1. **(40 pts)** A gas turbine powerplant generates electricity for a college campus, and the hot exhaust gas leaving the turbine is used to "cogenerate" steam for heating the buildings. The plant utilizes a simple Brayton cycle with 100 m$^3$/s of air entering the compressor at 20 °C and 100 kPa. The air is compressed to 700 kPa in an 85% efficient adiabatic compressor, and after passing through the combustor enters the adiabatic turbine at 1600 K and 700 kPa. The turbine, whose efficiency is 90%, expands the air to a pressure of 150 kPa, where it enters the steam generator (SG).

The steam generator is a large heat exchanger in which the hot turbine exhaust gives up energy to the feedwater; the air leaves the SG at 150 °C and 100 kPa. Feedwater enters the SG at 15 °C and 200 kPa, while saturated vapor at the same pressure leaves the SG. Determine:

(a) The net power output of the powerplant (kW) and its thermal efficiency. For properties at State 3, use $c_p=1.33$ kJ/kg-K and $k=1.274$. If time permits, explain how you could determine these values if they were not given to you (3 bonus points).

(b) The temperature of the air leaving the turbine (i.e., entering the steam generator);

(c) The mass flow rate of steam that can be produced in the steam generator (kg/hr). Use the average of the inlet and outlet temperatures to determine specific heat of the air in the SG.

You may assume that the air behaves as an ideal gas and determine the specific heats for each component at the entering temperature for that component, except as noted. Use the notation shown in the diagram below to designate states.
2. (10 pts)

(a) When the control on an electric heater is set to "low", the metal heating element glows a dull red, and when set on "high" the element glows bright yellow. Explain the difference in appearance of the heating element (both color and intensity) based on the blackbody radiation spectrum and the color/wavelength characteristics of visible radiation. Is all of the radiation emitted by the element for the low setting at one wavelength ("red") and at a different ("yellow") wavelength for the high setting? Explain.

(b) A large surface A has a radiation view factor $F_{A\rightarrow B} = 0.3$ relative to a small surface B some distance away. Is the view factor $F_{B\rightarrow A}$ also = 0.3? Explain why or why not.

3. (30 pts) An underwater power cable consists of a copper conductor 2 cm in diameter covered with three protective layers: a 1 cm thick layer of Teflon insulation, a 0.2 cm thick braided stainless steel jacket (AISI 304), and a 0.5 cm thick hard rubber water seal. Water at 27 °C is flowing over the outside of the cable at a velocity of 1.5 m/s.

(a) Determine the convective heat transfer coefficient, $h_{\text{conv}}$ on the outside of the cable. You may use Table A-18 to determine the properties of water at 300 K (note water temperatures in the table are in K, not °C).

(b) The maximum service temperature for Teflon insulation is 250 °C. If the resistance of the copper conductor is $0.7 \times 10^{-4}$ ohms/m, determine the maximum DC current (amps) that the cable can carry without damaging the insulation. Thermal properties for copper, Teflon, 304 stainless steel and hard rubber can be taken at 300 K from Tables A-14 and A-16c. For convenience, assume a 1 meter length of cable.

4. (20 pts) A circular hot-plate 0.25 m in diameter is placed face-up in a room whose ambient temperature (of both air and walls) is 24 °C. If the emissivity of the plate is 0.9, what is the required power input (watts) to maintain the surface at a temperature of 130 °C?

N.B.: Be sure to consider both convection and radiation, and be careful in how you define the characteristic length $\delta$ in determining dimensionless parameters.
1. Compressor: \[ \omega_c = h_2 - h_1 = C_p (T_2 - T_1) \]
\[ C_p = 1.005 \text{ kJ/kgK}, \quad k = 1.4 \text{ at } 20^\circ \text{C} \]

2. NKEPE

3. IG, Ideal

4. compressor:
\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 1.745 \quad \Rightarrow \quad T_2 = 511 \text{ K} \]

\[ \omega_{c_2} = 1.005 (511 - 293) = 219.3 \text{ kJ/kg} \]

\[ \omega_{c_a} = \frac{219.3}{1.4} = 157 \text{ kJ/kg} = C_p (T_{2a} - T_1) \]

\[ T_{2a} = 550 \text{ K} \]

Conductor:
\[ \frac{T_1}{T_H} = \frac{h_2 - h_1}{C_p (T_3 - T_2)} \]

