THERMAL/FLUIDS SYSTEMS

DOCTORAL QUALIFYING EXAMINATION

FLUIDS MECHANICS

September 8, 2003

READ THE FOLLOWING CAREFULLY BEFORE STARTING

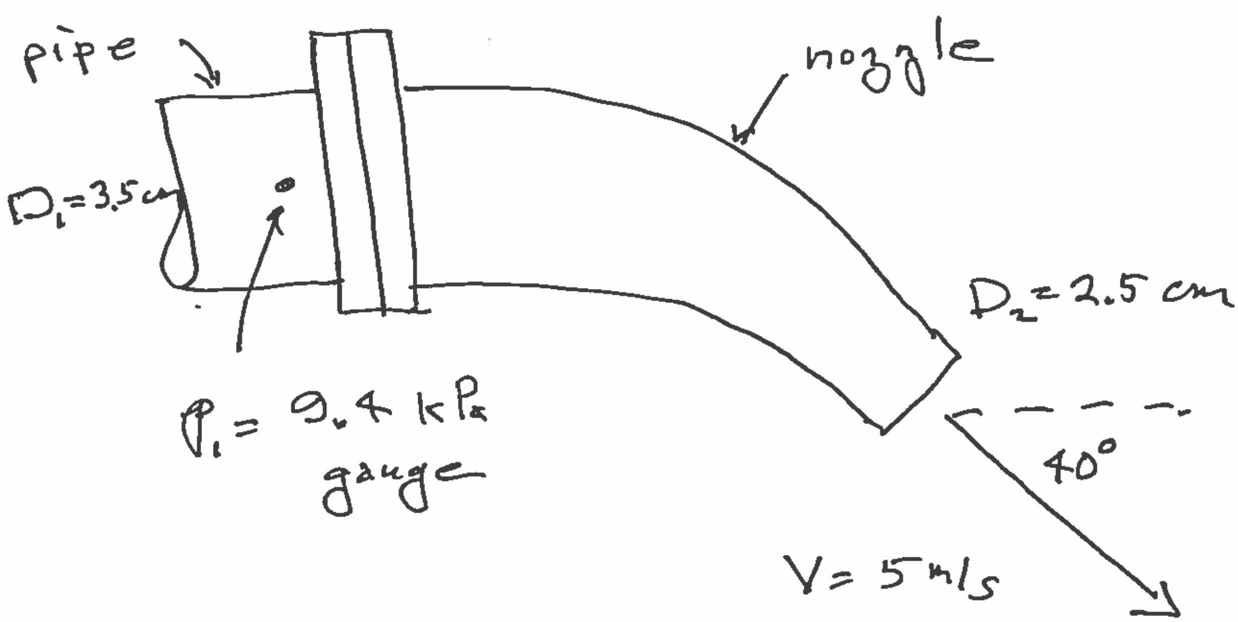
1. This is a 3 1/2 hour, closed book exam. Only reference material given out with the exam is allowed.
2. The exam includes 7 problems, and 5 of your questions will be graded.
   - You must work Problem 1 and Problem 2.
   - You may work any 3 of the remaining 5 problems.
3. Turn in solutions for only 5 problems. In the event you turn in more than 5 problems, the extra problems at the end of the exam package you turn in will be removed.
4. In addition to correctness, your answers will be judged for maturity and completeness. Show clearly any assumptions you make in order to complete a problem solution, and clearly explain your methodology.

5. Other:
   - Start each problem on a new page.
   - Write on only one side of the paper.
   - Put the last four digits of your student ID number on each page. (Do not put your name on any of the exam pages).
- Put the exam in order before turning it in.

1. (required). Oil (SG = 0.72) flows through the angled nozzle shown below. The inlet of the nozzle is 3.5 cm in diameter and the exit is 2.5 cm in diameter. The pressure at the inlet of the nozzle is P_1 = 9.4 kPa gage, and the velocity at the exit of the nozzle is 5 m/s.

