PROBLEM 12.43

KNOWN: An opaque surface with prescribed spectral, hemispherical reflectivity distribution is subjected to a prescribed spectral irradiation.

FIND: (a) The spectral, hemispherical absorptivity, (b) Total irradiation, (c) The absorbed radiant flux, and (d) Total, hemispherical absorptivity.

SCHEMATIC:

ASSUMPTIONS: (1) Surface is opaque.

ANALYSIS: (a) The spectral, hemispherical absorptivity, \( \alpha_\lambda \), for an opaque surface is given by Eq. 12.58,

\[ \alpha_\lambda = 1 - \rho_\lambda \]

which is shown as a dashed line on the \( \rho_\lambda \) distribution axes.

(b) The total irradiation, \( G \), follows from Eq. 12.16 which can be integrated by parts,

\[
G = \int_0^\infty G_\lambda \, d\lambda = \int_0^{5 \mu m} G_\lambda \, d\lambda + \int_{5 \mu m}^{10 \mu m} G_\lambda \, d\lambda + \int_{10 \mu m}^{20 \mu m} G_\lambda \, d\lambda
\]

\[
G = \frac{1}{2} \times 600 \frac{W}{m^2 \cdot \mu m} (5 - 0) \mu m + 600 \frac{W}{m^2 \cdot \mu m} (10 - 5) \mu m + \frac{1}{2} \times 600 \frac{W}{m^2 \cdot \mu m} \times (20 - 10) \mu m
\]

\[
G = 7500 \frac{W}{m^2}.
\]

(c) The absorbed irradiation follows from Eqs. 12.45 and 12.46 with the form

\[
G_{abs} = \int_0^\infty \alpha_\lambda G_\lambda \, d\lambda = \alpha_1 \int_0^{5 \mu m} G_\lambda \, d\lambda + \alpha_2 \int_{5 \mu m}^{10 \mu m} \alpha_\lambda G_\lambda \, d\lambda + \alpha_3 \int_{10 \mu m}^{20 \mu m} G_\lambda \, d\lambda.
\]

Noting that \( \alpha_1 = 1.0 \) for \( \lambda = 0 \to 5 \mu m \), \( G_{\lambda,2} = 600 \frac{W}{m^2 \cdot \mu m} \) for \( \lambda = 5 \to 10 \mu m \) and \( \alpha_3 = 0 \) for \( \lambda > 10 \mu m \), find that

\[
G_{abs} = 1.0 \left( 0.5 \times 600 \frac{W}{m^2 \cdot \mu m} \right) (5 - 0) \mu m + 600 \frac{W}{m^2 \cdot \mu m} (0.5 \times 0.5) (10 - 5) \mu m + 0
\]

\[
G_{abs} = 2250 \frac{W}{m^2}.
\]

(d) The total, hemispherical absorptivity is defined as the fraction of the total irradiation that is absorbed. From Eq. 12.45,

\[
\alpha = \frac{G_{abs}}{G} = \frac{2250 \frac{W}{m^2}}{7500 \frac{W}{m^2}} = 0.30.
\]

COMMENTS: Recognize that the total, hemispherical absorptivity, \( \alpha = 0.3 \), is for the given spectral irradiation. For a different \( G_{\lambda} \), one would then expect a different value for \( \alpha \).
**PROBLEM 12.59**

**KNOWN:** Spectrally selective, diffuse surface exposed to solar irradiation.

**FIND:** (a) Spectral transmissivity, $\tau_\lambda$, (b) Transmissivity, $\tau_S$, reflectivity, $\rho_S$, and absorptivity, $\alpha_S$, for solar irradiation, (c) Emissivity, $\varepsilon$, when surface is at $T_s = 350K$, (d) Net heat flux by radiation to the surface.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Surface is diffuse, (2) Spectral distribution of solar irradiation is proportional to $E_{\lambda,b}(\lambda, 5800K)$.

**ANALYSIS:**

(a) Conservation of radiant energy requires, according to Eq. 12.56, that $\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1$ or $\tau_\lambda = 1 - \rho_\lambda - \alpha_\lambda$. Hence, the spectral transmissivity appears as shown above (dashed line). Note that the surface is opaque for $\lambda > 1.38 \mu m$.

