Problem 2.116  At $t = 0$ a car starts from rest at point A. It moves toward the right and the tangential component of its acceleration is $a_t = 0.4t \text{ m/s}^2$. What is the magnitude of the car's acceleration when it reaches point B?

Solution  The velocity of the car along the path as a function of time is $v(t) = \int_0^t a_t \, dt = 0.2t^2$, since the car starts from rest. The distance traveled from point A is $s = \int_0^t v(t) \, dt = \frac{0.2}{3} t^3$. The distance is known from the sketch, $s = 200 + R\theta = 200 + 50\pi/2 = 278.54 \text{ m}$.

The time of travel is $t = \left(\frac{3s}{0.2}\right)^{1/3} = 16.11 \text{ s}$. The tangential acceleration at point B is $\ddot{a}_t = \left[0.4\ddot{g}\right]_{t=16.11} = 6.44\ddot{j} \text{ m/s}^2$. The velocity at point B is $\ddot{v} = \left[0.2t^2\right]_{t=16.11} = 51.88\ddot{j} \text{ m/s}$. The normal acceleration at point B is $\ddot{a}_n = \frac{\ddot{v}}{R} = -\frac{(51.88)^2}{50} = 53.83\ddot{i} \text{ m/s}^2$. The magnitude of the acceleration at point B is $|\ddot{a}| = \sqrt{6.44^2 + 53.83^2} = 54.22 \text{ m/s}^2$.

Problem 2.117  A group of engineering students constructs a sun-powered car and tests it on a circular track of 1000 ft radius. If the car starts from rest and the tangential acceleration component of its acceleration is given in terms of the car's velocity as $a_t = 2 - 0.1v \text{ ft/s}^2$. Determine $v$ and the magnitude of the car's acceleration 15 s after it starts.

Solution:  The acceleration is given by $a_t = \frac{dv}{dt} = 2 - 0.1v \text{ ft/s}^2$. Setting up the integral, we get $\int_0^v \frac{dv}{2 - 0.1v} = \int_0^t dt$. Integrating, we have $-\left(\frac{1}{0.1}\right)\ln(2 - 0.1 v) + \left(\frac{1}{0.1}\right)\ln(2) = 15$. Solving for $v$, we get $v = 15.54 \text{ ft/s}$. Substituting this back into the equation for the acceleration, we get $a_t = 0.446 \text{ ft/s}^2$. In the radial direction, $a_n = v^2 / r = (15.54)^2 / 1000 = 0.241 \text{ ft/s}^2$. The total acceleration is then given by $|\ddot{a}| = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.241)^2 + (0.446)^2} = 0.507 \text{ ft/s}^2$.

Problem 2.118  Suppose that the tangential component of acceleration in Problem 2.117 is $a_t = 2 - 0.008s \text{ ft/s}^2$, where $s$ is the distance the car travels along the track from the point where it starts from rest. Determine the velocity $v$ and the magnitude of the car's acceleration when it has traveled a distance $s = 100 \text{ ft}$.

Solution:  The acceleration is given by $a_t = \frac{dv}{dt} = v \frac{dv}{ds} = 2 - 0.008s \text{ ft/s}^2$. Setting up the integral, we get $\int_0^s v \, dv = \int_0^t (2 - 0.008s) \, ds$. Integrating, we get $\frac{v^2}{2} = \left(2s - 0.008 \frac{s^2}{2}\right)$. Evaluating at $s = 100 \text{ ft}$, we get $v = 17.89 \text{ ft/s}$. Substituting $s = 100 \text{ ft}$ into the acceleration relationship, we get $a_t = 1.2 \text{ ft/s}^2$. In the radial direction, $a_n = v^2 / r = (17.89)^2 / 1000 = 0.320 \text{ ft/s}^2$. The total acceleration is then given by $|\ddot{a}| = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.320)^2 + (1.2)^2} = 1.24 \text{ ft/s}^2$.
Problem 2.121  An astronaut candidate is to be tested in a centrifuge with a radius of 10 m. He will lose consciousness if his total horizontal acceleration reaches 14 g's. What is the maximum constant angular acceleration of the centrifuge, starting from rest, if he is not to lose consciousness within one minute?

