ADAPTED CONCEPT GENERATION AND COMPUTATIONAL TECHNIQUES FOR THE APPLICATION OF A TRANSFORMER DESIGN THEORY

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ABSTRACT

Transformers are a class of products with great potential in a number of markets and applications. These are systems that exhibit a change in state to facilitate new or enhanced product functionality. The historical children’s toys known as “transformers” provide a mental picture of this definition. Working examples are vertical lift aircraft that function as helicopters for take-offs but transform to propeller-driven airplanes for point-to-point travel. This paper builds on research into developing principles and design methodologies for the creation of transforming products. We summarize this research and demonstrate an approach for implementing Transformer Design Principles as part of an ideation and computational design process. An application to an Unmanned Aerial Vehicle (UAV-TACMAV) illustrates the utility of the approach.

KEYWORDS: Transformational Design Theory, Transforming Wing, Morphing Wing, Transformational Optimization, Metamodel Design

NOMENCLATURE

\( L \)  lift  \\
\( C_L \)  lift coefficient  \\
\( \rho \)  density of air  \\
\( V \)  velocity of wing relative to free stream velocity  \\
\( A \)  area of wing  \\
\( h \)  wing camber  \\
\( c \)  wing chord  \\
\( \alpha \)  angle of attack of wing relative to free stream  \\
\( D \)  total drag on wing  \\
\( C_D \)  drag coefficient  \\
\( C_{D,\text{min}} \)  minimum drag coefficient  \\
\( AR \)  aspect ratio of wing  \\
\( e \)  Oswalt’s efficiency factor  \\
\( L_{\text{min}} \)  minimum acceptable lift  \\
\( D_{\text{max}} \)  maximum acceptable drag

1. INTRODUCTION

A transformer is a system that exhibits a state change in order to facilitate new or enhance existing functionality. Transformational design facilitates the development of products...
with a much broader functional repertoire than traditional single-state designs.

Current design theory lacks a systematic methodology for the creation of products that have the ability to transform [20, 27]. A methodology is currently in development that will allow for the organized and reliable creation of transformers. The completed methodology will involve a set of principles and facilitators governing transformation and an associated process for their application to design problems. This paper contributes the first steps toward this process, namely, an ideation and computational approach to implement transformer theory. To illustrate the process and demonstrate utility, the transformation theory is applied to a specific design problem: the design of a lightweight, quickly deployable, easily operable, and low-storage volume wing for a micro aerial vehicle.

This work begins by describing the initial development of a transformational design methodology. Several of the transformational design principles and facilitators are listed and discussed in general terms. These are then applied to a design problem and used to create several concepts. The preliminary concepts are evaluated and one concept is selected for development. This idea is embodied as a CAD model which is then computationally optimized for performance.

2. MOTIVATION

2.1 Transformation

Transformational design greatly increases the ability of the designer to create products that satisfy a broad range of customer needs. Multifunctional devices can be designed that perform their tasks with greater efficiency than traditional devices, and functionality can be obtained that was previously unavailable. A systematic methodology to aid in the development of such devices is required to advance the science of design into this unexplored area.

The state changes that occur in transformers may occur in a variety of domains (chemical, electrical, etc.), but for the purposes of this research, only mechanical transformation is considered. Mechanical transformers exhibit a change in physical dimension and/or form to perform separate functions or enhance an existing function.

Devices exhibiting transformational qualities currently exist, but a methodology is desired to facilitate their creation. Such a methodology will provide designers with a set of comprehensive design principles and facilitators that can be used to produce transforming devices, and a process by which this transformation theory can be applied.

Transforming devices hold many advantages over single-state designs. The primary advantage is that the same device is able to perform multiple functions. By creating a transforming product that performs tasks usually requiring multiple devices, increases in efficiency can be realized throughout a wide range of customer needs. For example:

• Cost may be reduced since labor and material is typically less for a single device.
• Benefits may be achieved in weight-sensitive applications
• Benefits may be achieved by having one system that can accomplish the functions that may previously have been done by separate products.

• Scheduling and logistical issues may be simplified by using multifunctional devices.
• Certain transforming devices may perform functions between states that are not possible in single-state products.
• Deployment time may be reduced for many designs.
• The whole is greater than the sum of its individual parts. A single design that can transform to perform multiple functions may have an increased functional repertoire compared to several single-state devices.
• The functions that a single product can perform need not be related or require the same structural layout, as with other single-state multifunctional devices.