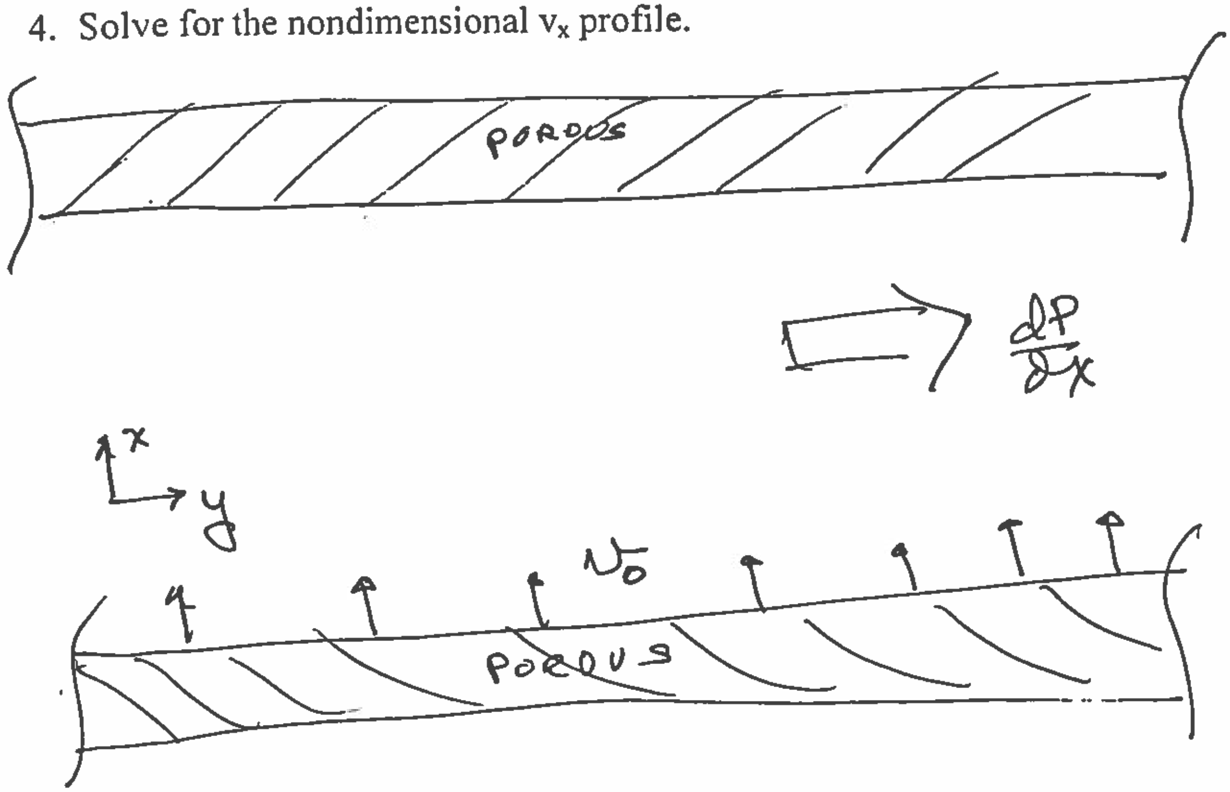
a. Verify that the given pressure at the inlet of the nozzle is reasonable (note any assumptions made in this analysis).

b. What is the <u>horizontal force</u> on the nozzle?



2. (required) Consider longitudinal flow of a liquid between two flat walls a distance H apart.. The flow is fully established and does not depend on the x-coordinate. There is a longitudinal pressure gradient dP/dx that drives the longitudinal flow. Both walls are porous and from the lower wall a uniform flow of fluid exits at velocity $v_y = v_o$

1. Find the y-direction velocity, v_y profile.
2. Write down the differential equation and boundary conditions that govern the longitudinal velocity v_x.
3. Introduce nondimensional variables and parameters into the problem
4. Solve for the nondimensional v_x profile.



3. Consider an infinite plate that is impulsively given a velocity U at time $t \geq 0$. The motion of the fluid above the plate (with constant ρ and μ) can be described by the following equation:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$$

subject to the following initial and boundary conditions:

$u(y,0) = 0$ [initial condition]

$u(0,t) = U$ [no-slip condition at wall]

$u(\infty,t) = 0$ [far field condition]

Since the flow is invariant in the x-direction, this problem has a similarity solution of the form:

$$\frac{u}{U} = f(y,t,\nu) = F(\eta)$$

where

$$\eta = \frac{y}{2\sqrt{\nu t}}.$$

Write the equation of motion and the corresponding initial and boundary conditions given above in terms of η and $F(\eta)$. Then, integrate the resulting equation to find $F(\eta)$.

NOTE: $erf(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta$

4. Assume that the velocity in the laminar boundary layer on a flat plate with constant free stream velocity U and thickness δ has the profile:

$$\frac{u}{U} = \sin\left(\frac{\pi y}{2\delta}\right)$$

Using the von Karman momentum integral equation (given below), show that

$$\frac{\delta}{x} = \frac{4.795}{\sqrt{\mathrm{Re}_x}}$$

and show that the skin friction coefficient is

$$C_f = \frac{0.655}{\sqrt{\mathrm{Re}_x}}.$$

where Re_x is the streamwise distance Reynolds number.

NOTE: The von Karman momentum integral equation is (Θ is momentum thickness, τ_o is the wall shear stress, and δ^* is the displacement thickness):

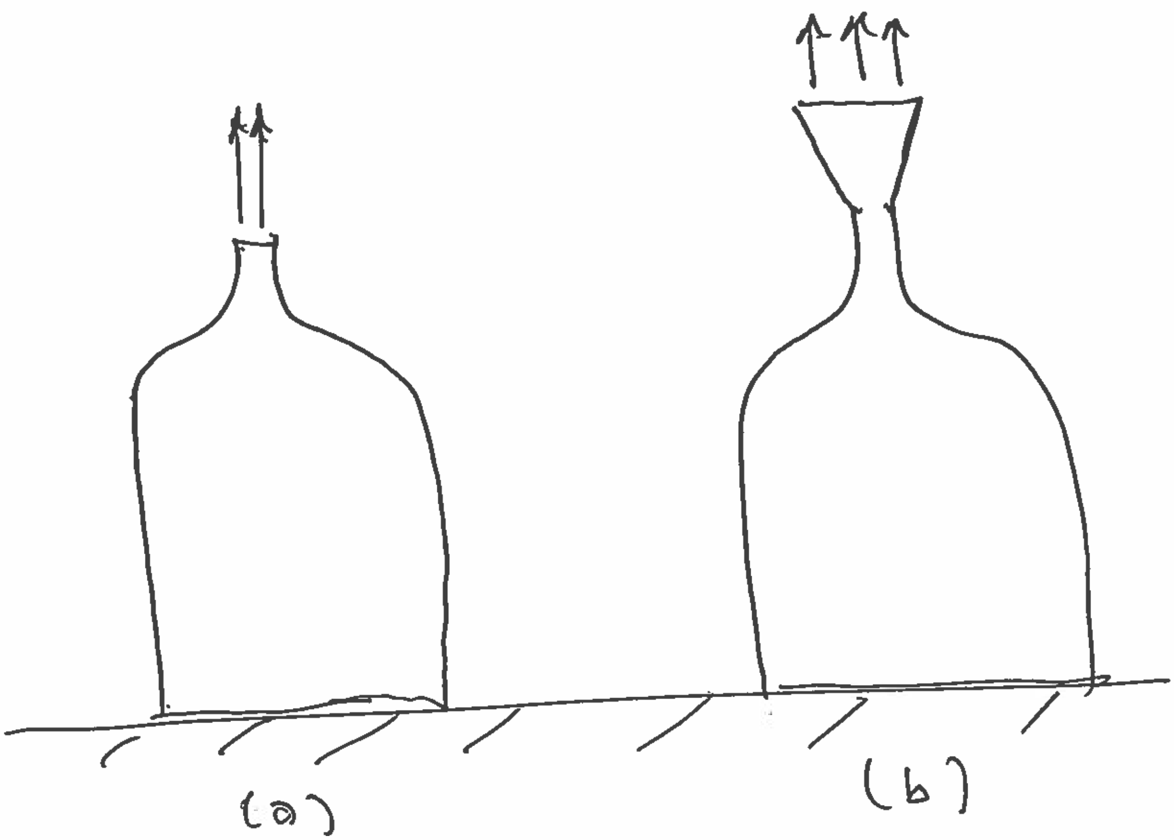
$$\frac{d}{dx}\left(U^2\theta\right) + \delta^* U\frac{dU}{dx} = \frac{\tau_0}{\rho}.$$

