_2 \]

- **Right Hand Side:**

\[
12U_2 \frac{dh}{dx} + 6(U_1 - U_2) \frac{dh}{dx} + 6h \frac{d(U_1 + U_2)}{dx}
\]

\[
= 6 \frac{d}{dx} \left[ h(U_1 + U_2) \right]
\]

- **Reynolds Equation:**

\[
\frac{d}{dx} \left( \frac{h^3 \frac{dp}{dx}}{\eta \frac{dx}{dx}} \right) = 6 \frac{d}{dx} \left[ h(U_1 + U_2) \right]
\]
Reynolds for Roller

\[ \frac{dp}{dx} = 6\eta(U_1 + U_2) \frac{h - h_*}{h^3} \]

- Integrate \((h_* = \text{arbitrary constant})\):

- \(dp/dx = 0\) at \(h = h_*\), where \(x = x_*\)

- About \(x = 0\), film thickness ``flattens``. Approximate: \(h_* \approx h_o\).

- Pressure boundary conditions:
  \[ p = p_a \text{ (or } p = 0\text{) at } x = \pm\infty \]
• Local coordinates: $x = R\theta$, for $h/R \ll 1$

• Film thickness: $h = h_g + h_d$

• Geometric film thickness $h_g$:

$$h_g(x) = h_o + R(1 - \cos \frac{x}{R}) \approx h_o + \frac{x^2}{2R}$$
• Deformation film thickness $h_d$, from relative squash of bodies:

$$h_d(x) = \left[ v_1(x) - v_1(0) \right] + \left[ v_2(x) - v_2(0) \right]$$

• From Flamant solution (2D Bousinesq):

$$\bar{v}_i(x') = -\frac{P(1+\nu)}{2\pi E} \left[ (\kappa + 1) \ln r - \frac{2y^2}{r^2} \right] | y = 0$$

$$r = \sqrt{x'x' + y^2}, \quad \kappa = 3 - 4\nu$$

• Let $x' = x - \xi$, $P = p(\xi)d\xi$, $\frac{1}{E} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$, $v_i(x) = \int_{-\infty}^{\infty} p(\xi)\bar{v}_i(x - \xi)d\xi$, then:

$$h_d(x) = -\frac{4}{\pi E} \int_{-\infty}^{\infty} p(\xi) \ln \frac{|\xi - x|}{|\xi|} d\xi$$

Young’s modulus: $E_i$, Poisson’s ratio: $\nu_i$
FLAMANT SOLUTION (2D)

- Point force $P$, normal to semi-infinite elastic space
- 2D: plane strain or plain stress

Elastic deformations: $(u, v)$ along $(x, y)$

$$u = -\frac{P}{4\pi\mu} \left\{ (\kappa - 1)\theta - \frac{2xy}{r^2} \right\}, \quad v = -\frac{P}{4\pi\mu} \left\{ (\kappa + 1)\log r - \frac{2y^2}{r^2} \right\}$$

Stresses:

$$\sigma_{xx} = -\frac{2P}{\pi} \left\{ \frac{y}{r^2} - \frac{y^3}{r^4} \right\}, \quad \sigma_{yy} = -\frac{2P}{\pi} \left\{ \frac{y^3}{r^4} \right\}, \quad \sigma_{xy} = -\frac{2P}{\pi} \left\{ \frac{xy^2}{r^4} \right\}$$

$\mu$: elastic shear modulus, $\nu$: Poisson's ratio
Elastic modulus: $E = 2(1 + \nu)\mu$
Dundurs constant: $\kappa = 3 - 4\nu$, plane strain,
$\kappa = (3 - \nu)/(1 + \nu)$, plane stress

$$r = \sqrt{x^2 + y^2}, \quad \tan \theta = x/y$$
Oil Rheology: Viscosity

• Viscosity: $\eta = \eta(p, T)$

• Pressure dependence: $\eta = \eta_p e^{\alpha(p-pr)}$

• Temperature dependence: $\eta = \eta_T e^{\beta(T-Tr)}$

• $\eta = \eta(p, T) = \eta_o e^{\alpha(p-pr)} e^{\beta(T-Tr)}$
FIGURE 13.6.
Viscosity versus temperature curves for typical SAE graded oils.
Final Reynolds Equation

\[
\frac{dp}{dx} = 6\eta(U_1 + U_2) \frac{x^2}{2R} + h_d(x) \left[ h_o - \frac{x^2}{2R} + h_d(x) \right]^{-3}
\]

- \( h_d(x) = -\frac{4}{\pi E} \int_{-\infty}^{\infty} p(\xi) \ln \left| \frac{\xi - x}{|\xi|} \right| d\xi \)

- \( \eta = \eta_o e^{\alpha(p(x) - p_r)} e^{\beta(T - T_r)} \)

- Nonlinear integral-differential equation!