From Table A-2, \( C_p = 1.04 \text{ at } 550 \text{ K} \)
\[ \frac{T_1}{T_H} = 1.04 (1600 - 550) = 1092 \text{ kJ/kg} \]

Turbine:
\[ \omega_{t_a} = C_p (T_3 - T_{4a}) = \frac{\omega_{c_a}}{C_p (T_5 - T_{4a})} \]

Need \( C_p \) & \( k \) at 1600 K (beyond range of A-2)

Use A-2c:
\[ C_p = 28.11 + 0.1967 \times 10^{-2} (1600) + 0.48 \times 10^{-5} (1600)^2 \]
\[ - 1.96 \times 10^{-7} (1600)^3 = 38.64 \text{ kJ/kgK} \]

\[ C_v = C_p - R = 30.32 \text{ kJ/kgK} \]

\[ C_p = \frac{38.64}{28.97} = 1.33 \text{ kJ/kgK} \]

\[ k = \frac{C_p}{C_v} = 1.274 \]

Isentropic process:
\[ \frac{T_{4a}}{T_3} = \left( \frac{P_4}{P_3} \right)^{\frac{k-1}{k}} = \left( \frac{150}{700} \right)^{0.215} \quad \Rightarrow \quad T_{4a} = 1149 \text{ K} \]

\[ \omega_{t_a} = [1.33 (1600 - 1149)] \times 0.90 = 539.8 \text{ kJ/kg} \]
(a) \( W_{\text{net}} = \dot{m} \left( w_{f_a} - w_c \right) \)

\[
M = \frac{V_i}{\dot{V}_i}
\]

Use IG law to find \( \dot{V}_i \):

\[
\dot{V}_i = \frac{RT_i}{P_i} = \left( \frac{8.3144 \text{ kJ/kmol K}}{28.97 \text{ kJ/kmol}} \right) \left( \frac{293 \text{ K}}{100 \text{ kJ/m}^2} \right)
\]

\[\dot{V}_i = 0.84 \text{ m}^3/\text{kg} \]

\[\dot{m} = \left( 100 \text{ m}^3/\text{s} \right) \left( 0.84 \text{ m}^3/\text{kg} \right) = 119 \text{ kg/s} \]

\[W_{\text{net}} = 119 \left( 539.8 - 258 \right) = 33,534 \text{ kW} \]

\[\eta_{th} = \frac{W_{\text{net}}}{\dot{q}_h} = \frac{539.8 - 258}{1092} \]

\[\eta_{th} = 0.258 = 25.8\% \]

(b) Turbine exhaust temperature \( T_{4_a} \):

\[w_{f_a} = 539.8 = c_p \left( T_3 - T_{4_a} \right) \Rightarrow T_{4_a} = 1194 \text{ K} \]

\[= 921^\circ \text{C} \]

(c) Energy balance on \( \leq \Sigma G \):

\[\dot{m}_a (h_4 - h_5) + \dot{m}_w (h_2 - h_1)_{sg} = 0 \]

\[\dot{m}_a c_p \left( T_4 - T_5 \right) \]

\[\dot{m}_w = \frac{M_w c_p \left( T_4 - T_5 \right)}{(h_2 - h_1)_{sg}} \]

Take \( c_p \) at \( \left( \frac{T_4 + T_5}{2} \right) = 809 \text{ K} \) \( \Rightarrow c_p \approx 1.10 \text{ kJ/kg K} \)

\[h_{1_{sg}} \approx h_f @ 15^\circ \text{C} = 62.99 \text{ kJ/kg} \]

\[h_{2_{sg}} = h_g @ 200 \text{ kPa} = 2706.7 \text{ kJ/kg} \]

\[M_w = \frac{\left( 119 \text{ kg/s} \right) \left( 1.10 \right) \left( 1194 - 423 \right)}{\left( 2706.7 - 62.99 \right)} = 38.18 \text{ kg/s} = 137,430 \text{ kcal/hr} \]
(2) The wavelength for peak radiation from a blackbody is shorter for higher temperatures, and the intensity is much greater \((\propto T^4)\). So on high settings, it is higher emission peak at shorter wavelength (yellow in visible spectrum).

At any temperature, blackbody emits over infinite range of wavelength, not just single wavelength.

