Modeling and Control of Analog Meter Dynamics

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Intro + Modeling + Control + Lab, LE, Summary
What was accomplished in the first part of this lab?

1. Developed a vision-based measurement system for tracking the angular deflection of the needle on an analog meter.
2. Used vision-based measurement to model the static gain, $K_{\theta/v}$, of the analog meter which relates the deflection to the input voltage.
3. Developed a quick ‘open loop’ controller to demonstrate ability to track a desired angular position, $\theta_d$.

- Find that using static gain may not be enough to help predict or control overshoot, or to estimate how long the system might oscillate before reaching a steady value; i.e., settling time.
- These are important dynamic characteristics of a system.
- There may also have been steady-state error.
Overshoot and steady-state error in the step response with open loop control. System appears underdamped.
**Typical specifications or metrics**

- Overshoot (measure of ‘relative stability’)
- Delay time, $T_d$ (50%)
- Rise time, $T_r$ (90%)
- Settling time, $T_s$ (within 2-5%)
- Dominant time constant, $\tau_d$ (63%)
To improve the response characteristics of a system, modify:
1. system parameters by design, or
2. control input(s).

In this follow on lab study:
1. model the system to understand which parameters could be changed to affect and improve response, and
2. study and experiment with control systems to determine if effective in this problem.
First review a dynamic model of the analog meter to understand key parameters

This insight can be used to determine whether design changes are feasible. The improved model can also be used to guide feedback control or open loop control design.

In the laboratory:

1. Modeling and estimating the system parameters
2. Implementing closed loop control
3. Adding feedforward control either as static gain or a dynamic system model
4. Compare how closed-loop versus open loop controls allow system response characteristics to be changed as needed
Transfer function form of the model

\[ G_{\theta/v}(s) = \frac{\theta_n}{v_{in}} = \frac{K_{\theta/v}\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

The following slides detail a derivation of this analog meter model both as state space model and transfer function (TF) as shown above.

Why use the TF model? The static experiments provide a measure of the static gain, so only two more parameters are needed – 3 total. Also, the step response looks like a classic 2\textsuperscript{nd} order system response.

The state space model needs 5 parameters, so it is not as convenient to use in this control study.

TF model only needs damping ratio and natural frequency, which can be readily determined in the laboratory.
Analog moving coil meter model

- The meter physical model is presented in two different versions: ‘full model’ and 2\textsuperscript{nd} order.
- The 2\textsuperscript{nd} order model neglects inductance, which is a good assumption for this application.
- The electromechanical (EM) torque can be modeled using a gyrator.
Recall the models shown before in schematic form:

**Electrical circuit model**

- $V_{in}$
- $i_m$
- $R_{series}$
- $R_m$

**Rotational system**

- Needle
- Moving coil mechanism is attached to needle
- Needle
- Flux path
- Permanent magnet
- Torsional spring is part of conductive path.
The EM torque induced on the moving coil is related to the current – a gyrator model

This slide summarizes the basic force-current relation in a conductor. In a bond graph, this can be modeled by a gyrator, which gives a net relation between force and current. For a motor or for the case of the rotational moving coil, this force is resolved into torque.

The differential force on a differential element of charge, \(dq\), is given by:

\[
d\mathbf{F} = dq \mathbf{v} \times \mathbf{B}
\]

where \(B\) is the magnetic field density, and \(i\) the current (moving charge).

It can be shown that the net effect of all charges in the conductor allow us to write:

\[
d\mathbf{F} = idl \times \mathbf{B}
\]

For a straight conductor of length \(l\) in a uniform magnetic field, you can integrate to find the total force:

\[
\mathbf{F} = il \times \mathbf{B}
\]

With angle \(\alpha\) between the vectors, you can arrive at the desired relation:

\[
\mathbf{F} = \left( B l \sin \alpha \right) \cdot i
\]
Analog moving coil meter bond graph

A bond graph of the meter can take the form shown below. The coil has resistance, $R_m$, and inductance, $L_m$. The needle has moment of inertia, $J_n$, and there is some damping, $B_n$, as well. The spring has stiffness, $K_s$. These are parameters for **linear** constitutive relations for each of the elements shown in this model. Note, the meter also has an external series resistor that is not shown here, but the value of that resistance can be added to $R_m$.