(b) The transmissivity to solar irradiation, $G_S$, follows from Eq. 12.55,

$$
\tau_S = \int_0^\infty \tau_\lambda G_{\lambda,S} d\lambda / G_S = \int_0^\infty \tau_\lambda E_{\lambda,b}(\lambda, 5800K) d\lambda / E_b(5800K)
$$

$$
\tau_S = \tau_{\lambda,b} \int_0^{1.38} E_{\lambda,b}(\lambda, 5800K) d\lambda / E_b(5800K) = \tau_{\lambda,b} F(0 \rightarrow \lambda_1) = 0.7 \times 8.56 = 0.599
$$

where $\lambda_1 T_S = 1.38 \times 5800 = 8000 \mu mK$ and from Table 12.1, $F(0 \rightarrow \lambda_1) = 0.856$. From Eqs. 12.52 and 12.57,

$$
\rho_S = \int_0^\infty \rho_\lambda G_{\lambda,S} d\lambda / G_S = \rho_{\lambda,b} F(0 \rightarrow \lambda_1) = 0.1 \times 0.856 = 0.086
$$

$$
\alpha_S = 1 - \rho_S - \tau_S = 1 - 0.086 - 0.599 = 0.315.
$$

(c) For the surface at $T_s = 350K$, the emissivity can be determined from Eq. 12.38. Since the surface is diffuse, according to Eq. 12.65, $\alpha_\lambda = \varepsilon_\lambda$, the expression has the form

$$
\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T_S) d\lambda / E_b(T_S) = \int_0^\infty \alpha_\lambda E_{\lambda,b}(350K) d\lambda / E_b(350K)
$$

$$
\varepsilon = \alpha_{\lambda,1} F(0 \rightarrow \lambda_1) + \alpha_{\lambda,2} \left[ 1 - F(0 \rightarrow 1.38 \mu m) \right] = \alpha_{\lambda,2} = 1
$$

where from Table 12.1 with $\lambda_1 T_S = 1.38 \times 350 = 483 \mu mK$, $F(0 \rightarrow \lambda_1, T_S) \approx 0$.

(d) The net heat flux by radiation to the surface is determined by a radiation balance

$$
q''_{rad} = G_S - \rho S G_S - \tau S G_S - E
$$

$$
q''_{rad} = \alpha S G_S - E
$$

$$
q''_{rad} = 0.315 \times 750 \text{ W/m}^2 - 1.0 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (350K)^4 \times 615 \text{ W/m}^2.
$$
**PROBLEM 12.65**

**KNOWN:** Opaque, horizontal plate, well insulated on backside, is subjected to a prescribed irradiation. Also known are the reflected irradiation, emissive power, plate temperature and convection coefficient for known air temperature.

**FIND:** (a) Emissivity, absorptivity and radiosity and (b) Net heat transfer per unit area of the plate.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Plate is insulated on backside, (2) Plate is opaque.

**ANALYSIS:** (a) The total, hemispherical emissivity of the plate according to Eq. 12.37 is

\[
e = \frac{E}{E_b(T_s)} = \frac{1200 \text{ W/m}^2}{5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \times (227 + 273) \text{ K}^4} = 0.34.
\]

The total, hemispherical absorptivity is related to the reflectivity by Eq. 12.57 for an opaque surface. That is, \( \alpha = 1 - \rho \). By definition, the reflectivity is the fraction of irradiation reflected, Eq. 12.51, such that

\[
\rho = 1 - \frac{G_{\text{ref}}}{G} = 1 - \frac{500}{2500} = 1 - 0.20 = 0.80.
\]

The radiosity, \( J \), is defined as the radiant flux leaving the surface by emission and reflection per unit area of the surface (see Section 12.24).

\[
J = \rho G + \varepsilon E_b = G_{\text{ref}} + E = 500 \text{ W/m}^2 + 1200 \text{ W/m}^2 = 1700 \text{ W/m}^2.
\]

(b) The net heat transfer is determined from an energy balance,

\[
q_{\text{net}} = q_{\text{in}} - q_{\text{out}} = G - G_{\text{ref}} - E - q_{\text{conv}}
\]

\[
q_{\text{net}} = (2500 - 500 - 1200) \text{ W/m}^2 - 15 \text{ W/m}^2 = -700 \text{ W/m}^2.
\]

An alternate approach to the energy balance using the radiosity,

\[
q_{\text{net}} = G - J - q_{\text{conv}}
\]

\[
q_{\text{net}} = (2500 - 1700 - 1500) \text{ W/m}^2 = -700 \text{ W/m}^2.
\]

**COMMENTS:** (1) Since the net heat rate per unit area is negative, energy must be added to the plate in order to maintain it at \( T_s = 227^\circ \text{C} \). (2) Note that \( \alpha \neq \varepsilon \). Hence, the plate is not a gray body. (3) Note the use of radiosity in performing energy balances. That is, considering only the radiation processes, \( q_{\text{net}} = G - J \).
PROBLEM 12.123

KNOWN: Amplifier operating and environmental conditions.

