Problem 2.4

A construction crane weighing 12,000 lbf fell from a height of 400 ft to the street below during a severe storm. For \( g = 32.05 \text{ ft/s}^2 \), determine mass, in lb, and the change in gravitational potential energy of the crane, in \( \text{ft} \cdot \text{lbf} \).

**KNOWN:** A crane of known weight falls from a known elevation to the street below.

**FIND:** Determine the change in gravitational potential energy of the crane.

**SCHMATIC AND GIVEN DATA:**

\[
\begin{align*}
F_{\text{grav}} &= 12,000 \text{ lbf} \\
g &= 32.05 \text{ ft/s}^2 \\
z_2 &= 0 \\
z_1 &= 400 \text{ ft}
\end{align*}
\]

**ENGINEERING MODEL:** (1) The crane is the closed system. (2) The acceleration of gravity is constant.

**ANALYSIS:**

To get the mass, note that \( F_{\text{grav}} = mg \). Thus

\[
m = \frac{F_{\text{grav}}}{g} = \frac{12000 \text{ lbf}}{32.05 \text{ ft/s}^2} \left( \frac{32.174 \text{ lb-ft/s}^2}{1 \text{ lbf}} \right) = 12,046 \text{ lb}
\]

The change in gravitational potential energy is

\[
\Delta PE = mg(z_2 - z_1) = F_{\text{grav}}\Delta z = (12000 \text{ lbf})(-400 \text{ ft}) = -4.8 \times 10^6 \text{ ft} \cdot \text{lbf}
\]
Problem 2.10

An object whose mass is 300 lb experiences changes in kinetic and potential energies owing to the action of a resultant force \( R \). The work done on the object by the resultant force is 140 lbf. There are no other interactions between the object and its surroundings. If the object’s elevation increases by 100 ft and its final velocity is 200 ft/s, what is the initial velocity, in ft/s? Let \( g = 32.2 \text{ ft/s}^2 \).

**KNOWN:** An object of known mass experiences changes in kinetic and potential energy due to the action of a resultant force. The final velocity, the change in elevation, and the work done by the force are given.

**FIND:** Determine the final velocity.

**SCHEMATIC AND GIVEN DATA:**

**ENGINEERING MODEL:** (1) The object is a closed system. (2) The force of gravity acts on the object, and \( g = 32.2 \text{ ft/s}^2 \). (3) The resultant force accounts for all interactions between the system and its surroundings.

**ANALYSIS:** By modeling assumption (3), the work of the resultant force must equal the sum of the changes in kinetic and gravitational potential energies. Thus, with Eq. 2.9

\[
\text{Work} = \frac{1}{2} m (V_2^2 - V_1^2) + mg(z_2 - z_1)
\]

Solving for \( V_1^2 \) and inserting values

\[
V_1^2 = \frac{2[mg(z_2 - z_1) - \text{Work}]}{m} + V_2^2
\]

First

\[
mg(z_2 - z_1) = (300 \text{ lb})(32.2 \text{ ft/s}^2)(100 \text{ ft}) \times \frac{1 \text{ lbf}}{32.2 \text{ lb-ft/s}^2} = \frac{1 \text{ Btu}}{778 \text{ ft-lbf}} = 38.6 \text{ Btu}
\]

So

\[
V_1^2 = \frac{2[38.6 - 140 \text{ BTU}]}{(300 \text{ lb})} \times \frac{778 \text{ ft-lbf}}{1 \text{ Btu}} \times \frac{32.2 \text{ lb-ft/s}^2}{1 \text{ lbf}} + 200^2 \text{ ft}^2/\text{s}^2 = 23065 \text{ ft}^2/\text{s}^2
\]

or

\[ V_1 = 151.9 \text{ ft/s} \]

1. The increase in velocity reflects the increase in kinetic energy of the object as a result of energy transferred to it by the work of the resultant force. Carefully observe that in Eq. 2.9 the work of the resultant force acting on the body is positive.
**Problem 2.15**

**Known:** Can moves down a surface that is inclined relative to the horizontal. The can is acted upon by a constant force parallel to the incline and by the force of gravity.