We seek a mathematical model that relates needle position (equal to spring deflection) to input voltage, $v_{in}$.

This model can be derived from the bond graph, or by application of Newton’s Laws (mechanical side) and KVL (circuit side).

See Appendix B for explanation of gyrator model for EM transduction.
Full 3\textsuperscript{rd} order model, with inductance

The state-space model for the meter, including the inductance, is 3\textsuperscript{rd} order.

\begin{equation*}
\begin{aligned}
\dot{h}_n &= J_n \omega_n = \text{needle angular momentum} \\
\lambda_m &= L_m i_m = \text{flux linkage} \\
\theta_n &= \text{angular position of needle/spring}
\end{aligned}
\end{equation*}

3 States:

\begin{equation*}
\begin{aligned}
\dot{h}_n &= J_n \dot{\omega}_n = T_m - K_s \theta_n - B \omega_n \\
\dot{\lambda}_m &= L_m \left( \frac{di_m}{dt} \right) = v_{in} - (R_m + R_s) i_m - v_{in} \\
\dot{\theta}_n &= \omega_n
\end{aligned}
\end{equation*}

3 State equations:

EM gyrator relations:

\begin{equation*}
\begin{aligned}
T_m &= r_m i_m \\
v_m &= r_m \omega_m
\end{aligned}
\end{equation*}

Note: the needle and the spring have the same velocity.

Also, can choose either the meter flux linkage or current as the state.
3rd order model state-space equations

In state space form:

\[
\begin{bmatrix}
\dot{\lambda}_m \\
\dot{h}_n \\
\dot{\theta}_n
\end{bmatrix} = 
\begin{bmatrix}
R_T & -r_m & 0 \\
\frac{L_m}{J_n} & -\frac{B_n}{J_n} & -K_s \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\lambda_m \\
h_n \\
\theta_n
\end{bmatrix} + 
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} v_{in}
\]

Output equation:

\[
y = \theta_n = 
\begin{bmatrix}
0 & 0 & 1
\end{bmatrix} 
\begin{bmatrix}
\lambda_m \\
h_n \\
\theta_n
\end{bmatrix} + 
\begin{bmatrix}
0
\end{bmatrix} v_{in}
\]
Analog moving coil meter bond graph – neglect inductance

If we neglect the inductance, we see that the model reduces to second order. Note the change in causality (if you understand bond graphs).

This assumption is reasonable given that we observe a step response in the experiments that looks 2\textsuperscript{nd} order, underdamped.

Now the only states of interest are the needle angle (related to spring deflection) and the needle rotational momentum.

\[
R_T = R_m + R_s
\]

See Appendix B for explanation of gyrator model for EM transduction.
2nd order model, neglecting inductance

The mathematical model for the meter, *neglecting inductance*,

States: \[
\begin{align*}
J_n \dot{\omega}_n &= \text{angular momentum} \\
\theta_n &= \text{angular position of needle/spring}
\end{align*}
\]

State equations: \[
\begin{align*}
\dot{J}_n &= T_m - K_s \theta_n - B_n \omega_n \\
\dot{\theta}_n &= \omega_n
\end{align*}
\]

EM gyrator relations: \[
\begin{align*}
T_m &= r_m i_m \\
v_m &= r_m \omega_m
\end{align*}
\]

where,

\[
i_{in} = i_R = \frac{(v_{in} - v_m)}{(R_m + R_s)}
\]

The meter current is now determined by the voltage drop, not by the inductor state.
2\textsuperscript{nd} order state-space equations

In state space form:

State equations:

\[
\begin{bmatrix}
\dot{h}_n \\
\dot{\theta}_n
\end{bmatrix} = \begin{bmatrix}
-B_n + \left( B_n + \frac{r_m^2}{R_T} \right) \frac{1}{J_n} & 0 \\
\frac{1}{J_n} & -K_s
\end{bmatrix}
\begin{bmatrix}
\dot{h}_n \\
\dot{\theta}_n
\end{bmatrix} + \begin{bmatrix}
\frac{r_m}{R_T} \\
0
\end{bmatrix} v_{in}
\]

Output equation:
\[
y = \theta_n = \begin{bmatrix}
0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{h}_n \\
\dot{\theta}_n
\end{bmatrix} + \begin{bmatrix}
0
\end{bmatrix} v_{in}
\]
Let’s convert this into a 2nd order ODE equation

First, consider just the first equation and write it in terms of the angle:

\[ h_n = J_n \omega_n = J_n \dot{\theta}_n \Rightarrow \dot{h}_n = J_n \ddot{\theta}_n \]

Substitute into the momentum equation:

\[ \Rightarrow J_n \ddot{\theta}_n + \left( B_n + \frac{r_m^2}{R_T} \right) \frac{1}{J_n} \left[ J_n \dot{\theta}_n \right] + K \dot{\theta} = \frac{r_m}{R_T} v_{in} \]

Remember that the angle and angular velocity are related, so write in terms of the angle. This gives the 2nd order ODE we want:

\[ \Rightarrow \ddot{\theta} + \left( B + \frac{r_m^2}{R_T} \right) \frac{1}{J_n} \dot{\theta} + \left( \frac{K \dot{\theta}}{J_n} \right) \theta = \frac{r_m}{J_n R_T} v_{in} = \left( \frac{K \dot{\theta}}{J_n} \right) \left( \frac{r_m}{K \dot{\theta}} v_{in} \right) \]

[\text{Modeling}]

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Relate to the static gain measured in lab

\[
\ddot{\theta}_n + \left( B_n + \frac{r_m^2}{R_T} \right) \frac{1}{J_n} \dot{\theta}_n + \left( \frac{K_s}{J_n} \right) \theta_n = \left( \frac{K_s}{J_n} \right) \left( \frac{r_m}{K_s R_T} \right) v_{in}
\]

For a constant input voltage, the steady-state angle (equilibrium) is found by making the derivative terms zero,

\[
\ddot{\theta}_n + \left( B_n + \frac{r_m^2}{R_T} \right) \frac{1}{J_n} \dot{\theta}_n + \frac{K_s}{J_n} \theta_n = 0 \quad \theta_n = \left( \frac{K_s}{J_n} \right) \left( \frac{r_m}{K_s R_T} \right) v_{in}
\]

Recall:

\[
\theta = \left[ \frac{90 \cdot \text{deg}}{15 \cdot \text{V}} \right] \cdot v_{in}
\]

⇒ \[\theta_{nss} = \left[ \frac{r_m}{K_s R_T} \right] v_{in} = K_{\theta/v} \cdot v_{in}\]

So, if you want a certain angle, you simply apply, \( v_{in} = K_{v/\theta} \theta_{desired} \)

Works well if parameters are known, remain constant, and dynamic effects are not significant.
Now write in **transfer function form**

Start in the standard form:
\[
\dot{\theta}_n + 2\zeta\omega_n \dot{\theta}_n + \omega_n^2 \theta_n = \omega_n^2 u(t)
\]

\[
u(t) = K_{\theta/v} v_{in}
\]

Transform to s-domain, and solve for angle-to-voltage relation:
\[
\frac{\theta_n}{v_{in}} = \frac{K_{\theta/v} \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]

You can see this model returns the static gain relation when you make \( s \) go to zero (i.e., steady-state).

