THERMAL/FLUIDS SYSTEMS

DOCTORAL QUALIFYING EXAMINATION

PART I

FLUIDS MECHANICS

January 15, 2003

READ THE FOLLOWING CAREFULLY BEFORE STARTING

1. This is a 3 1/2 hour, closed book exam. Only reference material given out with the exam is allowed.

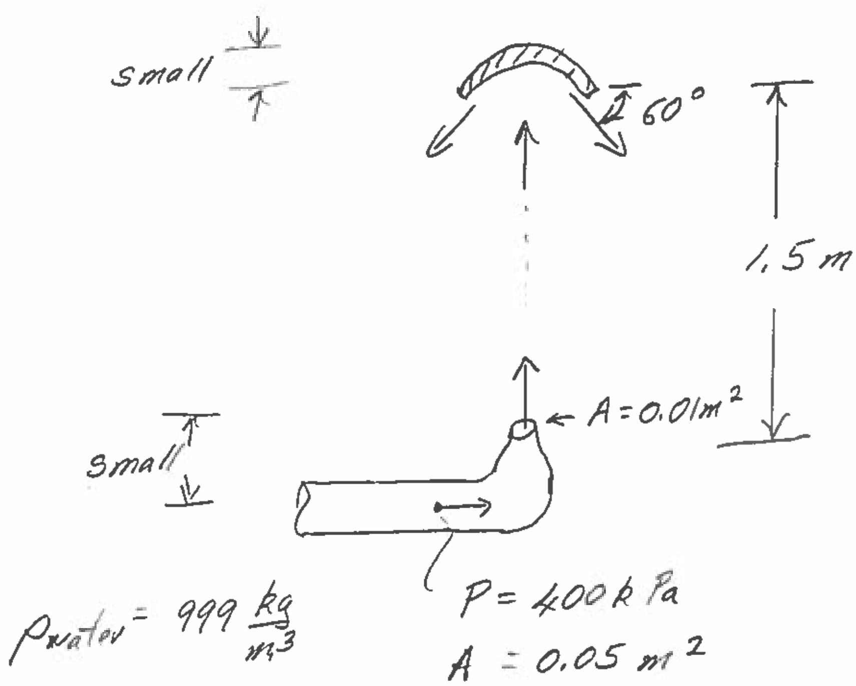
2. The exam includes 7 problems, and 5 of your questions will be graded.
   - You must work Problem 1 and Problem 2.
   - You may work any 3 of the remaining 5 problems.

3. Turn in solutions for only 5 problems. In the event you turn in more than 5 problems, the extra problems at the end of the exam package you turn in will be removed.

4. In addition to correctness, your answers will be judged for maturity and completeness. Show clearly any assumptions you make in order to complete a problem solution, and clearly explain your methodology.

5. Other:
   - Start each problem on a new page.
   - Write on only one side of the paper.
   - Put the last four digits of your student ID number on each page. (Do not put your name on any of the exam pages).
   - Put the exam in order before turning it in.

1. A nozzle with outlet area $0.01 m^2$ shoots water vertically upward. The pressure at the inlet to the nozzle is 400 kPa absolute and the area is $0.05 m^2$. At a height of 1.5 m the water encounters an axisymmetric cap with edge angle of 60° as shown. How heavy is the cap in order to remain at the 1.5 m position?



2. Consider laminar boundary layer flow over a flat plate, with constant free-stream velocity, U:

a) Write down the boundary layer equations for this flow.

b) The momentum integral for this flow is written as:

$$\frac{d}{dx}\int_0^\delta u(U-u)dy = \frac{\tau_0}{\rho}$$

Assuming an approximate velocity profile of the following form,

$$\frac{u}{U} = a_0 + a_1\frac{y}{\delta} + a_2\left(\frac{y}{\delta}\right)^2$$

evaluate the constants a_0, a_1, and a_2 and use the momentum integral to find an expression for how the boundary layer thickness, δ, and the shear stress on the surface, t_0, vary with x. Express your answer in non-dimensional form.

