Cash Flow Exercises

1. Consider the cash flow below.

![Cash Flow Diagram]

a. Find the uniform equivalent from 1 through 10.

This is a periodic series. The first runs from 1 through 5. To change a periodic series to a uniform series, you only need to change one of the cycles to a uniform series. The others have the same equivalent uniform series. For the example, a single cycle consists of a uniform series of 100 plus a gradient of 25. The net annual worth of that series is:
NAW = 100 + 25(A/G, i 5)

b. Find the present worth.

To compute the NPW of the series in the figure, ten payments would have to be brought to the present.
NPW = NAW*(P/A, i 10) or NPW = (100 + 25(A/G, i 5)) *(P/A, i 10)
Consider the cash flow below.

Find the equivalent value at time 0.

This is a periodic series occurring every three periods. To find the present worth, first change it to an equivalent uniform series. Only one cycle need be considered. The uniform series that runs from 1 through 3 equivalent to the single payment at time 3 is:

\[ A = 50 \times (A/F, i, 3) \]

The equivalent over an infinite time is:
\[ NPW = A \times (P/A, i, \infty) \]

From the table for limiting values of the factors we know that
\[ A \times (P/A, i, \infty) = 1/i \]

So, \[ NPW = 50 \times (A/F, i, 3) \times (P/A, i, \infty) = 50 \times (A/F, i, 3) / i \]
3. Find the equivalent value at time 10.

First move it to the time of the last payment with the \((F/A, i, 5)\) factor. This places a single value at time 4. Then move it to 10 with the \((F/P, i, 6)\) factor.

\[ F = 50(F/A, i, 5)(F/P, i, 6) \]

or

\[ F = 50(P/A, i, 5)(F/P, i, 11) \]
4. Write the formula for the NPW of this investment.

\[
\text{NPW} = -300 + 50(P/A, i, 7) + 50(P/G, i, 7) -100(P/G, i, 4)\times(P/F, i, 3)
\]

**Term** | **Explanation**
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NPW = -300 | This value is at time 0
+ 50(P/A, i 7) | The uniform series of 50 goes from 1 to 7
+ 50(P/G, i, 7) | The gradient increases the cash flow by 50 in each period. -100(P/G, i 4)*(P/F, i, 3)
-100(P/G, i 4)* (P/F, i, 3) | This subtracts a gradient of 100. Note that the first nonzero payment is at time 5. The other cash flows are 250 at this time. Subtracting 100 brings to 150 as shown. Similarly for the other times.

Other Solutions.
\[
\text{NPW} = -300 + 50(P/A, i, 7) + 50(P/G, i, 4) +100(P/F, i, 5)+ 50(P/F, i, 6)
\]
\[
\text{NPW} = -300 + 50(P/A, i, 7) + 50(P/G, i, 4) + (100(P/A, i, 3) - 50(P/G, i, 3))(P/F, i, 4)
\]
5. Find the equivalent value at time 11.

This is a cyclic cash flow. The cycle is 2, but the first payment is at 3. The uniform equivalent is \( A = 50(A/F,2) \), but it starts at 2 and ends at 11 (ten payments). Moving the series to time 11 requires the \((F/A,10)\).

\[
F = 50(A/F,2)(F/A,10)
\]
or
\[
F = 50(A/P,2)(F/A,8)+50
\]
6. Find the NAW.

The cash flow is a uniform series of 50 for ten periods. One payment is reversed. This simplest way to do this is subtract the uniform worth of the single out of sequence payment.

$$\text{NAW} = 50 - 100(P/F,5)(A/P,10)$$