

**THERMAL/FLUID SYSTEM
DOCTORAL QUALIFYING EXAMINATION**

HEAT TRANSFER

21 JANUARY 2003

READ THE FOLLOWING CAREFULLY BEFORE STARTING

1. This is a 3 ½ hour closed book exam. No external reference material is allowed, other than provided with the exam.
2. You will be graded on a total of five problems. Seven problems are presented to you.
 - You must work Problem 1 and Problem 2
 - You may work any three of the remaining five problems
3. Turn in solutions for only 5 problems. In the event you turn in more than 5 problems, the extra problems that are at the end of the package of exam papers you turn in will be removed.
4. In addition to correctness, your answers will be judged for maturity and completeness.
 - Show clearly any assumptions you make in order to complete a problem solution.
 - Start each problem on a new sheet of paper.
 - Write on only one side of the paper
 - Put the last four digit of your student ID on each page.
 - Put the exam in order before turning it in.
5. All notes and scratch paper associated with the exam must be turned in.

Problem 1 (Required)

Fluid enters a circular tube at a specified velocity and eventually obtains a thermally fully-developed flow condition. The Reynolds number of the flow is $Re = 100,000$, and the fluid Prandtl number is $Pr = 0.7$. The thermal boundary condition over the entire length of the pipe is constant wall temperature. The circular pipe has a diameter D and a length L .

a) Define the criterion for testing whether the fluid is in a thermally fully-developed state, and describe what experimental information would be required to prove that the flow is thermally fully developed.

b) If the thermal boundary condition is changed to that of constant wall heat flux, would the fully developed Nusselt number change, and if so by how much (what percentage)?

c) If the pipe were changed to a square duct with the same velocity, size ($D \times D$), and Pr , would the Nusselt number go up, stay the same, or be reduced. On what fact(s) do you base your answer?

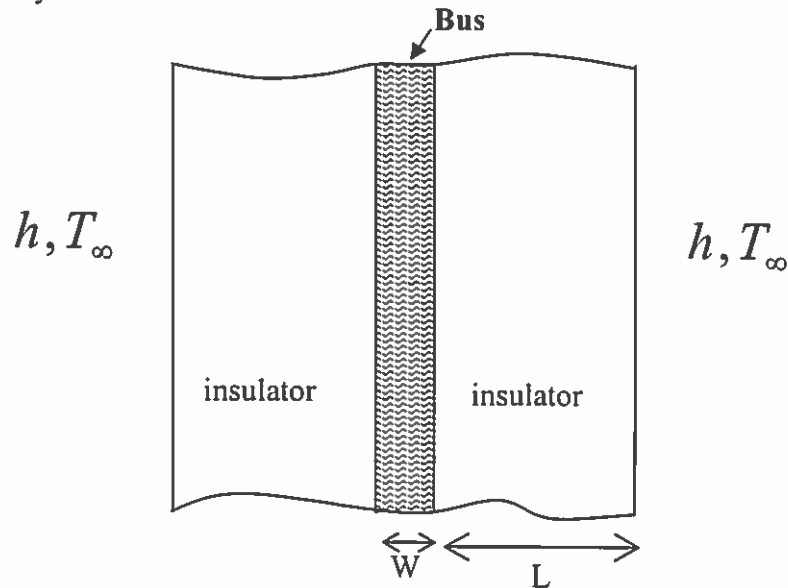
d) Consider now that the Prandtl number may be changed, but the velocity, diameter and Reynolds number remain the same as for the original circular tube. Sketch a plot of heat transfer coefficient (h) versus x/D for Prandtl numbers of 0.01, 0.7, and 100. Identify these sequentially as d1, d2, and d3.

Now assume the Reynolds number is reduced by a factor of 100. On the same plot, sketch the heat transfer coefficient (h) versus x/D for Prandtl numbers of 0.01, 0.7, and 100. Identify these sequentially as d4, d5, and d6. You will have a total of 6 lines on the one plot.

e) An engineer recommends changing the original design such that the diameter D is reduced by a factor of 10, but there will be 10 such tubes in parallel to keep the same tube surface area. The velocity remains the same and as a result, the Re will be substantially reduced. Is this design change appropriate from a total heat transfer point of view? Explain your reasoning, using equations and principles as appropriate.