A methodology for the design of transforming devices will include a set of principles that define transforming devices, and a collection of facilitators that can be used in their design. Collectively these principles and facilitators constitute a developing theory of design for transformation. Accompanying tools will be used to apply these to the design of transformers. Research has produced a number of principles and facilitators that can be employed to aid in the design of transforming devices. They were derived primarily through an inductive process that includes studying patents, products and natural analogies. This principle development process as well as a comprehensive description of the theory is described in a separate work [1]. A subset of these principles and facilitators is given below:

2.1.1 Principles

A transformation principle is a generalized directive to bring about a certain type of mechanical transformation. They are guidelines that, when embodied singly, create a transformation [1].

• Fuse/Divide – Make a single functional device become two or more devices, at least one of which has its own distinct functionality defined by the state of the transformer or vice versa. Several different individual parts or systems (which may or may not have distinct functionality) can combine to create a single separately functioning device. Similarly, a single functioning device or part may also decompose into two or more components. The memory module from the mp3 player in Figure 1 detaches and becomes a USB drive.

• Expand/Collapse – Change physical dimensions of an object along an axis, in a plane, or in 3-dimensional space.

Figure 1: Creative MuVo USB 2.0 [2]
A device is designed to expand and/or collapse, allowing for storage or altered functionality depending on the state of the device. Expansion/collapse can embody combinations of different mechanisms and/or flexible/elastic materials. Shown in Figure 2, Chuck Hoberman’s unfolding/deployable structure demonstrates this principle. It is a transforming device that uses the expansion/collapse principle.

![Figure 2: Hoberman Collapsible storage containers [3]](image)

Nature has its own share of transformers that use expansion/collapse to alter functionality by altering their configurations. Figure 3 shows a puffer fish which uses the flexibility of its skin as a membrane, which expands when it fills its body with water, to intimidate enemies. The armadillo in Figure 4 uses its ability to wrap around itself to reconfigure into a ball, only exposing its armor and therefore protecting itself. The wheel spider shown in Figure 5 collapses its legs towards its body to transform into a wheel structure to roll down sand dunes rather than crawl down.

![Figure 3: Puffer Fish [4]](image)

The bag in Figure 6 expands from a towel to a tote bag when the strings on periphery of the towel are pulled outward. In the tote configuration, the strings function as the shoulder straps.

![Figure 4: Armadillo [5]](image)

![Figure 5: Wheel Spider [6]](image)

2.1.2 Facilitators

A facilitator is a design architect that helps or aids in creating mechanical transformation. They aid in the design for transformation, but their implementation does not create transformation singly [1].

- **Shared power transmission** – Transmit power from a common source to perform different functions in different configurations. Power is transmitted from a single power source (i.e. an engine, motor, etc.) to perform separate tasks in different states. The Osprey™ shown in Figure 7 uses the same engines and propellers to lift off vertically and to fly horizontally.

- **Function Sharing** – Perform two or more discrete functions. A single part is designed so that it can perform different functions required by the different configurations of the device. The wheels on the car shown in Figure 8 are designed so that the rims double as propellers when the car transforms into a boat.

- **Furcate** – Change between two or more discrete stable states determined by boundary conditions. A product is designed with multiple stable states, and the transition between these states is defined by a set of boundary conditions imposed upon it. The common “slap bracelet” toy is an example of this facilitator; it is stable in its high-energy extended state until part of its cross-section is flattened, at which time it collapses to a lower-energy coiled state, shown sequentially in Figure 9.
A steel tape measure and Storable Tubular Elastic Membranes (STEM) [11] have similar bi-stable structural properties that make them capable of automatically varying their shape from a compact, packaged configuration to an expanded configuration. A tape spring and a STEM structure are shown in Figures 10 and 11.
Another interesting furcating product is a Phlat Ball, in Figure 12, which transforms into a disc and ball, both being stable states. It achieves this property by the use of its outer collapsible structure with flexible joints, a suction cup inside to hold the two inner surfaces of the product in the disc configuration and a spring that helps the disc pop back to ball configuration when forces are applied on the sides of the disc.

The Venus Fly Trap can close its leaves fast enough to snare a fly, shown in Figure 13. It does this by releasing stored elastic energy in its leaves, reversing its curvature allowing for a very quick reaction.

