ME 330
Fluid Mechanics
Solutions for Problem Set #1
Fox and McDonald, 5th Edition
Problems: 1.3, 1.29, 2.1 and 2.21
Given: Cylindrical tank, \( D = 150 \text{ mm} \times L = 1300 \text{ mm} \), contains nitrogen at pressure \( p = 204 \text{ atm} \) gage.

Find: the mass of nitrogen in the tank.

Solution:
Assume ideal gas behavior.

Basic equations:
\[
\begin{align*}
\rho &= pR \quad (p = \text{absolute pressure}) \\
\rho &= \frac{MRT}{V}
\end{align*}
\]

Substituting, we obtain:
\[
\rho = \frac{\frac{MRT}{V}}{\frac{D^2L}{4}}
\]

Since \( V = \frac{\pi D^2L}{4} \), then

\[
\rho = \frac{\pi MRT}{\pi D^2L}
\]

From Table A.6, \( R = 296.8 \text{ N.m/kg.K} \).

Assuming \( T = 20^\circ \text{C} = 293 \text{ K} \), then

\[
m = \frac{\pi}{4} \times (204 + 1) \times 101 \times 10^3 \times \frac{0.15}{0.05} \times \frac{(0.15)^2}{1.30} \times \frac{\text{kg.K}}{296.8 \text{ atm.K}} \times \frac{1}{293} \]

\[
m = 5.47 \text{ kg}
\]
Given: Density of mercury is \( \rho = 26.3 \text{ slug/ft}^3 \).

Acceleration of gravity on moon is \( g_m = 5.47 \text{ ft/s}^2 \).

Find: (a) Specific gravity of mercury.
(b) Specific volume of mercury, in \( \text{m}^3/\text{kg} \).
(c) Specific weight on Earth.
(d) Specific weight on moon.

Solution: Apply definitions: \( \gamma = \rho g \), \( \nu = 1/\rho \), \( SG = \rho / \rho_{H2O} \)

Thus \( SG = \frac{26.3 \text{ slug}}{\text{ft}^3} \times \frac{\text{ft}^3}{1.94 \text{ slug}} = 13.6 \)

\( \nu = \frac{\text{ft}^3}{26.3 \text{ slug}} \times (0.3048)^3 \text{ m}^3 \times \frac{\text{slug}}{\text{ft}^3} \times \frac{\text{lbm}}{32.2 \text{ lbm}} \times \frac{\text{lbm}}{0.4536 \text{ kg}} = 7.37 \times 10^{-5} \text{ m}^3/\text{kg} \)

On Earth,

\( \gamma_E = \frac{26.3 \text{ slug}}{\text{ft}^3} \times \frac{32.2 \text{ lb}}{\text{slug} \text{ ft}} \times \frac{\text{lb} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} = 847 \text{ lbf/ft}^3 \)

On the moon,

\( \gamma_m = \frac{26.3 \text{ slug}}{\text{ft}^3} \times \frac{5.47 \text{ ft}}{\text{sec}^2} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} = 144 \text{ lbf/ft}^3 \)

(Note that the mass-based quantities (\( SG \) and \( \nu \)) are independent of gravity.)
Given: Velocity fields listed below. (a and b are constants)

Determine: (a) Dimensions of each velocity field
       (b) Whether the flow is steady or unsteady

Solution:

The dimensions of each velocity field will be determined relative to an xyz coordinate system.