Problem 2 (Required)

A long flat electrical conducting bus of very large cross section is sandwiched between two insulators. Power is dissipated uniformly within the bus as a function of time, the volumetric heating rate in the bus being $\dot{S}'''(t)$. The insulators are cooled on their periphery by a convective flow. The bus has density ρ_b , thermal conductivity k_b , specific heat capacity c_b , and thickness W . Each layer of the insulating material has density ρ_i , thermal conductivity k_i , specific heat capacity c_i , and thickness L . At the initial time, all the elements are at the ambient temperature, T_∞ . Please answer the following questions about this system.



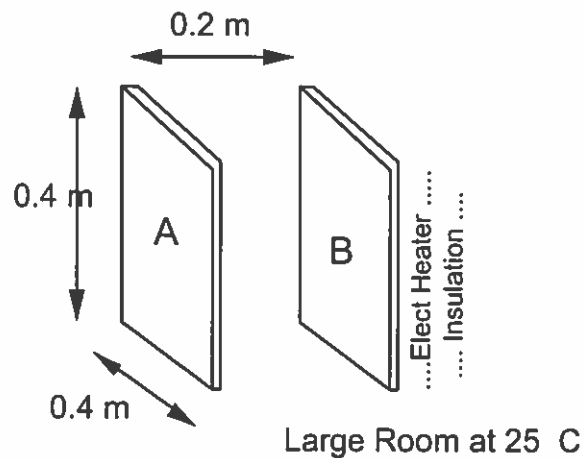
- Write the equations and boundary conditions that govern the evolution of the temperature in this system including the bus and the insulator.
- Discuss the validity and approximations that would be required to get a globally valid solution with a lumped temperature bus and spatially varying temperature in the insulator layers.
- If the heater is turned on at time $t = 0$ and the heating rate remains constant for $t > 0$, sketch the temperature distribution in both the insulator and the bus after the heater has been turned on for a short time and for a very long time. Do not assume lumped temperature in the bus. Identify these two cases sequentially as c1 and c2.
- After the heater has been turned on with a constant heating rate for a very long time, it is suddenly turned off. In the same plot that you make for c), sketch the temperature distribution in the bus and in the insulator when the average bus temperature cools down to 90% of its highest value, and when the average bus temperature reaches 10% of its highest value. Identify these two cases sequentially as d1 and d2.

Problem 3 (Optional)

Two plates (each 0.4 m square and separated by 0.2 m) are suspended in a large evacuated room at 25 C, as shown in sketch. Plate B with 0.7 emissivity is held at 300 C by a heater sandwiched between it and (perfect) insulation on its back (r.h.s.). Plate A, which faces plate B on its right and the room on its left, is black on both sides. Assume that the view factor between the two plates is $F_{A-B} = 0.45$

- (1) If plate A is maintained at 100 C by some active heating or cooling device, what are:
 - a. the heater power (Watts) required for plate B ?
 - b. the heating requirement or cooling requirement (Watts) for Plate A ?

- (2) Now assume the room has air at atmospheric pressure and 25 C, and plate A is not actively heater or cooled. Explain in detail with appropriate equations, assumptions and discussion (but do not actually calculate) how you would determine:
 - a. the heater power required for plate B
 - b. the temperature of plate A.



Problem 4 (Optional)

A thin rod of length L has its two ends connected to two walls which are maintained at temperatures T_1 and T_2 , respectively. The cross section of the rod is constant, of area A_c and perimeter P and the thermal conductivity of the rod is denoted by k . An electrical heating element is placed in the rod so that heat is generated uniformly along the length at a rate \dot{q} per unit volume. The rod loses heat to the air at T_∞ through a heat transfer coefficient, h . Derive expressions for (a) the temperature distribution along the rod, (b) the heat lost by the rod to the air.