An umbrella patent example, as shown in Figure 14, is an example of a furcating transforming product with two stable states – an open state and a closed state.

The proper employment of transformation principles and facilitators will result in increased efficiency of the design of transformers. Table 1 is a list of all principles and facilitators that have been developed to this point. While the applications of this theory can range across a wide range of products, this paper focuses on their application to a specific class of product: micro unmanned aerial vehicles.

2.2 Micro Aerial Vehicles

Micro aerial vehicles (MAV) are used extensively in military operations to perform functions such as reconnaissance and surveillance. These situations in which MAVs are used often require rapid deployment of the device, frequently in imperfect conditions. Current MAVs typically are stored in either a disassembled state that requires a series of assembly operations or in an assembled state that requires a relatively large storage volume. A lightweight, quickly deployable, and easily operable alternative that can be stored in a smaller volume is desired. Improvements in these areas will allow soldiers on the battlefield to function more efficiently both while deploying the MAV and transporting it [17].

The wing usually requires the greatest storage volume per unit mass of the plane and therefore presents the greatest challenge when attempting to reduce its stored volume. The goal of this application is to use the transformation theory that has been developed to design a wing that can be used in such a capacity and is more easily stored than current designs.
Figure 15: Problem Approach

After the development of several concepts, the premier idea is embodied and optimized. The design under consideration for optimization is a bi-stable wing design that is stable in its extended state and is also capable of maintaining a coiled configuration. This wing will provide a suitable lifting surface when extended and can be rapidly returned to its coiled state for storage when not in use. Optimization is required in order to ensure that the wing will provide the greatest range possible using this design. To accomplish this, a metamodel is developed using data from series of virtual experiments, because the analysis is otherwise very computationally intensive. Optimization is then performed to maximize the lift-to-drag ratio while maintaining other necessary flight characteristics. The final design is a quickly and easily deployable wing that greatly reduces storage volume with only slight degradation in flight performance.

3. RESEARCH OBJECTIVES

A set of transformational design principles and facilitators have been developed that provide insights into the nature of transformers. The question that naturally arises is: how is this theory mapped to working concepts? The objectives of this research is to demonstrate: (1) the application of transformational design theory to a practical design problem, (2) the use of transformation theory in the context of existing design techniques, and (3) the application of metamodeling to optimize embodiment decisions that arise in transformational design. In order to accomplish these overarching goals, a design problem is proposed: the development and optimization of a transforming, easily storable wing with acceptable aerodynamic properties. The wing concept will be designed using an experimental methodology which utilizes the transformational design principles and facilitators. The aims of optimization are to maximize the lift to drag ratio of a small wing while maintaining a predefined performance envelope.

Table 1: Transformation Principles and Facilitators [1]

<table>
<thead>
<tr>
<th>Principles</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expand/Collapse</td>
<td>Change physical dimensions of object along and axis, in a plane, or in three dimensional space</td>
</tr>
<tr>
<td>Expose/Cover</td>
<td>Expose/Cover a new surface to alter functionality</td>
</tr>
<tr>
<td>Fuse/Divide</td>
<td>Make single functional device become two or more devices, at least one of which has its own distinct functionality defined by the state of the transformer or vice versa</td>
</tr>
<tr>
<td>Common core structure</td>
<td>Compose devices with a core structure that remains the same, while the periphery reconfigures to alter the function of the device</td>
</tr>
<tr>
<td>Composite</td>
<td>Form a single part from two or more parts with distinct functionality</td>
</tr>
<tr>
<td>Conform with Structural Interfaces</td>
<td>Statically or dynamically constrain the motion of a component using structural interfaces</td>
</tr>
<tr>
<td>Enclosure</td>
<td>Manipulate object in two or three dimensions in order to enclose a three dimensional space</td>
</tr>
<tr>
<td>Flip</td>
<td>Perform separate functions based on orientation of the object</td>
</tr>
<tr>
<td>Function Sharing</td>
<td>Perform two or more discrete functions</td>
</tr>
<tr>
<td>Function</td>
<td>Change between two or more discrete stable states determined by boundary conditions</td>
</tr>
<tr>
<td>General Connections</td>
<td>Employ internal or external connections (structural, power) that can be used by different modules to perform different functions or perform the same function in a different way</td>
</tr>
<tr>
<td>Interchangeable transmissions</td>
<td>Use multiple transmissions to produce different motions</td>
</tr>
<tr>
<td>Material Flexibility</td>
<td>Change object dimensions with change in boundary conditions</td>
</tr>
<tr>
<td>Modularity</td>
<td>Localize related functions utilizing common signal, material, and force flows into subsystems (modules) which are easily integrated into the device and may be interchangeable</td>
</tr>
<tr>
<td>Nesting</td>
<td>Place an object inside another object wholly or partially wherein the internal geometry of the containing object is similar to the external geometry of the contained object</td>
</tr>
<tr>
<td>Segmentation</td>
<td>Divide single contiguous part into two or more parts</td>
</tr>
<tr>
<td>Shared Power Transmission</td>
<td>Transmit power from a common source to perform different functions in different configurations</td>
</tr>
<tr>
<td>Shelling</td>
<td>Embed functional element in a device which performs a different function</td>
</tr>
</tbody>
</table>
4. PROBLEM STATEMENT