5. A cylindrical tank with a flat bottom holds a liquid. The maximum deflection of the bottom δ is a function of tank diameter D, liquid height H, bottom metal thickness t, fluid specific weight γ = weight / unit volume, and the metal modulus of elasticity E $\sim M / (L T^2)$. Experiments will be done on a 1/5 scale model of the tank to determine if the deflection is excessive. What is a possible nondimensional form to display the test results.

6. The exhaust nozzle at the top of a compressed air tank is opened to release the air as shown in the schematic (a) below. The pressure in the tank is 250 kPa (gage) and the temperature is 300 K. The external air is at standard atmospheric pressure and a temperature of 295 K. The nozzle is a converging nozzle with a minimum area at the exit, with an exit diameter of 4.5 mm.

a. What is the exit velocity and mass flow rate? (Note any assumptions made in your analysis.)

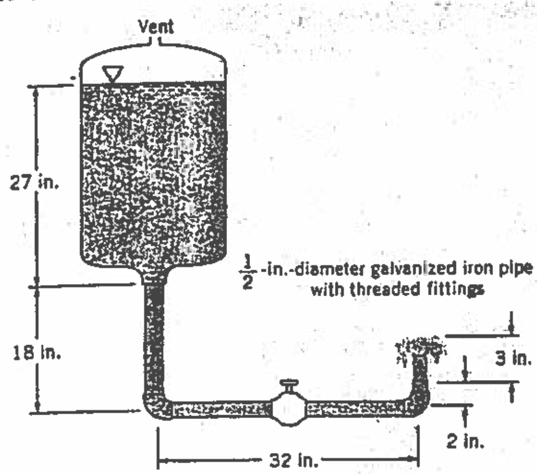
b. A diverging section is added to the exit as shown in schematic (b). If the diverging section increases the exit area by a factor of 4.0, how much does the mass flow change?



7. In the drinking water dispenser shown below, water flows from the container through the valve and jets up from the end of the pipe. The pipe is 1/2 inch inside diameter galvanized iron with an absolute surface roughness of

5×10^{-4} inches and all fittings are threaded. Entrance losses where the pipe enters the bottom of the container may be neglected.

Determine the required loss coefficient K_L for the valve to produce a "jet" of water 3 inches high at the pipe exit. Properties of water: $\rho = 62.4\ \text{lbm/ft}^3$, $\mu = 2.7\times10^{-5}$ lbf-s/ft^2. Units conversion factor: 1 lbf=32.2 lbm-ft/s^2.



C Basic Equations in Rectangular, Cylindrical, and Spherical Coordinates

Table C-1 The Equation of Continuity

Rectangular Coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Cylindrical Coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Spherical Coordinates (r, θ, φ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}(\rho v_\varphi) = 0$$

Adapted from Bird, Stewart, and Lightfoot (1960) by permission of John Wiley & Sons.

Table C-2 Components of the Rate-of-Strain Tensor $S_{ij} = \partial_{(i}v_{j)} = \frac{1}{2}\partial_i v_j + \frac{1}{2}\partial_j v_i$

Rectangular Coordinates (x, y, z):

$$S_{xx} = \frac{\partial v_x}{\partial x} \qquad S_{yx} = S_{xy} = \frac{1}{2}\left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right]$$

$$S_{yy} = \frac{\partial v_y}{\partial y} \qquad S_{zy} = S_{yz} = \frac{1}{2}\left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right]$$

$$S_{zz} = \frac{\partial v_z}{\partial z} \qquad S_{xz} = S_{zx} = \frac{1}{2}\left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right]$$

Cylindrical Coordinates (r, θ, z):

$$S_{rr} = \frac{\partial v_r}{\partial r} \qquad S_{\theta r} = S_{r\theta} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$S_{\theta\theta} = \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \qquad S_{z\theta} = S_{\theta z} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right]$$

$$S_{zz} = \frac{\partial v_z}{\partial z} \qquad S_{rz} = S_{zr} = \frac{1}{2}\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right]$$

Spherical Coordinates (r, θ, φ):

$$S_{rr} = \frac{\partial v_r}{\partial r} \qquad S_{\theta r} = S_{r\theta} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$S_{\theta\theta} = \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \qquad S_{\varphi\theta} = S_{\theta\varphi} = \frac{1}{2}\left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\varphi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \varphi}\right]$$

$$S_{\varphi\varphi} = \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r} \qquad S_{r\varphi} = S_{\varphi r} = \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \varphi} + r\frac{\partial}{\partial r}\left(\frac{v_\varphi}{r}\right)\right]$$

Adapted from Bird, Armstrong, and Hassager (1977) by permission of John Wiley & Sons.

