ME 330
Fluid Mechanics
Solutions for Problem Set #7
Fox and McDonald, 5th Edition
Problems: 6.59, 63
7.19, 31, 36, 51, 52
Given: Water flow from a kitchen faucet of 0.5 in. diameter at 2 gpm. Bottom of sink is 18 in. below faucet outlet.

Find: (a) If area of stream increases, decreases, or remains constant, and why.
(b) Expression for cross-section vs. $y$, measured above bottom.
(c) Force on plate held horizontal; variation with height, and why?

Solution: The water stream is accelerated by gravity. The area of the stream will decrease toward the sink bottom, because less area is needed to carry the same flow rate.

Apply Bernoulli to steady, incompressible, frictionless flow along a streamline:

Basic equation: \[ \frac{\partial}{\partial y} \left( \frac{P}{\rho} + \frac{V^2}{2} + g_3 \right) = -\frac{\partial}{\partial x} \left( \frac{P}{\rho} + \frac{V^2}{2} + g_3 \right) \]

But $p_1 = p = p_{atm}$, so
\[ \frac{V_1^2}{2} + gH = \frac{V^2}{2} + gy ; \ V = \left[ \frac{V_1^2 + 2g(H-y)}{V_1^2 + 2g(H-y)} \right]^{1/2} \]

For uniform flow, continuity reduces to $V, A_1 = VA$

\[ A = A_1 \frac{V_1}{V} = A_1 \frac{V}{\left[ \frac{V_1^2 + 2g(H-y)}{V_1^2 + 2g(H-y)} \right]^{1/2}} \]

Predict force on plate from $y$ component of momentum:

Basic equation: \[ F_y = \frac{\partial}{\partial x} \int_{CS} v \rho \, dA + \int_{CS} v \rho v \cdot dA \]

Since uniform, \[ R_y - W = v \left\{ -\rho v A_1 \right\} = -V \left\{ -\rho A \right\} = +VPA \]

\[ V = -V \]

Thus $R_y = W + VPA$

Since $V$ increases as $y$ decreases, $R_y$ varies in the same manner.
Given: Air jet, disk of diameter $D = 200$ mm, and manometer.

Find: (a) $\Delta h$
   (b) Force on disk
   (c) Sketch pressure distribution.

Solution: Apply Bernoulli, hydrostatic, and $x$ component of momentum. $V = 50$ m/s:

Basic equations:

$$\frac{\rho_0}{\rho} = \frac{p + \frac{V^2}{2}}{\Delta p} = \frac{5g\rho_m g \Delta h}{\Delta p}$$

$$F_{x,v} = \frac{\partial}{\partial t} \int_{CV} u e dt + \int_{CS} u \rho V \cdot \mathbf{V} \cdot \mathbf{d}A$$

Assumptions:
1. Steady flow
2. Incompressible flow
3. Flow along a streamline
4. No friction
5. Static liquid in manometer
6. $F_{x,v} = 0$
7. Uniform flow at each section

Then

$$\Delta p = p_0 - p = \frac{1}{2} \rho V^2 = \frac{5g \rho_m g \Delta h}{\Delta p}$$

$$\Delta h = \frac{\rho V^2}{25g \rho_m g \Delta h} = \frac{1}{2} \times 1.23 \text{ kg/m}^3 \times \frac{(50)^2 \text{ m}^2}{3 \pi \times (1.25) 999 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2} = 0.0896 \text{ m (89.6 mm)}$$

and

$$R_x = u_1 \{ -pV A \} + u_2 \{ pV A \} = -pV^2 A$$

$$u_1 = V \quad u_2 = 0$$

or

$$R_x = -k_x = pV^2 A = \frac{1.23 \text{ kg/m}^3 \times (50)^2 \text{ m}^2 \times \pi (0.010)^2 \text{ m}^2 \times \frac{N \text{m}^2}{4 \text{ kg/m}^3}}{10 \text{ kg/m}^2} = 0.242 \text{ N}$$

The pressure distribution is caused by streamline curvature:

Pressure is $p_0$ (gage) at center.