So, all we need is the damping ratio and the natural frequency to parameterize this dynamic model. These can be found either from the physical parameters or by experimentally determining the values in the lab. We’ll do the latter.
Pre-Lab (Experimental modeling)

1. **Build a model simulation.** Use a LabVIEW simulation of the analog meter model in TF form. Apply a step voltage input and compare the static gain and the dynamic model. For static gain, use the value you measured in lab. Use a value of 0.2 for damping ratio and 30 rad/s for natural frequency.

2. **Plan experiments to identify model parameters.** Briefly sketch out steps that need to be taken in lab to estimate the system damping and natural frequency for your specific system. Depending on how well you determined the static gain, you may also think about what you can do to improve your estimate of this value.
You can construct a model in the Control & Simulation Loop using blocks, formula nodes, and MathScript, as shown before. But you can also construct transfer function (TF) models. Look under: Simulation->Continuous to find Transfer Function. You need to define the TF as shown on next slide.
The “CD Construct Transfer Function Model” can be used to define models of the form we’ve constructed:

$$\frac{\theta_n}{v_{in}} = \frac{K_{\theta/v} \omega^2_n}{s^2 + 2\zeta \omega_n s + \omega^2_n}$$

Define the coefficients in symbolic form in ascending order on front panel.

In the variables cluster, you can assign numerical values to each symbolic parameter that is used in the symbolic numerator and denominator expressions.
The Control Design VIs are found under CD&S menu.
Estimating damping and natural frequency from log decrement

\[ T_d = \frac{9.16 \text{ s} - 8.94 \text{ s}}{9.38 - 9.16} \cdot \text{s} = 0.22 \text{ s} \]

\[ \omega_d = \frac{2 \pi}{T_d} = 28.56 \frac{1}{\text{s}} \]  
**Damped natural frequency**

\[ Kr = \frac{1}{2} \left( \ln \left( \frac{30.26 - 21.38}{23.95 - 21.38} \right) + \frac{1}{2} \ln \left( \frac{30.26 - 21.38}{22.11 - 21.38} \right) \right) = 1.248 \]  
**estimate slope of log decrement from 2 points**

\[ \zeta = \frac{Kr}{2 \sqrt{\frac{1}{\omega_d^2} + 4 \pi^2}} = 0.19412 \]  
(from solving for zeta in log decrement relation)

\[ \omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}} = 29.118 \frac{\text{rad}}{\text{s}} \]  
**Natural frequency**

\[ f_n = \frac{\omega_n}{2 \pi} = 4.634 \text{ Hz} \]

\[ \frac{y_s - y_0}{y_0 - y_0} = \frac{e^{-\zeta \omega_n t}}{\sqrt{1 - \zeta^2}} \sin \left( \omega_n \sqrt{1 - \zeta^2} \cdot t + \tan^{-1} \left( \frac{\sqrt{1 - \zeta^2}}{\zeta} \right) \right) \]

\[ x_q(t) = 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1 - \zeta^2}} \sin \left( \omega_n \sqrt{1 - \zeta^2} \cdot t + \tan^{-1} \left( \frac{\sqrt{1 - \zeta^2}}{\zeta} \right) \right) \]

Sanity check - compare model step response using estimated parameters to measured data

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Feedback control concepts

• Review basic feedback control concept and benefits
• Describe PID concept
• Show how open loop control is represented as a feedforward control
• Discuss simulation and implementation of feedback and feedforward control using LabVIEW
• Discuss some issues that can arise, such as delay from the vision acquisition and processing, device communications, etc.
Summary of Control Task

- Identify variables to control
- Identify any disturbances
- Identify what inputs can be manipulated
- Choose sensor(s) – ideally for all controlled variables
- Choose actuator(s)
- Power conditioning
- Logic
- Tuning and stability analysis
Closed-Loop Feedback Control

A standard closed-loop control diagram:

**Plant** – any physical system to be controlled
**Controller** – can generate inputs to the plant to achieve a desired objective
**Sensor** – means by which plant output is transformed to feedback information
Effect of Closing the Loop

• Advantages:
  – Provides for disturbance rejection
  – Reduces sensitivity to parameter variations
  – Use error signal for dynamic tracking
  – Enhance accuracy, extend bandwidth, etc.

