For this class you need to be comfortable with certain concepts from calculus. If you have problems with this assignment, you may want to come see me to discuss what things you need to review to prepare yourself for this class.

Compute the following (1 – 4):

1. \[ \sum_{k=1}^{\infty} \left(\frac{1}{4}\right)^k \] and \[ \sum_{k=1}^{\infty} k\left(\frac{1}{4}\right)^k \]

2. \[ \sum_{k=1}^{\infty} \frac{(1/5)^k}{k!} \] and \[ \sum_{k=1}^{\infty} \frac{k(1/5)^k}{k!} \]

3. \[ \int_{0}^{\infty} xe^{-x/5} \, dx \]

4. \[ \int_{0}^{2} \int_{0}^{1} \left(x^2 + \frac{1}{2}xy\right) \, dx \, dy \]

5. After retiring from their illustrious hip-hop careers, Eric B. and Rakim join the UT baseball team. Eric B. presents the following problem to Rakim: “Suppose I have a higher batting average than you during the first half of the season and I have a higher batting average than you during the second half of the season. Is it possible for you to have a higher batting average than me for the entire season?”

Rakim claims the answer is “yes.” Is he correct? If you believe so, give a numerical example of how this can happen. If not, explain your reasoning.

(Note that the batting average is the number of hits divided by the number of times at bat. We assume nothing about the number of at bats each player has had during the two halves of the season.)