Table C-3 Components of the Stress Tensor for Newtonian Fluids

Rectangular Coordinates (x, y, z)	Cylindrical Coordinates (r, θ, z)	Spherical Coordinates (r, θ, φ)
$\tau_{xx} = \mu\left[2\frac{\partial v_x}{\partial x} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{rr} = \mu\left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{rr} = \mu\left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{yy} = \mu\left[2\frac{\partial v_y}{\partial y} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{\theta\theta} = \mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{\theta\theta} = \mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{zz} = \mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{zz} = \mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{\varphi\varphi} = \mu\left[2\left(\frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right) - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$	$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$	$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$
$\tau_{yz} = \tau_{zy} = \mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$	$\tau_{\theta z} = \tau_{z\theta} = \mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right]$	$\tau_{\theta\varphi} = \tau_{\varphi\theta} = \mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\varphi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \varphi}\right]$
$\tau_{zx} = \tau_{xz} = \mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$	$\tau_{zr} = \tau_{rz} = \mu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$	$\tau_{\varphi r} = \tau_{r\varphi} = \mu\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \varphi} + r\frac{\partial}{\partial r}\left(\frac{v_\varphi}{r}\right)\right]$
$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$	$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$	$(\nabla \cdot \mathbf{v}) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi}$

Table C-4 Momentum Equations in Terms of τ[a]

Rectangular Coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\left[\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\left[\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right] - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\left[\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

Cylindrical Coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta r} + \frac{\partial}{\partial z}\tau_{zr} - \frac{\tau_{\theta\theta}}{r}\right] - \frac{\partial p}{\partial r} + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta\theta} + \frac{\partial}{\partial z}\tau_{z\theta} + \frac{\tau_{\theta r} - \tau_{r\theta}}{r}\right] - \frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta z} + \frac{\partial}{\partial z}\tau_{zz}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

Spherical Coordinates (r, θ, φ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\varphi}{r\sin\theta}\frac{\partial v_r}{\partial \varphi} - \frac{v_\theta^2 + v_\varphi^2}{r}\right) = -\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta r}\sin\theta)\right] + \left[\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\tau_{\varphi r} - \frac{\tau_{\theta\theta} + \tau_{\varphi\varphi}}{r}\right] - \frac{\partial p}{\partial r} + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\varphi}{r\sin\theta}\frac{\partial v_\theta}{\partial \varphi} + \frac{v_r v_\theta}{r} - \frac{v_\varphi^2\cot\theta}{r}\right) = -\left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta)\right]$$
$$+\left[\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\tau_{\varphi\theta} + \frac{(\tau_{\theta r} - \tau_{r\theta}) - \tau_{\varphi\varphi}\cot\theta}{r}\right] - \frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\varphi}{\partial t} + v_r\frac{\partial v_\varphi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\varphi}{\partial \theta} + \frac{v_\varphi}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi} + \frac{v_\varphi v_r}{r} + \frac{v_\theta v_\varphi}{r}\cot\theta\right) = -\left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\varphi}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\varphi}\sin\theta)\right]$$
$$+\left[\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\tau_{\varphi\varphi} + \frac{(\tau_{\varphi r} - \tau_{r\varphi}) + \tau_{\varphi\theta}\cot\theta}{r}\right] - \frac{1}{r\sin\theta}\frac{\partial p}{\partial \varphi} + \rho g_\varphi$$

[a] For symmetric τ set $\tau_{ij} = \tau_{ji}$.

Table C-5 Momentum Equations for a Newtonian Fluid with Constant Density (ρ) and Constant Viscosity (μ)

Rectangular Coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

Cylindrical Coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] - \frac{\partial p}{\partial r} + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] - \frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

Spherical Coordinates (r, θ, φ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\varphi}{r\sin\theta}\frac{\partial v_r}{\partial \varphi} - \frac{v_\theta^2 + v_\varphi^2}{r}\right)$$

$$= \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \varphi^2} - \frac{2}{r^2\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) - \frac{2}{r^2\sin\theta}\frac{\partial v_\varphi}{\partial \varphi}\right] - \frac{\partial p}{\partial r} + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\varphi}{r\sin\theta}\frac{\partial v_\theta}{\partial \varphi} + \frac{v_r v_\theta}{r} - \frac{v_\varphi^2\cot\theta}{r}\right)$$

$$= \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \varphi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\varphi}{\partial \varphi}\right] - \frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\varphi}{\partial t} + v_r\frac{\partial v_\varphi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\varphi}{\partial \theta} + \frac{v_\varphi}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi} + \frac{v_\varphi v_r}{r} + \frac{v_\theta v_\varphi}{r}\cot\theta\right)$$

$$= \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\varphi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\varphi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\varphi}{\partial \varphi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \varphi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \varphi}\right] - \frac{1}{r\sin\theta}\frac{\partial p}{\partial \varphi} + \rho g_\varphi$$

