Inventory Theory.S6
Variations on the \((s, Q)\) Model

Reorder Point Based on Inventory Position

In the foregoing, we have assumed that a replenishment order is to be placed whenever the inventory level reaches the reorder point. A more practical idea is to use the *inventory position* rather than the inventory level as an indicator. The inventory position is the inventory level plus the quantity on order. The difference is illustrated in Fig. 11. We note that inventory level is the same as inventory position when there are no outstanding orders. In the early cycles of the figure, the inventory level crosses the reorder point at the same time as the inventory position, and the same order pattern is obtained using either measure. Basing the order on the inventory level fails, however, when there is a lead time demand larger than the order quantity, as in the last cycle of the figure. In this case the inventory level falls below the reorder point and never reaches it again. Using the inventory position, however, allows two orders to be placed in quick succession, thus keeping the inventory in control.

![Diagram illustrating inventory position vs. inventory level](image)

Figure 11. Using Inventory Position as a Measure for Placing Orders

Using the inventory position in this manner, also allows us to drop the requirement that the lot size be very much greater than the average demand during the lead time. The results in the table can be used even in cases where the lot size is small in relation to the lead time demand. The primary assumption for the
derivations is that the probability of a stockout be small. This probability depends on the reorder point and not the lot size.

When the lot size is small, there may be many outstanding orders at any given time, emphasizing the need to track the inventory position. A particularly interesting case is when the lot size is 1. This implies that a replenishment order is placed whenever an item is withdrawn from inventory.

**Discrete Demand During the Lead Time**

The results of the table were derived for continuous distributions. In fact the items in an inventory are usually discrete, and a discrete demand distribution may be more appropriate. This is particularly true when the reorder point is relatively small. For the discrete distribution \( p(x) \) is the probability that the random demand during the lead time takes the value \( x \). \( F(x) \) is the probability that the demand is less than or equal to \( x \). The expected shortage and expected unit-time shortage are

\[
E_s = \sum_{x=s+1}^{\infty} (x-s)p(x) \quad (51)
\]

\[
T_s = \frac{1}{2a} \sum_{x=s+1}^{\infty} (x-s)^2p(x) \quad (52)
\]

**Table 2. The \((s, Q)\) Policy for Discrete Distributions**

<table>
<thead>
<tr>
<th>Situation</th>
<th>( C_s )</th>
<th>\textit{Optimum reorder point}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Cost per Stockout (( \pi_1 ))</td>
<td>( \pi_1 [1 - F(s)] )</td>
<td>( p(s^* - 1) &gt; \frac{hQ}{\pi_1 a} \geq p(s^*) ).</td>
</tr>
<tr>
<td>Charge per Unit Short (( \pi_2 ))</td>
<td>( \pi_2 E_s )</td>
<td>( F(s^<em>) \leq 1 - \frac{hQ}{\pi_2 a} &lt; F(s^</em>+1) )</td>
</tr>
<tr>
<td>Charge per Unit Short per Unit Time (( \pi_3 ))</td>
<td>( \pi_3 T_s )</td>
<td>( E_s(s^<em>-1) &gt; \frac{hQ}{\pi_3} \geq E_s(s^</em>)) )</td>
</tr>
<tr>
<td>Charge per Unit of Lost Sales (( \pi_L ))</td>
<td>( \pi_L E_s )</td>
<td>( F(s^<em>) \leq 1 - \frac{hQ}{hQ + \pi_L a} &lt; F(s^</em>+1) )</td>
</tr>
</tbody>
</table>

**Lead Time a Random Variable**

Previously we assumed that lead time is a constant. Indeed this is a very desirable characteristic of an inventory system. The lead time may actually be uncertain in duration due to variability in shipping times, material availability and supplier processing times.
Let lead time be a random variable $Y$ with p.d.f. $h(y)$, and let demand be the random variable, $X$, with p.d.f. $g(x, y)$. The demand distribution depends on the lead time. The distribution of demand during the lead time is\textsuperscript{1}

$$f(x) = \int_{0}^{\infty} g(x, y) h(y) dy.$$  \hfill (53)

This p.d.f. can then be used in the relations of Table 1 to determine approximate solutions.

\textsuperscript{1}Hadley and Whitten (1963) pg. 165.