Solution  The angular velocity is $\omega(t) = \alpha t$ for constant acceleration from rest. The tangential acceleration at the astronaut is $a_t = R\alpha$. The normal acceleration is $a_n = R\alpha^2 = R\alpha^2 t^2$. The magnitude of the acceleration is $|\ddot{a}| = \sqrt{R^2\alpha^2 + R^2\alpha^4 t^4} = 14(9.81) \text{ m/s}^2$. Square both sides and reduce: $R^2\alpha^4 + R^2\alpha^2 - 18862.3 = 0$. In canonical form: $\alpha^4 + 2b\alpha^2 + c = 0$, where $b = 3.858 \times 10^{-8}$ and $c = -1.455 \times 10^{-5}$. The solution is $\alpha^2 = \pm 0.003815$, from which $\alpha = \sqrt{0.003815} = 0.06177 \text{ rad/s}^2$ is the maximum constant acceleration from rest allowed.

Problem 2.122  After first-stage separation and before the second-stage engines have fired, a rocket is moving at $v = 3000 \text{ m/s}$ and the angle between its velocity vector and the vertical is 60°. Because aerodynamic forces are negligible, the rocket’s acceleration is that due to gravity, which is $9.50 \text{ m/s}^2$ at the rocket’s altitude. Determine (a) the normal and tangential components of the rocket’s acceleration, and (b) the instantaneous radius of curvature of the rocket’s path.

Solution:

The components of the acceleration are $a_T = g \cos(60°)$ toward the rear of the rocket, and $a_N = g \sin(60°)$ normal to the axis of the rocket directed 30 degrees away from straight down.

The normal acceleration is also given by $a_N = v^2 / r$, where $r$ is the radius of curvature of the path.

Substituting $g = 9.50 \text{ m/s}^2$ and $v = 3000 \text{ m/s}$ into these relations, we get $a_T = 4.75 \text{ m/s}^2$, $a_N = 8.23 \text{ m/s}^2$, and $r = 1094 \text{ km}$. 

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Problem 2.141  The radial line rotates with a constant angular velocity of 2 rad/s. Point P moves along the line at a constant speed of 4 m/s. Determine the magnitude of the velocity and acceleration of P when r = 2.

Solution  The angular velocity of the line is \( \frac{d\theta}{dt} = \omega = 2 \text{ rad/s} \), from

\[ \frac{d^2\theta}{dt^2} = 0. \]

The radial velocity of the point is \( \frac{dr}{dt} = 4 \text{ m/s} \), from

\[ \frac{d^2r}{dt^2} = 0. \]

The vector velocity is \( \vec{v} = \left( \frac{dr}{dt} \right) \vec{e}_r + r \left( \frac{d\theta}{dt} \right) \vec{e}_\theta = 4 \vec{e}_r + 4 \vec{e}_\theta \). The magnitude is

\[ |\vec{v}| = \sqrt{4^2 + 4^2} = 5.66 \text{ m/s} \]

The acceleration is \( \vec{a} = [-2(4)\vec{e}_r + (2)(2)\vec{e}_\theta] = -8\vec{e}_r + 16\vec{e}_\theta \). The magnitude is

\[ |\vec{a}| = \sqrt{8^2 + 16^2} = 17.89 \text{ m/s}^2 \]

Problem 2.142  At the instant shown, the coordinates of the slider A are \( x = 1.6 \text{ ft}, y = 1.0 \text{ ft} \), and its velocity and acceleration are \( \vec{v} = 10\hat{j} \text{ ft/s} \) and \( \vec{a} = -32.2\hat{j} \text{ ft/s}^2 \). Determine the slider’s velocity and acceleration in terms of polar coordinates.

Solution:  The equation relating the unit vectors in polar coordinates to those in cartesian coordinates are \( \vec{e}_r = \cos \theta \hat{i} + \sin \theta \hat{j} \) and \( \vec{e}_\theta = -\sin \theta \hat{i} + \cos \theta \hat{j} \). The inverse relationships are \( \hat{i} = \cos \theta \vec{e}_r - \sin \theta \vec{e}_\theta \) and \( \hat{j} = \sin \theta \vec{e}_r + \cos \theta \vec{e}_\theta \).

The angle \( \theta \) is determined from \( \tan \theta = \frac{y}{x} = \frac{1.0}{1.6} \). Hence, \( \theta = 32.01^\circ \). Then, in the general case,

\[ v_r = v_x \cos \theta + v_y \sin \theta \quad \text{and} \quad v_\theta = -v_x \sin \theta + v_y \cos \theta \]

Similar equations hold for the accelerations.