The project to which our transformation methodology was applied is the development of an easily storable wing for Tactical Mini UAV (TACMAV) shown in Figure 16. This lightweight battlefield surveillance MAV was developed by the Munitions Sector of the Air Force Research Laboratory (AFRL). It was designed for the United States Air Force (USAF) Battlefield Air Operations (BAO) kit. The TACMAV weighs less than 400 grams, and exemplifies the UAVs that are intended to replace current systems, which have similar capabilities but weigh two to three pounds. The TACMAV is powered by an electric motor and is hand-launched by the soldier. It uses a GPS/INS navigation system and is equipped with two TV cameras that provide real-time video transmitted to a PC-based ground station. The TACMAV is designed to complete hundreds of missions, but is inexpensive enough to be regarded as expendable. It is meant to provide the ability to be used by a covert land team. Some requirements of this MAV are to navigate, explore, and target, ultimately enhancing situational awareness, survivability, and mission success rates. The fact that this system will be deployed by foot soldiers in the field necessitates a lightweight, compact/collapsible structure that is easily setup.

The TACMAV’s airframe is constructed primarily of carbon fiber, resulting in a lightweight yet extremely robust structure. Carbon fiber also provides many characteristics that are beneficial to the production of MAV’s. These include the ability to form complex shapes at relatively low cost, while retaining superior strength and stiffness [19]. Other specifications of the TACMAV are shown in Table 2.

Currently the TACMAV’s wings curl underneath the fuselage for storage, but collapsibility is limited by the span on the wings since one wing impacts the underside of the other in this configuration. A separate tube is also required for storage in this configuration. A new design is proposed that will improve this collapsing mechanism by reducing storage volume and eliminating the need for a separate storage container.

The completed design should allow for the easy storage of the TACMAV without substantially degrading its performance with regard to lift and drag; these measures should not degrade by more than 20%. Additionally the wing should be lightweight, quickly deployable, and easy to use. The design is otherwise open-ended.

5. CONCEPT GENERATION AND SELECTION

The theory that has been developed provides insights into the nature of transformation; a methodology is required for their application to design problems. Concept generation tools were adapted to help map the principles and facilitators to working concepts. An extended mind mapping technique assists the designer in applying these ideas in practice (see fig. 17).

The traditional mind mapping approach is to write the problem to be solved in the center of a black sheet with a box around it. Ideas are generated to solve the central problem and are recorded in branches from the problem statement. As ideas are refined or spawn other ideas, these are connected to the parent idea on the map. The mind map helps organize the thoughts developed in a brainstorming session and helps the participants to create new ideas by branching off previous concepts [20].

This technique was adapted to aid in the generation of transformers. The basic process is the same, with the transformational design problem in the center of the map. The problem is stated in the form of the two (or more) objectives of the transformer, in this case Store / Fly. The designer then chooses design principles and facilitators that may be of use in the development of a transition between these two and places these as branches around the problem statement. Ideas are then generated that are specific to each principle and placed around them as branches. As with a traditional mind map, each new idea can spawn new branches of its own. Special attention should be paid to interactions between the ideas attached to different principles since transformers frequently arise from a combination of different principles.
A preliminary mind map for the storable MAV wing problem is shown in Figure 17. A collection of varying concepts were generated using this technique. Several of these are listed below:

- “Slap Bracelet” concept – Mimics the design of a “slap bracelet” toy. It was developed as using the principle Expand/Collapse and the facilitators Material Flexibility and Furcate.
- “Shape Memory Alloy” concept – Uses shape memory allow to expand the wing. For storage the wing can be folded to any convenient shape. It uses Material Flexibility to achieve Expand/Collapse.
- “Bird Wing” concept – Is a spring-loaded wing that unfolds like the wing of a bird. This concept uses the principles Fuse/Divide and Expand/Collapse and the facilitators Segmentation and Conform with Structural Interface.
- “Inflatable Wing” concept – Wing inflates for stiffness and can be deflated for storage. The impetus for this design was Expand/Collapse and it was facilitated by Material Flexibility.
- “Telescoping Wing” concept – Telescopes from completely or partially inside the fuselage of the plane. The design requires the principles Expand/Collapse and Fuse/Divide and the facilitators Segmentation and Nesting.

The “Slap Bracelet” concept, shown in Figure 18, was chosen based on its ease of use, speed of deployment, weight, feasibility, novelty, and the manner in which it embodies the transformation principles. This concept combines ideas generated by the principle expand/collapse and the facilitator furcate to produce a collapsible, bi-stable wing similar to the common “slap bracelet” toy. This design has two stable configurations: (1) fully extended in the shape of a wing and (2) coiled alongside the fuselage. Each state is stable from an energy standpoint. The current stable configuration depends on the boundary conditions imposed on the wing; if the wing is straightened it will remain rigidly straight until part of its cross-section is flattened. This is accomplished by constructing the wing in such a way that it has a natural curvature in both the transverse and longitudinal directions. These curvatures oppose one another (one up and one down). Because the wing can only curve in one direction at a time, the wing is always at a high-energy state in one dimension. The wing, in this view, is always stressed in either the longitudinal or transverse direction. The transition between states occurs when the wing’s cross section is flattened in one direction.
In the case of the wing concept, the transverse curvature is oriented downward at an angle to produce a thin airfoil shape, and the longitudinal curvature is oriented upward so that the wing rolls up next to the fuselage for storage.

6. PRELIMINARY EMBODIMENT

The concept was further developed as a bi-stable collapsible wing made primarily of carbon fiber composite. Carbon fiber was chosen due to its high flexibility, high strength-to-weight ratio, and resistance to fatigue. Three variants of this concept were developed: (1) a wing made entirely of carbon fiber, (2) a wing made of carbon fiber with metal structural inlays, and (3) a wing made of carbon fiber with a bi-stable metal inlay.

The wing composed entirely of carbon fiber is to be constructed using two composite pieces with identical perimeter values. One of these components is curved upward longitudinally and the other is curved transversely in the other direction. The two components are laminated together to create a bi-stable structure which functions substantially like a slap bracelet [10].

Figure 19 shows the wing before assembly. The top wing will be formed with a longitudinal curvature and the bottom wing will be formed with a transverse curvature, as shown in the figure. The two parts of the wing will then be laminated to form the completed part, shown in Figure 20.

The wing composed of carbon fiber with metal structural inlays is formed using two composite pieces with identical
perimeter values and one or more metal clock springs that span the length of the wing. The wing is laminated similarly to the all-composite design, with the clock springs contained between the two carbon fiber sheets. The carbon fiber sheets in this design both have transverse curvatures and the clock spring provides the force required to produce the longitudinal curvature and collapse the wing. Figure 21 shows the wing before assembly. The carbon fiber sheets are laminated together with the springs contained in between the two. The spring on the far side of the wing is shown in various stages of curling. Figure 22 shows the assembled wing.

The third wing concept that was explored consists of a bi-stable metal element laminated between two carbon fiber sheets. This design differs from the previous design because the metal element in this wing is itself a bifurcating element. The carbon fiber wing surface therefore does not need to provide the transverse curvature necessary to maintain the wing in an extended position; the metal insert provides the required stiffness in both directions. Otherwise this design is substantially the same as the clock spring concept.

The three wings share the same finished geometry, as defined by the nature of the slap bracelet. The required form is a straight wing with a circle segment cross section. All three wings were modeled in SolidWorks in order to verify geometry and perform analysis.