APPENDIX A

Isentropic Flow Tables ($\gamma = 1.4$)

M	P/P_t	T/T_t	A/A^*	M	P/P_t	T/T_t	A/A^*
0	1.0000	1.0000	∞	.60	.7840	.9328	1.1882
.01	.9999	1.0000	57.8738	.61	.7778	.9307	1.1767
.02	.9997	.9999	28.9421	.62	.7716	.9286	1.1657
.03	.9994	.9998	19.3005	.63	.7654	.9265	1.1552
.04	.9989	.9997	14.4815	.64	.7591	.9243	1.1452
.05	.9983	.9995	11.5914	.65	.7528	.9221	1.1356
.06	.9975	.9993	9.6659	.66	.7465	.9199	1.1265
.07	.9966	.9990	8.2915	.67	.7401	.9176	1.1179
.08	.9955	.9987	7.2616	.68	.7338	.9153	1.1097
.09	.9944	.9984	6.4613	.69	.7274	.9131	1.1018
.10	.9930	.9980	5.8218	.70	.7209	.9107	1.0944
.11	.9916	.9976	5.2992	.71	.7145	.9084	1.0873
.12	.9900	.9971	4.8643	.72	.7080	.9061	1.0806
.13	.9883	.9966	4.4969	.73	.7016	.9037	1.0742
.14	.9864	.9961	4.1824	.74	.6951	.9013	1.0681
.15	.9844	.9955	3.9103	.75	.6886	.8989	1.0624
.16	.9823	.9949	3.6727	.76	.6821	.8964	1.0570
.17	.9800	.9943	3.4635	.77	.6756	.8940	1.0519
.18	.9776	.9936	3.2779	.78	.6691	.8915	1.0471
.19	.9751	.9928	3.1123	.79	.6625	.8890	1.0425
.20	.9725	.9921	2.9635	.80	.6560	.8865	1.0382
.21	.9697	.9913	2.8293	.81	.6495	.8840	1.0342
.22	.9668	.9904	2.7076	.82	.6430	.8815	1.0305
.23	.9638	.9895	2.5968	.83	.6365	.8789	1.0270
.24	.9607	.9886	2.4956	.84	.6300	.8763	1.0237
.25	.9575	.9877	2.4027	.85	.6235	.8737	1.0207
.26	.9541	.9867	2.3173	.86	.6170	.8711	1.0179
.27	.9506	.9856	2.2385	.87	.6106	.8685	1.0153
.28	.9470	.9846	2.1656	.88	.6041	.8659	1.0129
.29	.9433	.9835	2.0979	.89	.5977	.8632	1.0108
.30	.9395	.9823	2.0351	.90	.5913	.8606	1.0089
.31	.9355	.9811	1.9765	.91	.5849	.8579	1.0071
.32	.9315	.9799	1.9219	.92	.5785	.8552	1.0056
.33	.9274	.9787	1.8707	.93	.5721	.8525	1.0043
.34	.9231	.9774	1.8229	.94	.5658	.8498	1.0031
.35	.9188	.9761	1.7780	.95	.5595	.8471	1.0022
.36	.9143	.9747	1.7358	.96	.5532	.8444	1.0014
.37	.9098	.9733	1.6961	.97	.5469	.8416	1.0008
.38	.9052	.9719	1.6587	.98	.5407	.8389	1.0003
.39	.9004	.9705	1.6234	.99	.5345	.8361	1.0001
.40	.8956	.9690	1.5901	1.00	.5283	.8333	1.000
.41	.8907	.9675	1.5587	1.01	.5221	.8306	1.000
.42	.8857	.9659	1.5289	1.02	.5160	.8278	1.000
.43	.8807	.9643	1.5007	1.03	.5099	.8250	1.001
.44	.8755	.9627	1.4740	1.04	.5039	.8222	1.001
.45	.8703	.9611	1.4487	1.05	.4979	.8193	1.002
.46	.8650	.9594	1.4246	1.06	.4919	.8165	1.003
.47	.8596	.9577	1.4018	1.07	.4860	.8137	1.004
.48	.8541	.9560	1.3801	1.08	.4800	.8108	1.005
.49	.8486	.9542	1.3595	1.09	.4742	.8080	1.006
.50	.8430	.9524	1.3398	1.10	.4684	.8052	1.008
.51	.8374	.9506	1.3212	1.11	.4626	.8023	1.010
.52	.8317	.9487	1.3034	1.12	.4568	.7994	1.011
.53	.8259	.9468	1.2865	1.13	.4511	.7966	1.013
.54	.8201	.9449	1.2703	1.14	.4455	.7937	1.015
.55	.8142	.9430	1.2550	1.15	.4398	.7908	1.017
.56	.8082	.9410	1.2403	1.16	.4343	.7879	1.020
.57	.8022	.9390	1.2263	1.17	.4287	.7851	1.022
.58	.7962	.9370	1.2130	1.18	.4232	.7822	1.025
.59	.7901	.9349	1.2003	1.19	.4178	.7793	1.028

APPENDIX A (continued)

Isentropic Flow Tables ($\gamma = 1.4$)