3. Consider the ideal case of incompressible flow around an infinitely long circular cylinder of diameter D. The freestream velocity, U, is normal to the axis of the cylinder and the cylinder is smooth. Answer the following questions:

a) Sketch C_D, the drag coefficient vs. Re, the Reynolds number from Re of 0.1 to Re of 1 million.
b) Why is the drag coefficient a function of Re?
c) What is the physical interpretation of Re?
d) Sketch C_p from the forward stagnation point to the rear "stagnation point" for both laminar and turbulent flow. Where C_p is the (local surface pressure minus the stagnation pressure) / freestream dynamic pressure.
e) Explain why the separation point for a turbulent boundary layer is further downstream than the separation point for a laminar boundary layer
f) What happens if we apply a "slightly" rough surface to the cylinder?
g) What is Stokes flow and show this region on your C_D vs. Re plot.
h) What is the so-called Karman vortex street?
i) At what freestream Mach number will compressible effects become important?
j) What is the dominant contribution to the drag force, i.e. is it related to skin friction or pressure, at a moderate Re of 100,000

4. A common method for determining the viscosity of a liquid is to drop a metal sphere of diameter **D** into the liquid and measure the time, **t**, required for it to fall a specified distance **h**. If the density of the sphere and the liquid are ρ_s and ρ, respectively, buoyancy forces on the sphere will be proportional to $\Delta\gamma = g(\rho_s - \rho)$, the difference in weight density between the sphere and the liquid; the viscosity, μ can be expressed as a function of **D**, $\Delta\gamma$, **h**, and **t**.

   (a) Would **h**, **D**, and ρ constitute a suitable set of repeating variables from which to form dimensionlesss parameters (Π groups)? Explain your answer.

   (b) Develop a dimensionless parameter for the viscosity, μ, using **t**, **h**, and $\Delta\gamma$ as non-dimensionalizing (repeating)variables.

   (c) In one such test, water at 20 °C (ρ=998 kg/m^3, μ=1x10^{-3} N-s/m^2) is used as a calibrating fluid, and it requires 1.3 seconds for a steel ball bearing (SG=7.83) to sink a distance of 50 cm. The test is repeated using the same ball bearing in a test liquid whose SG is 0.8 and the ball requires exactly 2 seconds to fall the same distance. What is the viscosity of the test liquid (N-s/m^2)?

5. Consider the five compressible flow cases listed below. In each case indicate whether the following quantities increase, decrease, or stay constant: *M, T_0, T, P_0, P, and V.*

   a. A converging-diverging nozzle operating at design conditions with a supersonic exit Mach number.

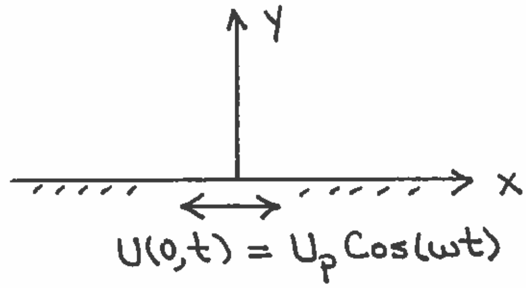
   b. Flow through a normal shock.

   c. flow in an adiabatic, constant area duct with friction, and with an inlet Mach number less than 1.0.

   d. flow in a frictionless, constant area duct with heat addition, and with an inlet Mach number less than 1.0.

   e. flow in a frictionless, constant area duct with heat addition, and with an inlet Mach number greater than 1.0.

6. An infinite flat plate undergoes a sinusoidal oscillation with frequency ω in the x-z plane as shown below. The fluid extends to infinity in both the x and y direction. Write down the simplified differential equation and boundary conditions and solve for the resulting velocity profile, u(y,t).



7. The differential equation for potential flow past a right circular cylinder is

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = 0$$

The relationship between velocity components and the stream function are

$$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad and \quad V_\theta = -\frac{\partial \psi}{\partial r}$$

The boundary condition at infinity is

$$V_r = -U_\infty Cos(\pi - \theta) = U_\infty Cos\theta \quad \text{as} \quad r \to \infty$$

while the surface boundary condition is

$$V_r(a,\theta) = 0$$

In addition the flow is symmetrical about the x-axis. Solve this system for the stream function and then find the surface velocity $V_\theta(a,\theta)$.