- Unknowns: \( h_o, p = p(x) \)

- Boundary conditions: \( p \big|_{x=\pm\infty} = p_a \)
Solution

- Eigenvalue problem: special value of parameter (eigenvalue) for solution to exist

  Eigenvalue: $h_o$

  Eigenfunction: $p = p(x)$

- Usually solved numerically (via finite differences)
Solution: EHD Pressures

- Pressure profile $p = p(x)$:
  - Approximately Hertzian
  - Pressure "spike" near outlet
- Film thickness $h$:
  - Roller flattens about contact zone
  - Nearly constant film thickness
  - Slight dimple forms near pressure spike

*Pressure measurements confirm spike!*
EHD Measurements

Film thickness:
- Sapphire roller against steel roller
- Polarized light through sapphire, observe interference fringes for film thickness

Temperature measurements:
- Infra-red through sapphire roller

---

Passing through the center of the contact area, the film thickness increases markedly. The maximum surface temperature rises of both the surfaces are about 100 K and the profiles cross at the center of the area. The temperature of the oil film, which is higher than that of both the surfaces, increases markedly at the dimple zone and reaches 357 K. The temperatures of both the surfaces also increase at the dimple zone. When the slip ratio is infinite, the film thickness is still about 0.1 \( \mu \)m at both the sides of the contact area and the maximum film thickness reaches 2.0 \( \mu \)m at the center of the contact area, where the oil temperature rise increases dramatically. For comparison between \( S = -8.0 \) and 8.0, it is worth noting that the film thickness for the slip ratio of -8.0 is above 0.1 \( \mu \)m thicker than that for the slip ratio of 8.0. This difference of the thickness is thicker than the accuracy for the film thickness. In addition, the maximum temperature rise in the oil film in case of the slip ratio of -8.0 is also higher than that of 8.0 as shown in Fig.4 and the difference for the temperatures is smaller than the temperature variation during the experiments.
Figures 5 to 7 show a series of results for the influence of the sliding speed at the slip ratio of infinity and load of 123 N. The effects of the sliding speed on the traction coefficient, the maximum temperature rise of the oil film and both the surfaces are illustrated in Fig.7. It can be seen that the maximum temperature rise of both the surfaces has a constant value of about 100 K whereas the maximum temperature rise of the oil film increases and thus reaches above 400 K. The traction coefficient demonstrates a gradual decrease and thus reaches a constant value of 0.02 asymptotically with increasing sliding speed.

It should be noted that the dimple varies in shape from an ellipse where the minor axis is aligned with the sliding direction, a circle and an ellipse where major axis is aligned with the sliding direction as the sliding speed increases in Fig.5. The depth of the dimple increases with increasing sliding speed to 2.0 m/s and when the sliding speed is beyond 2.0 m/s, the depth decreases gradually as seen in Fig.6. The film thicknesses at both the sides of the contact area increase and thus reach a constant value of about 0.6 m. This tendency of the variation of the film thickness has been obtained in the experimental results of Cameron [11], Dyson et al [14] and Shorgin et al [19] as averaged values.

The effects of the load on the maximum temperature rises, the traction coefficient and the film thickness are illustrated in Fig.8 and 9. As the load is increased, the size and depth of the dimple increases according to Fig.8. The thicknesses of both the sides of the contact area do not vary and have a constant value of about 0.6 m. The temperature rises increase with load and the increase in temperature rise of the oil film is much higher than that of the both the surfaces as shown in Fig.9. The traction coefficient decreases with increasing load.

CaF$_2$-steel contacts Figure 10 shows the interference fringe pattern of the film thickness, the profiles of the temperature rises and the film thickness and temperature distribution across the oil film for the CaF$_2$-steel contacts with simple sliding for a disk surface speed $\Delta u$ of 0.8 m/s against the stationary ball surface, the mean Hertzian pressure $p_m$ of 0.36 GPa using Santotrac100 oil. It can be seen easily that the dimple exits at the center of the contact. The temperature rise of the disk surface, which has lower thermal conductivity and is moving, is higher than that of the ball surface. The oil temperature rise increases at the dimple zone and reaches 211 K. As is seen in Fig.10, it is found that the heat is conducted