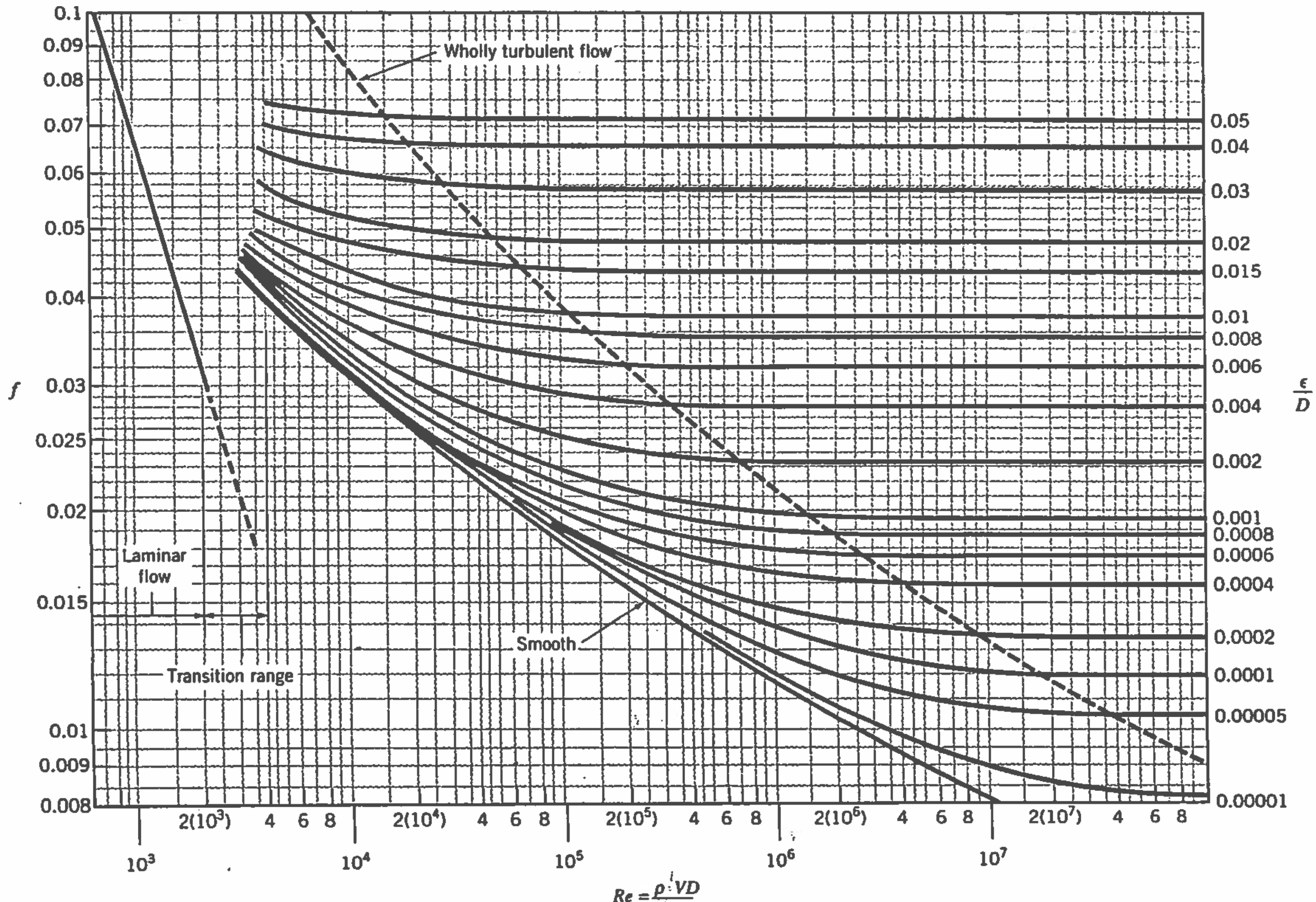
M	P/P_t	T/T_t	A/A^*
1.20	.4124	.7764	1.030
1.21	.4070	.7735	1.033
1.22	.4017	.7706	1.037
1.23	.3964	.7677	1.040
1.24	.3912	.7648	1.043
1.25	.3861	.7619	1.047
1.26	.3809	.7590	1.050
1.27	.3759	.7561	1.054
1.28	.3708	.7532	1.058
1.29	.3658	.7503	1.062
1.30	.3609	.7474	1.066
1.31	.3560	.7445	1.071
1.32	.3512	.7416	1.075
1.33	.3464	.7387	1.080
1.34	.3417	.7358	1.084
1.35	.3370	.7329	1.089
1.36	.3323	.7300	1.094
1.37	.3277	.7271	1.099
1.38	.3232	.7242	1.104
1.39	.3187	.7213	1.109
1.40	.3142	.7184	1.115
1.41	.3098	.7155	1.120
1.42	.3055	.7126	1.126
1.43	.3012	.7097	1.132
1.44	.2969	.7069	1.138
1.45	.2927	.7040	1.144
1.46	.2886	.7011	1.150
1.47	.2845	.6982	1.156
1.48	.2804	.6954	1.163
1.49	.2764	.6925	1.169
1.50	.2724	.6897	1.176
1.51	.2685	.6868	1.183
1.52	.2646	.6840	1.190
1.53	.2608	.6811	1.197
1.54	.2570	.6783	1.204
1.55	.2533	.6754	1.212
1.56	.2496	.6726	1.219
1.57	.2459	.6698	1.227
1.58	.2423	.6670	1.234
1.59	.2388	.6642	1.242
1.60	.2353	.6614	1.250
1.61	.2318	.6586	1.258
1.62	.2284	.6558	1.267
1.63	.2250	.6530	1.275
1.64	.2217	.6502	1.284
1.65	.2184	.6475	1.292
1.66	.2151	.6447	1.301
1.67	.2119	.6419	1.310
1.68	.2088	.6392	1.319
1.69	.2057	.6364	1.328
1.70	.2026	.6337	1.338
1.71	.1996	.6310	1.347
1.72	.1966	.6283	1.357
1.73	.1936	.6256	1.367
1.74	.1907	.6229	1.376
1.75	.1878	.6202	1.386
1.76	.1850	.6175	1.397
1.77	.1822	.6148	1.407
1.78	.1794	.6121	1.418
1.79	.1767	.6095	1.428
1.80	.1740	.6068	1.439
1.81	.1714	.6041	1.450
1.82	.1688	.6015	1.461
1.83	.1662	.5989	1.472
1.84	.1637	.5963	1.484