Substituting in numbers, and noting that \( v_x = a_x = 0 \), we get \( v_r = 5.30 \text{ ft/s}, \quad v_\theta = 8.48 \text{ ft/s}, \quad a_r = -17.07 \text{ ft/s}^2 \), and \( a_\theta = -27.31 \text{ ft/s}^2 \)

Problem 2.143  In Problem 2.142, determine \( \frac{d^2r}{dt^2} \) and \( \frac{d^2\theta}{dt^2} \).

Solution:  The total radial acceleration is \( a_r = \frac{d^2r}{dt^2} - r \left( \frac{d\theta}{dt} \right)^2 \) and the total circumferential acceleration is

\[ a_\theta = r \frac{d^2\theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt}. \]

The velocity relationships are \( v_r = \frac{dr}{dt} \) and \( v_\theta = r \frac{d\theta}{dt} \). From Problem 2.142, we have that \( v_r = 5.30 \text{ ft/s}, v_\theta = 8.48 \text{ ft/s}, a_r = -17.07 \text{ ft/s}^2 \), and \( a_\theta = -27.31 \text{ ft/s}^2 \). Also, note that

\[ r = \sqrt{x^2 + y^2} = 1.89 \text{ ft}. \]

From the equations for the velocity components, we get \( \frac{dr}{dt} = v_r = 5.30 \text{ ft/s} \) and \( r \frac{d\theta}{dt} = v_\theta = 8.48 \text{ ft/s} \). Since we know \( r \), we can determine \( \frac{d\theta}{dt} = 4.49 \text{ rad/s} \). We now go to the acceleration equations. We know everything in the \( a_r \) equation except \( \frac{d^2r}{dt^2} \) and we know everything in the \( a_\theta \) equation except \( \frac{d^2\theta}{dt^2} \). Evaluating these, we get \( \frac{d^2r}{dt^2} = 21.0 \text{ ft/s}^2 \) and \( \frac{d^2\theta}{dt^2} = -39.6 \text{ rad/s}^2 \)

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Problem 2.161  A charged particle P in a magnetic field moves along the spiral path described by \( r = 1 \, m \), \( \theta = 2z \, \text{rad} \), where \( z \) is in meters. The particle moves along the path in the direction shown with constant speed \( \vec{v} = 1 \, \text{km} / \text{s} \). What is the velocity of the particle in terms of cylindrical coordinates?

Solution  The radial velocity is zero, since the path has a constant radius.

The magnitude of the velocity is \( v = \sqrt{r^2 \left( \frac{d\theta}{dt} \right)^2 + \left( \frac{dz}{dt} \right)^2} = 1000 \, \text{m/s} \). The angular velocity is

\[
\frac{d\theta}{dt} = 2 \frac{dz}{dt}.
\]

Substitute: \( v = \sqrt{r^2 \left( \frac{d\theta}{dt} \right)^2 + \frac{1}{4} \left( \frac{d\theta}{dt} \right)^2} = \left( \frac{d\theta}{dt} \right) \sqrt{r^2 + \frac{1}{4}} = \sqrt{1.25} \), from which

\[
\frac{d\theta}{dt} = \frac{1000}{\sqrt{1.25}} = 894.4 \, \text{rad/s},
\]

from which the transverse velocity is \( v_\theta = r \left( \frac{d\theta}{dt} \right) = 894.4 \, \text{m/s} \). The velocity along the cylindrical axis is \( \frac{dz}{dt} = \frac{1}{2} \left( \frac{d\theta}{dt} \right) = 447.2 \, \text{m/s} \). The velocity vector:

\[\vec{v} = 894.4 \vec{e}_\theta + 447.2 \vec{e}_z\]

Problem 2.162  Two cars approach an intersection. Car A is going 20 m/s and is accelerating at 2 m/s\(^2\), and car B is going 10 m/s and is decelerating at 3 m/s\(^2\). In terms of the earth fixed coordinate system shown, determine the velocity of car A relative to car B and the velocity of car B relative to car A.

Solution  The velocity of car A is . The velocity of car B is \( \vec{v}_B = \vec{j} \, 10 \, \text{m/s} \). The relative velocity is

\[\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B = -\vec{i} \, 20 - \vec{j} \, 10 \, \text{m/s} \]

\[\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = 20 \vec{i} + 10 \vec{j}\]

Problem 2.163  In Problem 2.162, determine the accelerations of car A relative to car B, and the accelerations of car B relative car A.