Pressure is zero (gage) at edges.
Given: Jet pump shown.
\[ \Delta p = f(\rho, V, d, D, \mu, \alpha) \]
\[ \begin{align*}
\Delta p \quad \rho \quad V \quad d \quad D \quad \mu \quad \alpha \\
\frac{M}{L^2} \quad \frac{M}{L^3} \quad \frac{L}{t} \quad L \quad L \quad \frac{M}{L^2} \quad \frac{L^3}{t}
\end{align*} \]

Find: (a) Number of independent dimensionless parameters.
(b) Obtain parameters containing (1) \( \alpha \) and (c) \( \mu \).

Solution: Apply the Buckingham \( \Pi \)- theorem.

1. \( \Delta p \quad \rho \quad V \quad d \quad D \quad \mu \quad \alpha \quad n = 7 \) parameters.
2. Select \( M_1, L, t \) as primary dimensions.
3. \( \frac{M}{L^2} \quad \frac{M}{L^3} \quad \frac{L}{t} \quad L \quad L \quad \frac{M}{L^2} \quad \frac{L^3}{t} \)
   \( r = 3 \) primary dimensions.
4. \( m = 7 - 3 = 4 \) dimensionless parameters result.
5. Set up dimensional equations:

\[ \Pi_1 = \rho^a V^b d^c \alpha^d \quad \Pi_2 = \rho^a V^b d^c \mu \]

Equating exponents:
\[ M: a = 0 \quad \alpha = 0 \]
\[ L: -3a + b + c + 3 = 0 \quad \Pi_1 \]
\[ t: -b - 1 = 0 \quad \Pi_2 \]

\[ \therefore c = 3a - b - 3 = -2 \]

For completeness,
\[ \Pi_3 = \rho^a V^b d^c \Delta p = \frac{\Delta p}{\rho V^2} \]
\[ \Pi_4 = \rho^a V^b d^c \delta = \frac{\delta}{d} \]

6. Check using \( F \) dimensions:
\[ \Pi_1 = \frac{L^3}{t} \frac{1}{L^3} = [1] \]
\[ \Pi_2 = \frac{Ft^2}{L^2} \frac{L^4}{Ft^2} \frac{L}{L} = [1] \]

The functional relationship would be
\[ \Pi_3 = f(\Pi_1, \Pi_2, \Pi_4) \]
\[ \frac{\Delta p}{\rho V^2} = f\left( \frac{\alpha}{V d^2}, \frac{\mu}{\rho V d}, \frac{\delta}{d} \right) \]
Given: Measurements of drag force are made on a model car in a towing tank filled with freshwater; \( h_n L_n = 1.5 \). The dimensionless force ratio becomes constant at model test speeds above \( V_n = 4 \text{ m/s} \). At this speed, the drag force on the model is \( F_{D_m} = 1.82 \text{ N} \).

Find: (a) State conditions required to assure dynamic similarity between model and prototype.
(b) Determine required speed ratio \( V_n / V_p \) to assure dynamically similar conditions.
(c) Calculate expected prototype drag when operating in air at speed, \( V_p = 80 \text{ km/hr} \).

Solution:
(a) The flows must be geometrically and kinematically similar and have equal Reynolds numbers to be dynamically similar.
   - Geometric similarity requires true model in all respects.
   - Kinematic similarity requires same flow pattern; i.e., no free-surface effects or cavitation.
   - The problem may be stated as \( F_D = F(\rho, V, l, \mu) \).
   - Dimensional analysis gives \( \rho V^2 l^2 = f(\rho V l) = g(Re) \).