• Disadvantages:
  – Can lead to excess oscillation or instability
The closed-loop control TF

\[ c = \frac{G_c G_p}{1 + G_c G_p H} \cdot r + \frac{G_p}{1 + G_c G_p H} \cdot u_d \]

Using block diagram algebra, you can solve for output.

More opportunities to design how the system responds to the reference input \( (r) \) as well as to disturbances \( (u_d) \).
Why derive a closed-loop transfer function (TF)?

You can prove some of the properties we discussed.

1. First, show how the closed loop TF is derived
2. Use the TF to show effect on a closed loop controller output if there is a variation in the system parameters or a variation in the disturbance

These simple relations show that a controller relies on proper tuning of the controller. The mathematical models can be used for this purpose.
Basic control actions used in $G_c(s)$

- Proportional (P) control
- Integral (I) control
- Derivative (D) control
- Combination: PI, PD, PID

Most industrial controllers (well over 90 to 95%) you will run across will be of a PID type.
Example: proportional control

\[ E(s) \xrightarrow{G_c(s)} U(s) \]

\[ G_c(s) = K_p = \text{constant} \]

\[ \frac{X_c}{X_R} = \frac{G}{1+GH} = \frac{K_p G_p \cdot 1}{1+K_p G_p \cdot 1} = \frac{K_p}{ms^2 + bs + k + K_p} \]

Closed-loop

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Forming I, D, and PID

**Integral control:**
\[ u = \frac{K_I}{T_I} \int e \, dt \Rightarrow U(s) = \frac{K_I}{T_I s} E(s) \]

Integral control reduces or eliminates steady-state error, but has reduced stability.

**Derivative control:**
\[ u = K_D T_D \frac{de}{dt} \Rightarrow U(s) = K_D T_D s E(s) \]

Derivative control yields an increase in effective damping, improving stability.

**PID control:**
\[ U(s) = K \left( 1 + \frac{1}{T_I s} + T_D s \right) E(s) \]

or,
\[ U(s) = \left( K_p + \frac{K_I}{s} + K_D s \right) E(s) \]

The basic idea is to tune the controller by choosing the gains so specifications are met.
Feedforward Control

• One of the key attributes feedback control adds to a system is the ability to diminish the influence of disturbances.
• If you know or measure the disturbances before they influence a system, it might be possible to nullify the effect on system performance.
• One way of doing this is to use feedforward control.
Feedback with Feedforward

\[ R(s) \xrightarrow{+} E(s) \xrightarrow{+} G_c(s) \xrightarrow{+} U(s) \xrightarrow{+} G_p(s) \xrightarrow{} C(s) \]

Feedforward

\[ G_{FF}(s) \]

Disturbances

\[ D(s) \xrightarrow{} G_p(s) \]

\[ Y(s) \xrightarrow{+} H(s) \xrightarrow{} \]

Control
Stability

- The stability of a system refers to its ability or tendency to seek a condition of static equilibrium after it has been disturbed.

- If given a small perturbation from the equilibrium, it is stable if it returns.
3. **Simulate feedback.** Use the model of the meter movement to **simulate a model** of closed-loop feedback control using the Control and Simulation Module. The idea is to simulate the system operating with various types of reference inputs. Start with zero and introduce step input changes to test the response. Experiment with the PID gains. Try to determine the PID gains that will give minimal overshoot.
Direct feedback from the TF output represents perfect measurement of the angle
What to do with a model

• Use the model developed and add a control system to experiment with the parameters, study the response, and come up with possible design solutions

• You can define critical metrics to quantify the performance, and justify building the actual system

• Simulations might reveal issues with practical implementation: does it take too much power? can you actually measure the variable you need to feed back, etc.
The PID VI in LabVIEW

We can simulate instead!