**Find:** Can’s change in kinetic energy, in J, and whether it is increasing or decreasing. If friction between the can and the inclined surface were significant, what effect would that have on the value of the change in kinetic energy?

**Schematic and Given Data:**

![Schematic of can moving down an incline](image)

**Engineering Model:**
(1) The can is a closed system.
(2) The acceleration of gravity is constant.
(3) The applied force is constant.
(4) Ignore friction between the can and inclined surface.

**Analysis:**

Begin with Eq. 2.6

\[
\int_{s_i}^{s_f} F \cdot ds = \frac{1}{2} m (V_f^2 - V_i^2) = \Delta KE
\]  

(1)

From the free body diagram in the schematic, the dot product can be expressed as

\[
F \cdot ds = (R + mg \sin 20^\circ) ds
\]

Substituting into Eq. (1)

\[
\int_{s_i}^{s_f} F \cdot ds = \int_{s_i}^{s_f} [(R + mg \sin 20^\circ) ds = \Delta KE
\]

(2)

Since \(\Delta z = \Delta s \sin 20^\circ\), Eq. (2) becomes

\[
\int_{s_i}^{s_f} (R) ds + (mg) \Delta z = (R) \Delta s + (mg) \Delta z = \Delta KE
\]

(3)

Evaluate \(\Delta s\)

\[
\Delta s = \frac{\Delta z}{\sin 20^\circ} = \frac{1.5 \text{ m}}{0.342} = 4.39 \text{ m}
\]
Substituting all known and calculated data into Eq. (3)

\[ \Delta KE = (0.05 \text{N})(4.39 \text{m}) \left( \frac{1 \text{J}}{1 \text{N} \cdot \text{m}} \right) + (0.4 \text{kg}) \left( \frac{9.8 \text{m}}{\text{s}^2} \right)(1.5 \text{m}) \left( \frac{1 \text{N} \cdot \text{m}}{1 \text{kg} \cdot \text{m} \cdot \text{s}^2} \right) \left( \frac{1 \text{J}}{1 \text{N} \cdot \text{m}} \right) = \]

\[ \Delta KE = 0.22 \text{ J} + 5.88 \text{ J} = 6.1 \text{ J} \]

Which corresponds to an increase in kinetic energy.

If friction were significant, the magnitude of the net force acting in the direction of motion would be less, and thus the kinetic energy change would be less than calculated.

1. Observe that in the absence of the force \( R \) the can is acted on only by gravity, and the can’s change in kinetic energy is 5.88 J.
Problem 2.29

Nitrogen (N\textsubscript{2}) gas within a piston-cylinder assembly undergoes a process from \( p_1 = 20 \text{ bar}, \ V_1 = 0.5 \text{ m}^3 \) to a state where \( V_2 = 2.75 \text{ m}^3 \). The relationship between pressure and volume during the process is \( pV^{1.35} = \text{constant} \). For the N\textsubscript{2}, determine (a) the pressure at state 2, in bar, and (b) the work, in kJ.

**KNOWN:** N\textsubscript{2} gas within a piston-cylinder assembly undergoes a process where the \( p-V \) relation is \( pV^{1.35} = \text{constant} \). Data are given at the initial and final states.

**FIND:** Determine the pressure at the final state and the work.

**SCHEMATIC AND GIVEN DATA:**

<table>
<thead>
<tr>
<th>N\textsubscript{2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( pV^{1.35} = \text{constant} )</td>
</tr>
<tr>
<td>( p_1 = 20 \text{ bar}, \ V_1 = 0.5 \text{ m}^3 )</td>
</tr>
<tr>
<td>( V_2 = 2.75 \text{ m}^3 )</td>
</tr>
</tbody>
</table>

**ENGINEERING MODEL:**
1. The N\textsubscript{2} is the closed system.
2. The \( p-V \) relation is specified for the process.
3. Volume change is the only work mode.