(b) Matching Reynolds numbers between model and prototype gives:
   \[ \frac{\rho_n V_n^2 l_n}{\mu_n} = \frac{\rho_p V_p^2 l_p}{\mu_p} \]
   Assume \( T = 20^\circ \text{C} \)
   \[ \frac{V_n}{V_p} = \frac{V_n}{V_p} \frac{\rho_p}{\rho_n} \frac{\mu_n}{\mu_p} \]
   \[ \frac{V_n}{V_p} = \frac{2 \times 10^{-3} \text{ m/s}}{1.5 \times 10^{-3} \text{ m/s}} = 0.333 \]

(c) For dynamically similar conditions, \( \frac{\rho_n V_n^2 l_n}{\mu_n} = \frac{\rho_p V_p^2 l_p}{\mu_p} \)
   \[ F_{D_p} = F_{D_m} \rho_n \frac{V_n^2}{V_p^2} \frac{l_n}{l_p} \frac{\mu_p}{\mu_n} \]
   \[ = 1.82 \text{ N} \times 1.20 \times \left( \frac{10^3 \text{ m/s}}{1.5 \times 10^3 \text{ m/s}} \right) \left( \frac{80 \text{ km/hr}}{1.82 \text{ N}} \right) \left( \frac{3600 \text{ s}}{1 \text{ km/hr}} \right) \left( \frac{1}{5} \right) \]
   \[ F_{D_p} = 21.4 \text{ N} \]
Given: Fluid dynamic characteristics of a golf ball are to be tested using a model in a wind tunnel.
Dependent variables: $F_D$, $F_L$
Independent variables should include $\omega$, $d$ (dimple depth)
Golf pro can hit prototype ($r = 1.65$ in) at $V = 240$ fps
and $\omega = 9000$ rpm. Prototype is to be modeled in wind tunnel with $V = 80$ fps.

Find: (a) suitable dimensionless parameters
(b) required diameter of model
(c) required rotational speed of model

Solution: Assume the functional dependence to be given by

$$F_D = F_D(D, \rho, \omega, d, p, \mu) \quad \text{and} \quad F_L = F_L(D, \rho, \omega, d, p, \mu)$$

From the Buckingham $\pi$-Theorem, for $n=7$ and $n=3$, we would expect four dimensionless groups

$$\frac{F_D}{\rho V^2 D^2} = \pi_1 \left( \frac{\rho}{\mu}, \frac{D}{d}, \frac{V}{\omega} \right) \quad \text{and} \quad \frac{F_L}{\rho V^2 D^2} = \pi_2 \left( \frac{\rho}{\mu}, \frac{D}{d}, \frac{V}{\omega} \right)$$

To determine the required diameter of the model,

$$\frac{\rho V^2 D^2}{\mu} = \left( \frac{\rho V^2 D^2}{\mu} \right)_m \quad \Rightarrow \quad D_m = 3 \times 1.65 \text{ in} = 5.04 \text{ in.}$$

To determine the required rotational speed of the model,

$$\frac{\omega}{V} = \frac{\omega_m}{V_m} \quad \Rightarrow \quad \omega_m = \omega \left( \frac{V_m}{V} \right) = \frac{1}{3} \times 9000 = 3000 \text{ rpm}$$
Given: Power, P, to drive a fan depends on $P$, $Q$, $D$, and $w$.

<table>
<thead>
<tr>
<th>Condition</th>
<th>D (in.)</th>
<th>$Q$ (cfm)</th>
<th>$W$ (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>800</td>
<td>2400</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>?</td>
<td>1850</td>
</tr>
</tbody>
</table>

Find: Volume flow rate at condition 2, for dynamic similarity.

**Solution:** Step 1: $P = \rho Q D W$

Step 2: $MLT$ (3):

| $M$: $a + 1 = 0$ | $a = -1$ | $M$: $a + 0 = 0$ | $a = 0$
|-------------------|----------|-------------------|----------|
| $L$: $-3a + c + 2 = 0$ | $c = 3a - 2 = -5$ | $L$: $-3a + c + 3 = 0$ | $c = -3$
| $t$: $-b - 3 = 0$ | $b = -3$ | $t$: $-b - 1 = 0$ | $b = -1$

$\Pi_1 = \frac{P}{\rho w^3 D^3}$

$\Pi_2 = \frac{Q}{W D^3}$

$\Pi_1 = \frac{P}{\rho w^3 D^3}$

$\Pi_2 = \frac{Q}{W D^3}$

Thus $\Pi_1 = f(\Pi_2)$ or $\frac{P}{\rho w^3 D^3} = f\left(\frac{Q}{W D^3}\right)$