Solution  Car A is accelerating in the positive x direction; car B is accelerating in the negative y direction:

\[\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B = 2 \vec{i} - (-3 \vec{j}) = 2 \vec{i} + 3 \vec{j} \, \text{m/s}^2\]

\[\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A = -2 \vec{i} - 3 \vec{j} \, \text{m/s}^2\]
Problem 2.169  The bar rotates about the fixed point O with a constant angular velocity of 2 rad/s. Point A moves outward along the bar at a constant rate of 100 mm/s. Point B is a fixed point on the bar. In terms of the nonrotating reference frame with origin O, what is the magnitude of the velocity of point A relative to point B?

Solution  Use polar coordinates: The radial velocities:
\[
\vec{v}_A = \frac{dr}{dt} \hat{e}_r + r \frac{d\theta}{dt} \hat{e}_\theta = 100 \hat{e}_r + 80(2) \hat{e}_\theta = 100 \hat{e}_r + 160 \hat{e}_\theta \quad (mm/s)
\]
\[
\vec{v}_B = r \frac{d\theta}{dt} = 200(2) \hat{e}_\theta = 400 \hat{e}_\theta \quad mm/s. The relative velocity in polar coordinates: \( \vec{v}_{A/B} = \vec{v}_A - \vec{v}_B = 100 \hat{e}_r - 240 \hat{e}_\theta \quad (mm/s) \). The magnitude of the relative velocity: \( |\vec{v}_{A/B}| = \sqrt{100^2 + (240)^2} = 260 \frac{mm}{s} \)

Problem 2.170  In Problem 2.169, what is the magnitude of the acceleration of point B relative to point A at the instant shown?

Solution  Use polar coordinates, the radial accelerations:
\[
\vec{a}_A = \left( \frac{d^2 r}{dt^2} - r \left( \frac{d\theta}{dt} \right)^2 \right) \hat{e}_r + \left( r \frac{d^2 \theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right) \hat{e}_\theta = -80(4) \hat{e}_r + 2(100)2 \hat{e}_\theta.
\]
\[
\vec{a}_B = \left( \frac{d^2 r}{dt^2} - r \left( \frac{d\theta}{dt} \right)^2 \right) \hat{e}_r + \left( r \frac{d^2 \theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right) \hat{e}_\theta = -200(4) \hat{e}_r. The relative acceleration is.
\]
\[
\vec{a}_{B/A} = (-800 + 320) \hat{e}_r - (400) \hat{e}_\theta. The magnitude is \( |\vec{a}_{B/A}| = \sqrt{480^2 + 400^2} = 625 \frac{mm}{s^2} \)

Problem 2.171  The bars OA and OB are each 400 mm long and rotate in the x-y plane. OA has a counterclockwise angular velocity of 10 rad/s and a counterclockwise angular acceleration of 2 rad/s^2. AB has a counterclockwise angular velocity of 5 rad/s relative to the nonrotating coordinate system. What is the velocity of point B relative to point A?

Solution  Use polar coordinates. The velocity of B relative to A is
\[
\vec{v}_{B/A} = \left( \frac{dr}{dt} \right) \hat{e}_r + \left( \frac{d\theta}{dt} \right) \hat{e}_\theta = 400(5) \hat{e}_\theta \quad (mm/s). In cartesian coordinates:
\]
(See Problem 2.133) \( \hat{e}_\theta = -\hat{i} \sin \theta + \hat{j} \cos \theta \), from which the relative velocity in cartesian coordinates is
\[
\vec{v}_{B/A} = (400)(5)\left( -\hat{i} \sin 60^\circ + \hat{j} \cos 60^\circ \right) = -1732.1 \hat{i} + 1000 \hat{j} \quad (mm/s)
\]

Problem 2.172  In Problem 2.171, what is the acceleration of point B relative to point A?

Solution  Use polar coordinates: The acceleration of B relative to A is
\[
\vec{a}_{B/A} = \left( \frac{d^2 r}{dt^2} - r \left( \frac{d\theta}{dt} \right)^2 \right) \hat{e}_r + \left( r \frac{d^2 \theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right) \hat{e}_\theta = -400(25) \hat{e}_r. In cartesian coordinates:
\]
\( \hat{e}_r = \hat{i} \cos \theta + \hat{j} \sin \theta \), from which \( \vec{a}_{B/A} = -5000 \hat{i} - 8660 \hat{j} \quad (mm/s^2) \)
Problem 2.182  In Problem 2.181, at what point on the \( x \) axis should the player aim the puck's velocity vector relative to him so that it enters the goal? 