7. DETAILED EMBODIMENT

A technique is required to make design decisions regarding the design parameters associated with the transforming wing concept. The development of analytical and experimental models is a necessary step in the development process. Due to the complex nature of the problem posed by the application of the design theory, a combination of analytically derived models and experimentally derived computational metamodels is proposed.

Because the design transforms, multiple states must be considered for model derivation. In the case of the collapsible wing, and for the purposes of demonstration, one state of the wing is analyzed: the in-flight configuration. Ultimately models developed for each state of the transformer will guide the designer in decisions related to the interactions between states. They are necessary to provide an understanding of the whole system and will dictate the successful application of transformational design theory.

For the extended state of the collapsible wing design, a model describing the flight characteristics of the wing is essential. Due to the complexity in the equations of flight mechanics, an analytical model was not used. A computational fluid dynamics (CFD) program was used instead to produce a metamodel of the system by application of a series of virtual experiments. An approximate analytical model was derived to provide justifiable point selection for the experiments and to verify the results.

The goal of modeling the wing is to provide a mathematical equation(s) that can be used to optimize its flight parameters; the lift to drag ratio must be maximized with respect to a performance envelope defined by the original TACMAV performance. The variables of the design problem are summarized in Table 3. The metamodel must accurately captures the relationships between these variables and the lift and drag of the system.

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord (in)</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Camber (in)</td>
<td>0</td>
<td>0.45</td>
</tr>
<tr>
<td>Angle of Attack (deg)</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>Lift (lb)</td>
<td>1.83</td>
<td>N/A</td>
</tr>
<tr>
<td>Drag (lb)</td>
<td>N/A</td>
<td>0.188</td>
</tr>
</tbody>
</table>

7.1 Analytical Model

An analytical model is required in order to determine appropriate values for variables in our metamodel analysis, and to validate the CFD models. This model was developed using equations derived from Prandtl’s lifting line theory and thin airfoil theory. While the results are not exact, they provide us with an estimation of lift and drag that can be compared with those obtained from our CFD models to ensure realistic results. Figure 23 shows the wing cross-section and details the system variables.

To calculate the lift for a given wing, thin airfoil theory was used [21]. This theory assumes that the chord of an airfoil is much greater than its thickness, and restricts the design of an airfoil to low camber and angle of attack. These assumptions are particularly valid for our design as we are designing the wing as a thin sheet of carbon fiber. The theory further assumes that the velocity perpendicular to the surface of the wing is zero at all points. This allows for calculation of the local wind velocity at any point along the 2-dimensional wing profile. A further assumption is made that the wing is parabolic in shape rather than a circle segment. This simplifies the lift calculation and is acceptable because for wings with such low camber the difference in geometries is negligible. The lift for this wing design is calculated in Equation 1:

\[ L = C_L \rho \frac{V^2}{2} A = \pi \left( \frac{2h}{c} + \alpha \right) \rho V^2 A \] (1)

Drag was calculated using Prandtl’s lifting line theory [22, 23]. This theory is based on the presence of trailing vortices
created by the wing as it travels through the air. Equation 2 shows the analytical formula for drag. This equation calculates the total drag on the wing although effects in 3-dimensions, such as wing tip vortex drag and drag at the fuselage are estimated using efficiency factors for straight wing aircraft.

$$D = C_D \rho \frac{V^2}{2} A = \left[ C_{D,\text{min}} + \frac{AR \left( 2 \pi \left( \frac{2h}{c} + \alpha \right) \frac{1}{AR + 2} \right)}{\pi e} \right] \rho \frac{V^2}{2} A$$  \hspace{1cm} (2)

Although these calculations provide us with a general guide that can be used to determine the rough accuracy of our CFD models, they are also very limited, necessitating a more accurate representation of our system. The most glaring limitation of these models is the unbounded nature of the formulas. In practice, wing performance degrades as parameters approach certain upper and lower bounds. Our model fails to capture this phenomenon. For example, in our model lift increases as angle of attack increases, regardless of the initial angle. In practice the angle of attack increases the lift up to the wing’s stall angle, at which time the lift begins to decrease. If the angle continues to increase, the aircraft will lose lift very rapidly. Our model cannot account for these more complex interactions, and will increase unbounded in these situations. Similar incidents occur with the other parameters.