<table>
<thead>
<tr>
<th>FIELD</th>
<th>DIMENSIONS</th>
<th>STEADY OR UNSTEADY</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ( \vec{V} = [ae^{-bt}]\hat{i} )</td>
<td>one-dimensional</td>
<td>steady ( \vec{V} = \nabla (x) )</td>
</tr>
<tr>
<td>(2) ( \vec{V} = ax\hat{i} + by\hat{j} )</td>
<td>one-dimensional</td>
<td>steady ( \vec{V} = \nabla (y) )</td>
</tr>
<tr>
<td>(3) ( \vec{V} = [axe^{-bt}]\hat{i} )</td>
<td>one-dimensional</td>
<td>unsteady ( \vec{V} = \nabla (z) )</td>
</tr>
<tr>
<td>(4) ( \vec{V} = ax\hat{i} - by\hat{j} )</td>
<td>two-dimensional</td>
<td>steady ( \vec{V} = \nabla (x, y) )</td>
</tr>
<tr>
<td>(5) ( \vec{V} = (axt)\hat{i} - byt\hat{j} )</td>
<td>two-dimensional</td>
<td>unsteady ( \vec{V} = \nabla (x, y) )</td>
</tr>
<tr>
<td>(6) ( \vec{V} = ax^2 + by^2\hat{j} )</td>
<td>two-dimensional</td>
<td>steady ( \vec{V} = \nabla (y, z) )</td>
</tr>
<tr>
<td>(7) ( \vec{V} = ax(t^2)\hat{i} (1/2)^{1/3} )</td>
<td>three-dimensional</td>
<td>steady ( \vec{V} = \nabla (x, y, z) )</td>
</tr>
<tr>
<td>(8) ( \vec{V} = ax\hat{i} - by\hat{j} + t\hat{k} )</td>
<td>three-dimensional</td>
<td>unsteady ( \vec{V} = \nabla (x, y, z) )</td>
</tr>
</tbody>
</table>
Given: Velocity field in xy plane, \( \vec{V} = a \vec{i} + bx \vec{j} \), where \( a = 2 \text{ m/s} \) and \( b = 1 \text{ s}^{-1} \).

Find: (a) Equation for streamline through \( (x, y) = (2, 5) \).
(b) At \( t = 2 \), coordinates of particle \((0, 4)\) at \( t = 0 \).
(c) At \( t = 3 \), coordinates of particle \((1, 4.25)\) at \( t = 1 \).
(d) Compare pathline, streamline, streakline.

**Solution:** For a streamline \( \frac{dx}{u} = \frac{dy}{v} \)

For \( \vec{V} = a \vec{i} + bx \vec{j} \), \( u = a \) and \( v = bx \), so \( \frac{dx}{a} = \frac{dy}{bx} \) or

\( xdx = \frac{a}{b} dy \)

Integrating

\( \frac{x^2}{2} = \frac{a}{b} y + c' \) or \( y = \frac{b}{2a} x^2 + c \)

Evaluating \( c \) at \( (x, y) = (2, 5) \),

\( c = y - \frac{b}{2a} x^2 = 5m - \frac{1}{2} \times \frac{1}{5} \times \frac{5}{2} (2m)^2 = 4m \)

Streamline through \( (x, y) = (2, 5) \) is \( y = \frac{x^2}{4} + 4 \)  \( \text{(a)} \)

To locate particles, derive parametric equations

\( u_p = \frac{dx}{dt} = a, \quad dx = adt, \) and \( x - x_0 = at - t_0 \)

\( v_p = \frac{dy}{dt} = bx, \quad dy = bxdt = b(x_0 + at - t_0) \)

\( y - y_0 = bx_0(t - t_0) + \frac{a}{2} (t^2 - t_0^2) - at_0(t - t_0) \)

For the particle at \( (x_0, y_0) = (0, 4) \) at \( t = 0 \),

\( x = 0 + at \) \( \quad \text{so at } t = 2 \), \( x = \frac{2m}{3} \times 2 \times 5 = 4 \text{ m} \)

\( y = 4 + \frac{at^2}{2} \) \( \quad \text{so at } t = 2 \), \( y = 4 + \frac{1}{2} \times \frac{2m \times (2)^2}{5} \times 5^2 \)

\( y = 8 \text{ m} \)  \( \text{(b)} \)
For the particle at \((x, y) = (1, 4.25)\) at \(t = 1\) s,
\[
\begin{align*}
\chi &= \chi_0 + a(t-t_0) = 1 + a(t-1) \\
\text{so at } t = 3 \text{ s, } \chi &= 1 + \frac{2 \cdot m}{3} (3-1) s = 5 m
\end{align*}
\]
\[
\begin{align*}
y &= y_0 + bx_0(t-t_0) + \frac{a}{2} (t^2 - t_0^2) - at_0(t-t_0) \\
&= 4.25 + \frac{1}{3} \times 1 m x (t-1) + \frac{1}{2} \times \frac{2 \cdot m}{3} (t^2 - 1) - 2 \frac{m}{3} \times 1 s (t-1)
\end{align*}
\]
So at \(t = 3\) s, \(y = 4.25 + 2 + 8 - 4 = 10.25 m\)

All these points lie on the same streamline, as shown below:

For this steady flow, streamlines, pathlines, and streaklines coincide, as expected.