**PID Academic**

\[
\frac{U(s)}{E(s)} = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{T_f s + 1}\right)
\]

**PID Parallel**

\[
\frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + \frac{K_d s}{T_f s + 1}
\]
Influence of time-delay

In our laboratory system, a USB web-cam is used to provide position feedback, so in this case we can have a delay in the feedback. This means we can model the measurement of the meter needle by,

\[ \theta(t - \tau_d) \]

where \( \tau_d \) is the delay time in seconds.

Delays can be represented by the transfer function,

\[ G_d(s) = e^{-\tau_d s} \]

Time delays arise in many practical systems due to processes, hardware, sensing, and also if you have human-in-the-loop actions.
Simulating a time delay

You can model the delay using a transport delay in the current analog meter simulation model.

Getting control systems to work with time delays can be troublesome because it turns out that this effect can cause the system to become unstable.

Using a simulation model can provide some insight for tuning your control system. Often you will find that you must ‘dial back’ your gains to make a system stable when there is a delay introduced.
Pre-Lab: Simulate effect of time-delay

4. **Simulate feedback with influence of time delay.** Experiment with a time delay, which is expected due to vision, computing, etc. You can study this effect in the simulation by using a “transport delay” VI (under Continuous VIs in the simulation pallet). Assume that the time delay might vary from 1/10 to 1/30 seconds. It should be found that significantly different controller gains are needed than when there is no delay. Find gains for PID that will given minimal overshoot and fastest response time.
Adding a transport delay to simulate time delay in the measurement feedback
Laboratory Objectives

1. Conduct feedforward experiments if these were not completed last week.
2. Implement feedback control for analog meter position. Test your feedback control using a constant reference angle. Demonstrate that you can make changes of about +/-20 degrees, for example, and use these tests to tune the controller. Make a note of how effectively your simulations enabled you to narrow in to a proper range of values for proportional and integral gain.
3. Implement a signal to track using the signal generator VI and demonstrate that you can
   a. Stabilize the needle position at a value of 10 degrees
   b. Introduce tracking of a reference signal. First use a 0.1 Hz sinusoidal signal about the 10 degree position. Use an amplitude of 5 degrees and 8 degrees (peaks 5 or 8 above and below 10 degree position).
   c. Switch the tracking signal from a sinusoidal signal to a square wave, and repeat the test in 'b'.
4. The TA may also evaluate the ability of your controller to: a) regulate position when there are changes in system parameters, and/or b) given external disturbances.
Laboratory Evaluation

1. Summarize your feedback control implementation. Report on how well your simulations supported your lab work; i.e., were you able to implement and tune the feedback controller in a reasonable amount of time, and did the results match. Compare the controller gain values and the simulation and lab results.

2. Report on results from testing the controller when changes were made to the system parameters and/or given external disturbances.

3. Summarize the experience with using open-loop feedforward control versus feedback control. Discuss how effectively you were able to demonstrate each case. If results were not conclusive, what did you determine were the main obstacles that prevented your group from completing the work? How would you improve the laboratory protocol?
Summary comments:

1. Measuring damping ratio and natural frequency will help determine how to improve the response.
2. Make sure to check static gain using a dc voltage input. Is a linear model valid?
3. Test open loop FF with step inputs. Use a pre-filter to shape the control input.
4. Combine FF with PID FB if the FF works well but there is steady-state error. The integral control can close up the steady-state error. A FB controller may not work well with FF if the FF is not pre-filtered and the controller/actuator cannot update fast enough to damp the overshoot. It may be better to try FB alone. This could be the case with the myDAQ which may have slow response combined with vision. Results might be better with a different controller (e.g., myRIO).
5. Another reasonable solution is to use PID FB alone and tune to get an overdamped response – if your goal is to have no overshoot. You may try comparing to simulation results, but how well it compares will depend on whether you modeled the system and any delays accurately.