Another limitation of these models is in the assumptions made during calculation. Many simplifications were made to allow for calculation of our values in the absence of experimental data. Among these are a Fourier Series expansion used to estimate the value $C_L$, and the approximations for $C_{D,\text{min}}$ and $e$ in the calculation of $C_D$.

The model also makes assumptions about the type of flow and the shape of the wing. Due to the condition in thin airfoil theory that wind velocity perpendicular to the surface of the wing is zero, the model essentially assumes laminar flow. This does not occur in practice. The models also will only work for a straight-winged, thin airfoil design. Our wing design necessitates the same wing profile, but for other others such as swept wings or thicker airfoils, these estimates would be significantly less accurate. A CFD model was developed to more accurately represent the system.

7.2 Metamodel

In order to examine the mechanics of a transforming device and to produce an optimally functioning product, a mathematical model of the system is required: in this case a model of the flight performance of the wing. Due to the complexity of the system, a basic analytical model could not be constructed with the desired accuracy; a more comprehensive algorithm was required to account for complexities in the physical system such as turbulence and flow separation. Therefore a series of analyses were performed in Cosmos FloWorks, and a metamodel of the system was created [24, 29]. The metamodel was then optimized for the maximum value of lift-to-drag ratio. The constraints for modeling and optimization were determined based on the parameters of the original TACMAV and system variables were determined from the original dimensions and varied within reasonable limits.

Design variables for this problem were the chord, camber, and angle of attack of the wing. These variables were chosen because together they define the lift and drag characteristics of the predefined wing profile. Since our basic wing shape was determined as a requirement of our design concept, these variables control the flight characteristics of our final optimized design. By varying these values, we can obtain the best performance available in our design space.

The original TACMAV has an average chord of 4.2 inches. During the CFD analysis, the chord was varied between 3 and 6 inches in step sizes of 1.5 inches. These values were chosen because we wish to maintain the current lift and drag values for the TACMAV, and we don’t expect to drastically improve efficiency over the existing design. This is because the current TACMAV does not face the same profile constraints as our design and can therefore be constructed using more efficiently shaped airfoils. Since the length will not increase greatly in our design, we will likely require a similar chord to achieve equivalent values for lift and drag. The range of 3 to 6 inches will allow us to explore the design space based on changes in the efficiency of the wing within reasonable limits. The upper bound was set in order to keep the rolled storage volume of the wing to a size compatible with the current aircraft fuselage.

Camber was varied between 0.00 and 0.45 inches in step sizes of 0.15 inch. These values were chosen based on the theoretical behavior seen in the analytical model of our system since higher cambers will likely produce drag well above the acceptable bound.

For a typical airfoil, lift increases linearly for small angles of attack, but lift decreases and drag increases very quickly as the wing approaches the stall angle. The angle of attack for our tests varied between 0° and 14° in step sizes of 2°. These values were chosen because the theoretical behavior of the system at higher angles of attack will produce drag above the acceptable bound. Because the stall angle is an unknown quantity, the 2° step size is fine enough that we should capture the stall angle if it occurs below 14°.

The important constraints in our design optimization are wing length, velocity, minimum lift, and maximum drag. These values were determined based on the original design of the TACMAV [19], and were varied to allow for some performance degradation with our design. Generally speaking we wish to maintain or improve the performance of the plane.

The wing length was given an upper bound of 24 inches. The current wing length of the TACMAV is 20 inches, and since an increase in weight or size is undesirable, the length will not be allowed to increase more than 20% above the original value. Velocity was defined as 35 mph. This is the average cruising speed of the TACMAV under nominal conditions.

Minimum acceptable values for lift and drag derived from the literature [19]. Experimental values for the lift and drag on the original TACMAV were provided and used to calculate the lower and upper bounds for lift and drag, respectively. The experimental lift value was decreased by 20% and the experimental drag was increased by 20% to allow for some minor degradation in performance for the new wing design. The final calculated values were $L_{\text{min}} = 1.83 \text{ lb}$ and $D_{\text{max}} = 0.188 \text{ lb}$. These values are still well within the performance...
requirements of the TACMAV and were used in our optimization algorithm.

Since the behavior of the system was unknown beforehand, we employed an exhaustive sampling approach to ensure that the metamodel captured output variations over small changes in the design space. We discretized each design variable, as shown in Table 4, and conducted a CFD analysis for each combination of design variable values. A total of 96 tests were conducted. This exhaustive experimental design is more computationally expensive than approaches such as fractional factorial designs, but yields better initial coverage of the design space. A comprehensive metamodel was constructed from this data from which optimal values could be obtained.