**ANALYSIS:**
(a) \( p_1 V_1^n = p_2 V_2^n \quad \rightarrow \quad p_2 = p_1 \left( \frac{V_1}{V_2} \right)^n \); \( n = 1.35 \). Thus

\[
p_2 = (20 \text{ bar}) \left( \frac{0.5 \text{ m}^3}{2.75 \text{ m}^3} \right)^{1.35} = 2 \text{ bar}
\]

(b) Since volume change is the only work mode, Eq. 2.17 applies. Following the procedure of part (a) of Example 2.1, we have

\[
W = \frac{p_2 V_2 - p_1 V_1}{1-n} = \frac{(2 \text{ bar})(2.75 \text{ m}^3) - (20)(0.5)}{1-1.35} \left[ \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right] \left[ \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right] = 1285.7 \text{ kJ}
\]

\[\rightarrow\]
PROBLEM 2.33

Known: A gas contained within a piston-cylinder assembly undergoes three processes in series. State data are provided.

Find: Sketch the processes in series on p-V coordinates and evaluate the work for each process, in kJ.

Diagram & Given Data:

**Note:** Minus signs signify energy transfer by work to the gas.

**Analysis:** The work is given by Eq. 2.17: \( W = \int P \, dV \)

Process 1-2: \( V \) is constant. Thus, the piston does not move, and \( W_{12} = 0 \).

Process 2-3:

\[
W_{23} = \int_{V_2}^{V_3} p \, dV = C \ln \frac{V_3}{V_2} = \frac{P_2 V_2}{V_2} \ln \frac{V_3}{V_2} = (2 \times 10^5 \text{ N/m}^2 \times 4 \text{ m}^3) \left[ \ln \left( \frac{2}{4} \right) \right] = -554.5 \text{ kJ}
\]

Process 3-4:

\[
W_{34} = P (V_4 - V_3) = (4 \times 10^5 \text{ N/m}^2 \times 4 \text{ m}^3) \left[ \ln \left( \frac{2}{4} \right) \right] = -400 \text{ kJ}
\]

**Engineering Model:**

1. The gas within the piston-cylinder is the closed system.
2. The gas experiences three processes in series, as shown in the sketch.
**PROBLEM 2.36**

**KNOWN:** Operating data is provided for a belt sander.

**FIND:** Evaluate the power transmitted by the belt to the surface and the work done in one minute of sanding.

**SCHEMATIC & GIVEN DATA:**

Belt speed = 1500 ft/min
Normal force on sander = 151 lb

![Belt Sander Diagram](image)

**Coefficient of friction is 0.2**

**ENGR. MODEL**

1. The force exerted by the belt is related to the normal force, $F_n$, by the coefficient of friction:
   
   $F = \text{(coeff. of friction)} F_n = 0.2 F_n$

**ANALYSIS:** Using Eq. 2.13, the power, $\dot{W}$, transmitted is:

   $\dot{W} = F \cdot v = 0.2 F_n \cdot v$

   or

   $\dot{W} = 0.2 \times (151 \text{ lb})(1500 \text{ ft/min}) \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1 \text{ Btu}}{778 \text{ ft-lb}}$

   $\dot{W} = 0.096 \text{ Btu/s}$

   or

   $\dot{W} = 0.096 \text{ Btu/s} \times \frac{3600 \text{ s}}{1 \text{ hr}} \times \frac{1 \text{ hp}}{2385 \text{ Btu/hr}}$

   $\dot{W} = 0.136 \text{ hp}$

(b) In one minute of sanding, the work done on the surface is:

   $W = (0.096 \text{ Btu/s}) \times \frac{60 \text{ sec}}{1 \text{ min}}$

   $W = 5.76 \text{ Btu}$