Problem 5 (Optional)

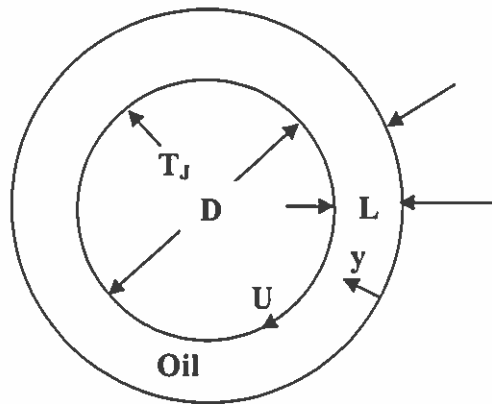
Consider a lightly loaded journal bearing using oil having the constant properties: viscosity $\mu = 10^{-2}$ kg/s-m; thermal conductivity $k = 0.15$ W/m-K.

a) Make suitable assumptions and derive the following expression for the oil temperature

$$T(y) = T_B + \frac{\mu}{2k} U^2 \left[\frac{y}{L} - \left(\frac{y}{L} \right)^2 \right] + (T_J - T_B) \frac{y}{L}$$

where T_B is the bearing temperature, T_J is the journal temperature, U is the tangential velocity of the journal and L is the bearing-journal gap dimension. Note that $L \ll D$, where D is the diameter of the journal. (see figure below)

b) If the journal and the bearing are each maintained at a temperature of 40°C , what is the maximum temperature in the oil when the journal is rotating at 10 m/s?



Problem 6 (Optional)

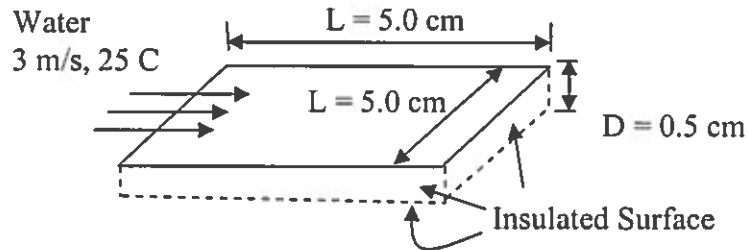
In convective heat transfer, the Reynold's analogy is an important concept in relating heat and momentum transport.

- a) For laminar external boundary layer flow over a flat plate, write the exact boundary layer forms of the momentum and energy equations that are required to develop the proof of the Reynold's analogy. Do not non-dimensionalize the equations.
- b) Write the boundary conditions for part a).
- c) Write the Reynold's analogy that represents the solution to parts a) and b). Be sure to define whatever symbols you use.
- d) Assume the flow is a turbulent boundary layer. Discuss how your answers to parts a), b), and c) compare or differ. That is, does a similar analogy exist for turbulent flow? What assumptions are required?

Problem 7 (Optional)

A thin, flat wafer of Nichrome after manufacturing is embedded in an insulated surface. Initially, the temperature of the wafer is 200 C. The wafer is then cooled down by exposing its top surface to a steady stream of water at 3 m/s and 25 C.

Below is a schematic of the wafer.



The properties of the wafer and the water are listed below.

| | ρ (kg/m ³) | k (W/mK) | C_p (KJ/kgK) | ν (m ² /s) | Pr |
|----------|-----------------------------|------------|----------------|---------------------------|-------|
| Nichrome | 8400 | 12 | 0.42 | ----- | ----- |
| Water | 1000 | 0.613 | 4.20 | 1.00 E-6 | 5.85 |

In your analysis below, assume the spatially averaged Nusselt number is:

$$\overline{Nu}_L = 25$$

- In this analysis, you will treat the wafer cooling as a lumped system. Is this a reasonable approximation? (Justify your answer.)
- From basic principles, develop the governing equation for the wafer temperature assuming lumped analysis and constant properties. Then integrate the equation analytically in order to derive an expression for the wafer temperature as a function of time.
- What is the time constant for this wafer?
- Estimate the time it takes for this wafer to complete 90% of its cooling process.