FIND: (a) Power generation when $T_s = 58^\circ C$ with diffuse coating $\varepsilon = 0.5$, (b) Diffuse coating from among three (A, B, C) which will give greatest reduction in $T_s$, and (c) Surface temperature for the conditions with coating chosen in part (b).

SCHEMATIC:

ASSUMPTIONS: (1) Environmental conditions remain the same with all surface coatings, (2) Coatings A, B, C are opaque, diffuse.

ANALYSIS: (a) Performing an energy balance on the amplifier’s exposed surface, 

\[ \hat{E}_{in} - \hat{E}_{out} = 0 \]

Find

\[
\begin{align*}
P_e + A_S \left[ \alpha_S G_S + \alpha_{sky} G_{sky} - \varepsilon E_b - 4_{conv} \right] &= 0 \\
P_e &= A_S \left[ \varepsilon T_s^4 + h (T_s - T_\infty) - \alpha_S G_S - \alpha_{sky} \sigma T^4_{sky} \right] \\
P_e &= 0.13 \times 0.13 \text{ m}^2 \left[ 0.5 \times 331^4 + 15 \times (331 - 300) - 0.5 \times 800 - 0.5 \times \sigma (253)^4 \right] \text{W/m}^2 \\
P_e &= 0.0169 \text{ m}^2 \left[ 0.5 \times 680.6 + 465 - 0.5 \times 800 - 0.5 \times 232.3 \right] \text{W/m}^2 = 4887 \text{ W}. <
\end{align*}
\]

(b) From above, recognize that we seek a coating with low $\alpha_S$ and high $\varepsilon$ to decrease $T_s$. Further, recognize that $\alpha_S$ is determined by values of $\alpha_\lambda = \varepsilon_\lambda$ for $\lambda < 3 \text{ \mu m}$ and $\varepsilon$ by values of $\varepsilon_\lambda$ for $\lambda > 3 \text{ \mu m}$. Find approximate values as

\begin{tabular}{|c|c|c|c|}
\hline
Coating & A & B & C \\
\hline
$\varepsilon$ & 0.5 & 0.3 & 0.6 \\
$\alpha_S$ & 0.8 & 0.3 & 0.2 \\
$\alpha_S/\varepsilon$ & 1.6 & 1 & 0.333 \\
\hline
\end{tabular}

Note also that $\alpha_{sky} = \varepsilon$. We conclude that coating C is likely to give the lowest $T_s$ since its $\alpha_S/\varepsilon$ is substantially lower than for B and C. While $\alpha_{sky}$ for C is twice that of B, because $G_{sky}$ is nearly 25% that of $G_S$, we expect coating C to give the lowest $T_s$.

(c) With the values of $\alpha_S$, $\alpha_{sky}$ and $\varepsilon$ for coating C from part (b), rewrite the energy balance as

\[
\begin{align*}
P_e / A_s + \alpha_S G_S + \alpha_{sky} \sigma T^4_{sky} - \varepsilon \sigma T^4_s - h (T_s - T_\infty) &= 0 \\
4.887 \text{ W} / (0.13 \text{ m})^2 + 0.2 \times 800 \text{ W} / \text{m}^2 + 0.6 \times 232.3 \text{ W} / \text{m}^2 - 0.6 \times \sigma T^4_s - 15 (T_s - 300) &= 0
\end{align*}
\]

Using trial-and-error, find $T_s = 316.5 \text{ K} = 43.5^\circ C$. <

COMMENTS: (1) Using coatings A and B, find $T_s = 71$ and $54^\circ C$, respectively. (2) For more precise values of $\alpha_S$, $\alpha_{sky}$ and $\varepsilon$, use $T_S = 43.5^\circ C$. For example, at $\lambda T_s = 3 \times (43.5 + 273) = 950 \text{ \mu m.K}$, $F_0 \lambda T = 0.000$ while at $\lambda T_{solar} = 3 \times 5800 = 17.400 \text{ \mu m.K}$, $F_0 \lambda T = 0.98$; we conclude little effect will be seen.