(b) It is not true that \(F_{AB} = F_{BA}\) because the objects are not of the same size (i.e., area). However, the reciprocity relation says \(A_{A} F_{AB} = A_{B} F_{BA}\); so if the areas were known, \(F_{BA}\) could be determined.
(a) To determine heat, need to first determine $Re_p = \frac{VD}{\mu}$ & $Pr$

For $H_2O$ at 300 K: $\nu = 0.86 \times 10^{-6}$ m$^2/s$, $Pr = 5.88$, $k = 0.608$ W/mK

Diameter of cable $D = 2(1+1+0.2+0.5) = 5.4$ cm

$Re_p = \frac{1.5 \text{ m/s})(5.4 \times 10^{-2} \text{ m})}{0.86 \times 10^{-6} \text{ m}^2/\text{s}} = 94,186$

From Table 10-3, for $40,000 < Re_p < 400,000$

$Nu_p = 0.027 \frac{Re_p^{1/3}}{Pr} = 49.16 \frac{hD}{k}$

$\eta = \frac{(49.16)(0.608 \text{ W/mK})}{(5.4 \times 10^{-2} \text{ m}) \text{ conv}} = 5535 \text{ W/mK}$

(b) Thermal resistance network:

\[ T_1 \xrightarrow{R_{Tet}} T_2 \xrightarrow{R_{SS}} T_3 \xrightarrow{R_{Rub}} T_4 \xrightarrow{R_{conv}} T_0 \]

Conduction resistances: Assume 300K for properties; all $K$'s in W/mK

$K_{Tet} = 0.35$, $K_{Rub} = 0.16$, $K_{SS} = 14.9$

For 1 m length of cable:

$R_{Tet} = \frac{ln(1.5/1.3)}{2\pi (1.3) K_{Tet}} = \frac{ln(2/1)}{2\pi (0.35)} = 0.315 \text{ oc/watt}$

$R_{SS} = \frac{2\pi (14.9)}{2\pi (2.2/2)} = 0.001 \text{ oc/watt}$

$R_{Rub} = \frac{2\pi (2.7/2.2)}{2\pi (0.16)} = 0.204 \text{ oc/watt}$

$R_{conv} = \frac{1}{hA} = \frac{(5535)(2\pi \times 2.7 \times 10^{-2} \times 1 \text{ m})}{1 \text{ m}} = 0.001 \text{ oc/watt}$
Key - Problem 3

\[ Q = I^2 R = \sum R = \frac{(T_T - T_0)}{0.521 \, ^\circ C/W} \]

= 428 watts (per 1 m length)

\[ R = 0.7 \times 10^{-4} \, \Omega/m \]

\[ I^2 = \frac{428}{0.7 \times 10^{-4}} = 6.115 \times 10^6 \]

\[ I = 2473 \, \text{amps} \]
Natural convection: Need first to find $Gr$. For properties, use $T_f = \frac{130+24}{2} = 77°C$

Table A-19 for air @ 350 K:

$\nu = 2.06 \times 10^{-5} \text{ m}^2/\text{s}$, $Pr = 0.706$, $k = 0.0297 \text{ W/m}$

$B = \frac{1}{T} = 0.00286/\text{K}$

$Ra = \frac{g \beta (T_s - T_\infty) B}{Pr}$

$Ra = \frac{(9.8 \text{ m/s}^2)(0.025 \text{m}^3)(106 \text{K})(0.00286/\text{K})}{(2.06 \times 10^{-5} \text{ m}^2/\text{s})^2} = \frac{(0.706)}{0.0625 \text{m}^2/\text{s}^2}$

$Ra = 1.207 \times 10^6$

From Table 11-1, for an upward-facing plate with $Ra < 10^7$, $Nu = 0.54 Ra^{1/4}$

$Nu = 0.54 (1.207 \times 10^6)^{0.25} = 17.9$

$h = \frac{Nu k}{\delta} = \frac{17.9 (0.0297)}{0.0625} = 8.5 \text{ W/m}^2\text{K}$

$Q_{conv} = h A (T_s - T_\infty) = (8.5) \left( \frac{\pi (0.25)^2}{4} \right) (130 - 24)$

$Q_{conv} = 44.2 \text{ W}$

Radiation: $Q_{rad} = \sigma A (T_s^4 - T_\infty^4)$

$Q_{rad} = (5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4)(0.9) \left( \frac{\pi (0.25)^2}{4} \right) (403^4 - 297^4) \text{ K}$

$Q_{rad} = 46.6 \text{ W}$

$Q_{total} = Q_{conv} + Q_{rad} = 90.8 \text{ W}$