M	P/P_t	T/T_t	A/A^*
1.85	.1612	.5936	1.495
1.86	.1587	.5910	1.507
1.87	.1563	.5884	1.519
1.88	.1539	.5859	1.531
1.89	.1516	.5833	1.543
1.90	.1492	.5807	1.555
1.91	.1470	.5782	1.568
1.92	.1447	.5756	1.580
1.93	.1425	.5731	1.593
1.94	.1403	.5705	1.606
1.95	.1381	.5680	1.619
1.96	.1360	.5655	1.633
1.97	.1339	.5630	1.646
1.98	.1318	.5605	1.660
1.99	.1298	.5580	1.674
2.00	.1278	.5556	1.688
2.01	.1258	.5531	1.702
2.02	.1239	.5506	1.716
2.03	.1220	.5482	1.730
2.04	.1201	.5458	1.745
2.05	.1182	.5433	1.760
2.06	.1164	.5409	1.775
2.07	.1146	.5385	1.790
2.08	.1128	.5361	1.806
2.09	.1111	.5337	1.821
2.10	.1094	.5313	1.837
2.11	.1077	.5290	1.853
2.12	.1060	.5266	1.869
2.13	.1043	.5243	1.885
2.14	.1027	.5219	1.902
2.15	.1011	.5196	1.919
2.16	.9956 -1	.5173	1.935
2.17	.9802 -1	.5150	1.953
2.18	.9649 -1	.5127	1.970
2.19	.9500 -1	.5104	1.987
2.20	.9352 -1	.5081	2.005
2.21	.9207 -1	.5059	2.023
2.22	.9064 -1	.5036	2.041
2.23	.8923 -1	.5014	2.059
2.24	.8785 -1	.4991	2.078
2.25	.8648 -1	.4969	2.096
2.26	.8514 -1	.4947	2.115
2.27	.8382 -1	.4925	2.134
2.28	.8251 -1	.4903	2.154
2.29	.8123 -1	.4881	2.173
2.30	.7997 -1	.4859	2.193
2.31	.7873 -1	.4837	2.213
2.32	.7751 -1	.4816	2.233
2.33	.7631 -1	.4794	2.254
2.34	.7512 -1	.4773	2.274
2.35	.7396 -1	.4752	2.295
2.36	.7281 -1	.4731	2.316
2.37	.7168 -1	.4709	2.338
2.38	.7057 -1	.4688	2.359
2.39	.6948 -1	.4668	2.381
2.40	.6840 -1	.4647	2.403
2.41	.6734 -1	.4626	2.425
2.42	.6630 -1	.4606	2.448
2.43	.6527 -1	.4585	2.471
2.44	.6426 -1	.4565	2.494
2.45	.6327 -1	.4544	2.517
2.46	.6229 -1	.4524	2.540
2.47	.6133 -1	.4504	2.564
2.48	.6038 -1	.4484	2.588
2.49	.5945 -1	.4464	2.612

Isentropic Flow Tables ($\gamma = 1.4$)

M	P/P_t	T/T_t	A/A^*
2.50	.5853 -1	.4444	2.637
2.51	.5762 -1	.4425	2.661
2.52	.5674 -1	.4405	2.686
2.53	.5586 -1	.4386	2.712
2.54	.5500 -1	.4366	2.737
2.55	.5415 -1	.4347	2.763
2.56	.5332 -1	.4328	2.789
2.57	.5250 -1	.4309	2.815
2.58	.5169 -1	.4289	2.842
2.59	.5090 -1	.4271	2.869
2.60	.5012 -1	.4252	2.896
2.61	.4935 -1	.4233	2.923
2.62	.4859 -1	.4214	2.951
2.63	.4784 -1	.4196	2.979
2.64	.4711 -1	.4177	3.007
2.65	.4639 -1	.4159	3.036
2.66	.4568 -1	.4141	3.065
2.67	.4498 -1	.4122	3.094
2.68	.4429 -1	.4104	3.123
2.69	.4362 -1	.4086	3.153
2.70	.4295 -1	.4068	3.183
2.71	.4229 -1	.4051	3.213
2.72	.4165 -1	.4033	3.244
2.73	.4102 -1	.4015	3.275
2.74	.4039 -1	.3998	3.306
2.75	.3978 -1	.3980	3.338
2.76	.3917 -1	.3963	3.370
2.77	.3858 -1	.3945	3.402
2.78	.3799 -1	.3928	3.434
2.79	.3742 -1	.3911	3.467
2.80	.3685 -1	.3894	3.500
2.81	.3629 -1	.3877	3.534
2.82	.3574 -1	.3860	3.567
2.83	.3520 -1	.3844	3.601
2.84	.3467 -1	.3827	3.636
2.85	.3415 -1	.3810	3.671
2.86	.3363 -1	.3794	3.706
2.87	.3312 -1	.3777	3.741
2.88	.3263 -1	.3761	3.777
2.89	.3213 -1	.3745	3.813
2.90	.3165 -1	.3729	3.850
2.91	.3118 -1	.3712	3.887
2.92	.3071 -1	.3696	3.924
2.93	.3025 -1	.3681	3.961
2.94	.2980 -1	.3665	3.999
2.95	.2935 -1	.3649	4.038
2.96	.2891 -1	.3633	4.076
2.97	.2848 -1	.3618	4.115
2.98	.2805 -1	.3602	4.155
2.99	.2764 -1	.3587	4.194
3.00	.2722 -1	.3571	4.235
3.01	.2682 -1	.3556	4.275
3.02	.2642 -1	.3541	4.316
3.03	.2603 -1	.3526	4.357
3.04	.2564 -1	.3511	4.399
3.05	.2526 -1	.3496	4.441
3.06	.2489 -1	.3481	4.483
3.07	.2452 -1	.3466	4.526
3.08	.2416 -1	.3452	4.570
3.09	.2380 -1	.3437	4.613
3.10	.2345 -1	.3422	4.657
3.11	.2310 -1	.3408	4.702
3.12	.2276 -1	.3393	4.747
3.13	.2243 -1	.3379	4.792
3.14	.2210 -1	.3365	4.838