**Solution**  Use the solution to Problem 2.181. The velocity of the puck relative to the player is 
\[
\vec{v}_{p/h} = -V(\vec{i} \cos \theta + \vec{k} \sin \theta) \quad (ft / s).
\]
The velocity of the player relative to the rink is 
\[
\vec{v}_{h/r} = 4\vec{i} - 20\vec{k} \quad (ft / s).
\]
The velocity of the puck relative to the rink is 
\[
\vec{v}_{p/r} = \vec{v}_{p/h} + \vec{v}_{h/r} = (4 - V \cos \theta)\vec{i} - (20 + V \sin \theta)\vec{k}.
\]
The path of the puck is 
\[
x(t) = (4 - V \cos \theta)t + 12 \quad ft, \quad z(t) = -(20 + V \sin \theta)t + 12.
\]
The \( x \) axis crossing occurs when \( z(T) = 0 \), from which 
\[
T = \frac{12}{20 + 100 \sin \theta} \quad s, \quad \text{from which} \quad x = \left( \frac{4 - 100 \cos \theta}{20 + 100 \sin \theta} + 1 \right) 12. \quad \text{Check: This should reduce to} \quad x = 3.17 \quad ft \quad (\text{the solution to Problem 2.181}) \quad \text{when} \quad \theta = 45^\circ, \quad \text{and it does. check.}
\]
To hit the center of the goal, \( x = 0 \), from which \( (\cos \theta - \sin \theta) = 0.24 \). This can be solved by iteration to obtain \( \theta = 35.23^\circ \). 
The aiming point is the point that the puck would strike if the player were stationary, hence the velocity relative to the player is used. The path relative to the player is 
\[
x_{p/h}(t) = -100 \cos 35.23^\circ \quad t + 12, \quad \text{and} \quad z_{p/h}(t) = -100 \sin 35.23^\circ \quad t + 12.
\]
At \( z_{p/h}(T) = 0 \), 
\[
T = \frac{12}{100 \sin 35.23^\circ} = 0.208 \quad s, \quad \text{from which} \quad x_{p/h}(T) = -4.993 \quad ft.
\]
is the aiming point.

Problem 2.183  An airplane flies in a jet stream flowing east at 100 miles per hour. The airplane's airspeed (its velocity relative to the air) is 500 miles per hour toward the northwest. What are magnitude and direction of airplane's velocity relative to the earth? 

**Solution**  The airplane's relative velocity is 
\[
\vec{v}_{a/r} = V(\vec{i} \cos \theta + \vec{j} \sin \theta), \quad \text{where} \quad V = 500 \quad mi / hr, \quad \text{and}
\]
\( \theta = 135^\circ \). The velocity of the air relative to the ground is 
\( \vec{v}_{w/g} = 100\vec{i} \quad (mi / hr) \). The velocity of the airplane relative to the ground is 
\[
\vec{v}_{a/g} = \vec{v}_{a/r} + \vec{v}_{w/g} = (100 + 500 \cos \theta)\vec{i} + 500 \sin \theta \vec{j} = -253.6\vec{i} + 353.6\vec{j}.
\]
The magnitude is 
\[
|\vec{v}_{w/g}| = \sqrt{253.6^2 + 353.6^2} = 435.1 \quad mi / hr \quad \text{The direction is} \quad \beta = \tan^{-1} \left( \frac{353.6}{-253.6} \right) + 180^\circ = 125.65^\circ,
\]
where \( 180^\circ \) must be added because the angle is in the second quadrant. (The compass direction on a four point compass is \( 125.65^\circ - 90^\circ = 35.65^\circ \) West of North.)

Problem 2.184  In Problem 2.183, if the pilot wants to fly toward a city that is northwest of his current position, what direction must he point the airplane, and what will be the magnitude of his velocity relative to the earth? 