The model used to represent the physical system is Cosmos FloWorks running on the Solidworks platform. This program integrates directly with the CAD program to provide CFD analysis for 2 and 3-dimensional parts. 96 wing shapes were developed in Solidworks, each based on the original concept. 2-dimensional analysis was run to decrease computation time and expedite results; because our design is a straight wing with a constant cross-section, this is a valid approach. The wing analyzed was a unit span so the lift for the entire wing can be determined by multiplying by the total wingspan.

<table>
<thead>
<tr>
<th>Table 4: CFD Input Variables</th>
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<tbody>
<tr>
<td>Chord (in)</td>
</tr>
<tr>
<td>3.0</td>
</tr>
<tr>
<td>4.5</td>
</tr>
<tr>
<td>6.0</td>
</tr>
<tr>
<td>-</td>
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</table>

Using the collection of data from the complete set of tests, metamodels were constructed relating lift and drag to the input variables, chord, camber, and angle of attack. The models were fit using nonlinear regression. The derived formulas are shown in Equations 3 and 4, for drag, $D$, and lift, $L$, respectively.

\[
D = 0.00837 - 0.00276 * c - 0.00150 * h \\
- 0.00111 * \alpha + 0.00167 * c * h + 0.000284 * c * \alpha \\
- 0.000545 * h * \alpha + 0.000219 * c^2 + 0.0153 * h^2 \\
+ 0.0000101 * \alpha^2
\]

\[
L = 0.00489 + 0.00261 * c + 0.000245 * h \\
+ 0.00161 * \alpha + 0.0292 * c * h + 0.00101 * c * \alpha \\
+ 0.00492 * h * \alpha - 0.000209 * c^2 - 0.185 * h^2 \\
- 0.0000556 * \alpha^2
\]

The model provided sufficiently accurate results based on the measured data to provide a mathematical model for the evaluation and optimization of the transforming wing. Table 5 shows the correlation coefficients for lift and drag, indicating close agreement with the experimental CFD data. The analytical model also verifies the metamodel, although much higher correlation was achieved at lower values of camber and angle of attack. This result is expected since the effects that the model fails to capture (turbulence, flow separation, etc.) occur increasingly with higher cambers and steeper angles of attack. The analytical model does however show strong correspondence to the general trends exhibited in the metamodel, so it would appear to support the results of the CFD even in light of its limitations.

<table>
<thead>
<tr>
<th>Table 5: Correlation Coefficients for Wing Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation Coefficient</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>92.57%</td>
</tr>
</tbody>
</table>

### 7.3 Optimization

Once a firm understanding of the system is gained from the model, optimization is required to produce the most efficiently performing design possible [28, 30]. This optimization guides the designer in making decisions that affect the different states of the wing. In this case the aerodynamic qualities of the wing are inspected and the efficiency of the wing is maximized with regard to the adjustable variables.

The goal of optimization was to maximize the lift to drag ratio while maintaining a minimum set of flight characteristics. The function chosen for optimization was $L/D$ as defined by the metamodel. The optimization is formulated in Equations 5 and 6.

\[
\text{Maximize:}
L/D = 0.584 - 0.945 * c - 0.164 * h \\
- 1.45 * \alpha + 17.4 * c * h + 3.57 * c * \alpha \\
- 9.04 * h * \alpha - 0.955 * c^2 - 12.0 * h^2 \\
- 0.550 * \alpha^2
\]

Subject to:

\[
3" \leq c \leq 6"
\]

\[
0" \leq h \leq 0.45"
\]

\[
0^\circ \leq \alpha \leq 14^\circ
\]

\[
\text{Lift} \geq 1.83 \text{ lb}
\]

\[
\text{Drag} \leq 0.188 \text{ lb}
\]

The optimization problem was solved using the Generalized Reduced Gradient (GRG) solver in Excel [25] and the solutions were validated using a Sequential Quadratic Programming (SQP) algorithm in MATLAB [26]. Our solution gives a lift to drag ratio of 13.4. The optimal values for chord, camber, and angle of attack are shown in Table 6.

<table>
<thead>
<tr>
<th>Table 6: Optimized Wing Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>6 in</td>
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REFERENCES