M	P/P_t	T/T_t	A/A^*
3.15	.2177 -1	.3351	4.884
3.16	.2146 -1	.3337	4.930
3.17	.2114 -1	.3323	4.977
3.18	.2083 -1	.3309	5.025
3.19	.2053 -1	.3295	5.073
3.20	.2023 -1	.3281	5.121
3.21	.1993 -1	.3267	5.170
3.22	.1964 -1	.3253	5.219
3.23	.1936 -1	.3240	5.268
3.24	.1908 -1	.3226	5.319
3.25	.1880 -1	.3213	5.369
3.26	.1853 -1	.3199	5.420
3.27	.1826 -1	.3186	5.472
3.28	.1799 -1	.3173	5.523
3.29	.1773 -1	.3160	5.576
3.30	.1748 -1	.3147	5.629
3.31	.1722 -1	.3134	5.682
3.32	.1698 -1	.3121	5.736
3.33	.1673 -1	.3108	5.790
3.34	.1649 -1	.3095	5.845
3.35	.1625 -1	.3082	5.900
3.36	.1602 -1	.3069	5.956
3.37	.1579 -1	.3057	6.012
3.38	.1557 -1	.3044	6.069
3.39	.1534 -1	.3032	6.126
3.40	.1512 -1	.3019	6.184
3.41	.1491 -1	.3007	6.242
3.42	.1470 -1	.2995	6.301
3.43	.1449 -1	.2982	6.360
3.44	.1428 -1	.2970	6.420
3.45	.1408 -1	.2958	6.480
3.46	.1388 -1	.2946	6.541
3.47	.1368 -1	.2934	6.602
3.48	.1349 -1	.2922	6.664
3.49	.1330 -1	.2910	6.727
3.50	.1311 -1	.2899	6.790
3.51	.1293 -1	.2887	6.853
3.52	.1274 -1	.2875	6.917
3.53	.1256 -1	.2864	6.982
3.54	.1239 -1	.2852	7.047
3.55	.1221 -1	.2841	7.113
3.56	.1204 -1	.2829	7.179
3.57	.1188 -1	.2818	7.246
3.58	.1171 -1	.2806	7.313
3.59	.1155 -1	.2795	7.382
3.60	.1138 -1	.2784	7.450
3.61	.1123 -1	.2773	7.519
3.62	.1107 -1	.2762	7.589
3.63	.1092 -1	.2751	7.659
3.64	.1076 -1	.2740	7.730
3.65	.1062 -1	.2729	7.802
3.66	.1047 -1	.2718	7.874
3.67	.1032 -1	.2707	7.947
3.68	.1018 -1	.2697	8.020
3.69	.1004 -1	.2686	8.094
3.70	.9903 -2	.2675	8.169
3.71	.9767 -2	.2665	8.244
3.72	.9633 -2	.2654	8.320
3.73	.9500 -2	.2644	8.397
3.74	.9370 -2	.2633	8.474
3.75	.9242 -2	.2623	8.552
3.76	.9116 -2	.2613	8.630
3.77	.8991 -2	.2602	8.709
3.78	.8869 -2	.2592	8.789
3.79	.8748 -2	.2582	8.870

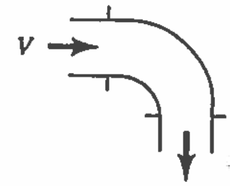


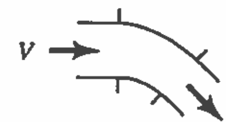
■ FIGURE 8.20 **Friction factor as a function of Reynolds number and relative roughness for round pipes—the Moody chart. (Data from Ref. 7 with permission.)**

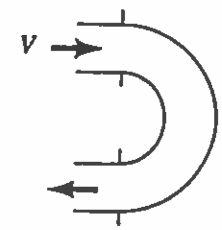
■ **TABLE 8.2**

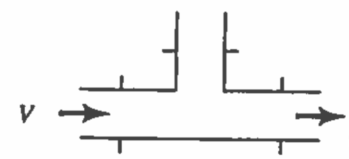
Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$ **(Data from Refs. 5, 10, 27)**

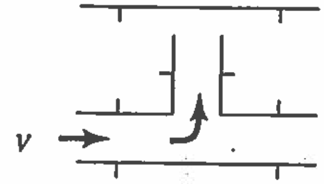
Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
***e. Valves**	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210

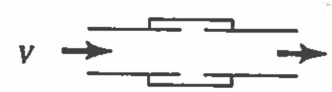












*See Fig. 8.36 for typical valve geometry