**Solution**  The velocity relative to the earth: 
\[
\vec{v}_{a/g} = \vec{v}_{a/w} + \vec{v}_{w/g}
\]
\[
\vec{v}_{a/g} = 500(\vec{i} \cos \theta + \vec{j} \sin \theta) + 100\vec{i} = V(\vec{i} \cos 135^\circ + \vec{j} \sin 135^\circ), \quad \text{from which} \quad \sin \theta + \cos \theta = -0.2 \quad \text{Solve:} \quad \theta = 143.1^\circ \quad (53.1^\circ \text{ West of North }), \quad \text{and} \quad V = 424.26 \quad mi / hr
\]
Problem 2.185 A river flows north at 3 m/s. (Assume that the current is uniform.) If you want to travel in a straight line from point C to point D in a boat that moves a constant speed of 10 m/s relative to the water, in what direction do you point the boat? How long does it take to make the crossing?

Solution The direction of travel from C to D is \( \beta = \tan^{-1}\left(\frac{400}{500}\right) = 38.66^\circ \)

north of east. The velocity relative to the earth is \( \vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE} \),

\[
\vec{v}_{BE} = V_{BE} (\hat{i} \cos \beta + \hat{j} \sin \beta) = V_{BW} (\hat{i} \cos \theta + \hat{j} \sin \theta) + 3\hat{j} \text{ (m/s)}
\]

Reduce: \( V_{BE} \cos \beta = V_{BW} \cos \theta \), \( V_{BE} \sin \beta = V_{BW} \sin \theta + 3 \), from which

\[
\tan \beta = \frac{V_{BW} \sin \theta + 3}{V_{BW} \cos \theta}. \text{ Solve: } \theta = \beta - \sin^{-1}\left(\frac{3 \cos \beta}{10}\right) = 38.66 - \sin^{-1}(0.234) = 25.1^\circ \text{ north of east (64.9^\circ)}
\]

east of north) and \( V_{BE} = V_{BW} \left(\frac{\cos \theta}{\cos \beta}\right) = 10 \left(\frac{0.9054}{0.7809}\right) = 11.6 \text{ m/s} \). The distance is

\[
L = \sqrt{400^2 + 500^2} = 640.3 \text{ m}, \text{ and the time of travel is } t = \frac{L}{V} = 55.2 \text{ s}
\]

Problem 2.186 In Problem 2.185, what is the minimum boat speed relative to the water necessary to make the trip from point C to D?

Solution The strategy is (a) to show that when the boat must follow a given straight line path, the magnitude of the boat velocity relative to the water is a minimum when the boat heading is normal to the desired path, and then (b) to determine the value of this minimum.

From the solution to Problem 2.185, the velocity along the desired path relative to the earth is

\[
\vec{v}_{BE} = V_{BE} (\hat{i} \cos \beta + \hat{j} \sin \beta) = V_{BW} (\hat{i} \cos \theta + \hat{j} \sin \theta) + 3\hat{j} \]. Equate the components, and take the ratio to eliminate \( V_{BE} \) to obtain \( \tan \beta = \frac{V_{BW} \sin \theta + 3}{V_{BW} \cos \theta} \). Solve: \( V_{BW} = \frac{3 \cos \beta}{\sin(\beta - \theta)} \). The minimum occurs when \( \frac{dV_{BW}}{d\theta} = \frac{\cos(\beta - \theta)}{\sin^2(\beta - \theta)} (3 \cos \beta) = 0 \). Since \( \beta \) is a constant angle along the desired path, this minimum is satisfied when \( \cos(\beta - \theta) = 0 \), or \( \beta - \theta = \pm \frac{\pi}{2} \). From physical considerations, the boat will cross only for an easterly heading, hence the positive sign is applicable, and

\[
\theta = \beta - \frac{\pi}{2} = 38.66^\circ - 90^\circ = -51.34^\circ, \text{ which is perpendicular to the path. That this is indeed a minimum:}
\]

\[
\left[ \frac{d^2V_{BW}}{d\theta^2} \right]_{\beta - \theta = \frac{\pi}{2}} = \left[ -\frac{6 \cos \beta \cos^2(\beta - \theta)}{\sin^3(\beta - \theta)} + \frac{3 \cos \beta}{\sin(\beta - \theta)} \right]_{\beta - \theta = \frac{\pi}{2}} = 3 \cos \beta > 0 \text{ The value of the minimum for this heading is}
\]

\[
V_{BW} = \left[ \frac{3 \cos \beta}{\sin(\beta - \theta)} \right]_{\beta - \theta = \frac{\pi}{2}} = 3 \cos \beta = 2.34 \text{ m/s}
\]