For dynamic similarity, need geometric and kinematic similarity and

$$\frac{Q_1}{W D_1^3} = \frac{Q_2}{W D_2^3}$$

Thus

$$Q_2 = Q_1 \frac{W_2}{W_1} \left(\frac{D_2}{D_1}\right)^3 = 800 \text{ cfm} \frac{1850 \text{ rpm}}{2400 \text{ rpm}} \frac{(16 \text{ in.})^3}{(8 \text{ in.})^3} = 4930 \text{ cfm}$$
Given: Information relating to geometrically similar model test of centrifugal pump:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Prototype</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Rise</td>
<td>$\Delta p$</td>
<td>29.3 kPa</td>
</tr>
<tr>
<td>Volume Flow Rate</td>
<td>$Q$</td>
<td>1.25 m³/min</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>800 kg/m³</td>
</tr>
<tr>
<td>Angular Speed</td>
<td>$\omega$</td>
<td>999 kg/m³</td>
</tr>
<tr>
<td>Diameter</td>
<td>$D$</td>
<td>183 rad/s</td>
</tr>
</tbody>
</table>

Find: Missing values for dynamically similar conditions.

Solution: Apply Buckingham $\pi$-theorem. Assume $\Delta p = f(Q, \rho, \omega, D)$

1. $\Delta p$ $Q$ $\rho$ $\omega$ $D$ $n = 5$ parameters
2. Choose $M, L, T$ as fundamental dimensions.
3. $\frac{M}{L^2}$ $\frac{L^3}{T^2}$ $\frac{L}{T}$ $M$ $L$ $T$ $r = 3$ primary dimensions
4. Let $\rho, \omega$, and $D$ be repeating variables. $m = r = 3$
5. Then $n - m = 5 - 3 = 2$ dimensionless parameters result.
6. Check:

   $M_1 = \frac{Q^a \rho^b \omega^c D^d \Delta p}{L^e}$

   $M_1 = (\frac{L^2}{M^3})^{a} (\frac{T}{L})^{b} (\frac{L}{T})^{c} = M^{a} L^{b} T^{c} L^{d} = M^{a} L^{b} T^{c}$

   $\pi_1 = \frac{\Delta p}{\rho \omega^2 D^2}$

   $M_2 = \frac{Q^a \rho^b \omega^c D^d \Delta p}{L^e}$

   $M_2 = (\frac{L^2}{M^3})^{a} (\frac{T}{L})^{b} (\frac{L}{T})^{c} = M^{a} L^{b} T^{c}$

   $\pi_2 = \frac{Q}{\rho \omega D^3}$

Thus $\pi_1 = f(\pi_2)$ for this situation. Flows are geometrically similar. Assume kinematic similarity. Then for dynamic similarity, if $\pi_2 m = \pi_2 p$, then $\pi_1 m = \pi_1 p$.

$\pi_1 m = \frac{G_m}{\omega m D_m^3} = \pi_1 p = \frac{Q_p}{\omega p D_p^3}$; $Q_m = Q_p (\frac{\omega m}{\omega p}) (\frac{D_m}{D_p})^3 = Q_p (\frac{360}{183}) (\frac{150}{50})^3 = 0.0743 Q_p$

$Q_m = 0.0743 \times 1.25 m^3/min = 0.928 m^3/min$

$\pi_1 m = \frac{\Delta p_m}{\rho m \omega_m^2 D_m^2} = \pi_1 p = \frac{\Delta p_p}{\rho p \omega_p^2 D_p^2}$; $\Delta p_m = \Delta p_p (\frac{\omega_p}{\omega m}) (\frac{D_p}{D_m})^2$

$\Delta p_m = \Delta p_p \left(\frac{183}{360}\right) (\frac{50}{150})^2 = 1.79 \times 29.3 kPa = 52.5 kPa$

{This result neglects any effect of viscosity.}